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Number of ATM Machines Required To Bring Down Waiting Time of Customers Using Queueing Model

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ABSTRACT

Automated Teller Machine (ATM) is one of the finest electronic banking systems used in the banking industry. In various commercial banks, operating a single ATM machine results in delays in customer service due to high demand. Customers spending time duration to access services at banks with a single ATM facility is becoming a significant problem and a justification for the rationale behind such waiting times. Keeping customers in a queue for an extended period of time might have a negative impact on business growth. Motivation behind the present study lies in the importance of customer satisfaction in the banking sector and the potential benefits of implementing efficient queuing systems. This paper examined the service effectiveness of the banks using single server queueing models. To ascertain how to reduce the waiting time, estimated service time, inter-arrival time, enhance their customer service level and traffic intensity of customers in a queue of banks the data was gathered from secondary sources over a ten-day period having a single ATM machine. Using M/M/1 queueing model in which a first come first served policy was used, for customer arrival and service times were exponentially dispersed. These findings demonstrated that a greater number of ATM's will result in break down customer wait times, less ATM overuse, higher customer satisfaction to increase and reduce their all performance.

KEYWORDS: Queueing theory; Arrivals; Waiting time; Service time; M/M/1; ATM's; Customer satisfaction.

MSC (2020): 60K25; 60K30; 90B22; 68M20.

1. INTRODUCTION

A queueing system includes customers coming, waiting for service if it is not provided immediately and then departing the system once they have been serviced. Queueing is the process of joining a line or waiting. When customer's demand service, they have to wait because there is more customer available than there are servers or the facility has issues or takes longer than the allocated time to serve a customer. The main issue though, is that banks are unable to quickly adapt their service offerings to meet the needs of their customer's. The issue is that customer's wait times for services are becoming a significant cause for concern because there are insufficient facilities and explanations for why they must wait. A shortage of personnel may be the cause of lengthy wait times before service, which costs businesses and properties a lot of clients. One of the most important service facilities in the banking industry is the Automated Teller Machines (ATM). Many banks introduced ATM in an effort to reduce waiting lines, however the frequent failure of these computerization and networking arrangements has prevented the much-needed outcome. As a result, many banks continue to have lengthy lines and customers are still waiting for assistance. It has been observed that the Indian economic system has grown rapidly over the last ten years. In order to draw customer, banking companies in the financial sector frequently introduce new concepts.

In 1987, $HSBC$ Bank installed the country's first ATM at its Mumbai branch. The purpose of the ATM's introduction was to assist customers in times of need when they needed to make cash deposits or withdrawals outside of regular business hours. In order to save customers time while waiting inside the bank, the next advancement in the ATM industry is the implementation of an inquiry system to determine the account balance and statement. Customers can save time by having access to important services like money transfers, bill payment and other financial services. Choodambigai (2011) analysis on related research work seeks to determine how many customers uses ATM's. As a result, the majority of customers uses ATM's to withdraw money and very few use them to confirm their account information. Only public sector banks are the focus of this investigation. The study also attempts to investigate how customers perceive ATM and credit card services. This study compares the use of ATM's and other services in the public and private sectors of banks. The analysis only included two banks: SBI and $ICICI$. According to the results, private sector banks frequently run out of currency and public sector banks dispense obsolete currencies. Consequently, each bank has its own constraints.

Tuli *et.al* (2012) emphasizes the paper is solely on the customers who uses the ATM, without taking into account the opinions of bank employees. Premalatha and Sundaram (2012) had focused on Customer satisfaction with ATM's is the main focus of this article and it has been observed that age, safety, gender, and tangibility all have a substantial impact. However, there is no meaningful correlation between job and contentment. Customers therefore anticipate safety, certainty and convenience while using ATM's and the banker is advised to give users accurate and safe information to boost dependability. Chattopadhyay and Sarelelimath (2012) had concentrated on customer preference towards use of ATM services in Pune city. They evaluated problems and awareness of ATM services offered by banks in Pune city. Both primary as well as secondary data was used under the study. Bishnoi (2013) demonstrates the rise in ATM client usage, it does not identify the causes or contributing factors. Hota (2013) in his article, "Growth of ATM Industry in India" put light on White ATM's and Brown ATM's in India and their growth in the country. The article concluded that the ATM's are growing in India and slowly banks are shifting from multi-vendor to multi-channel integration. Odusina (2014) attempted to investigate ATM usage and customer satisfaction in Nigeria. Even if there are more ATM's installed in Nigeria, it came to light that the needs of the customers were not being adequately satisfied. In the study, three banks-First Bank, Guaranty Trust Bank and Skye Bank in Ogun State, Metropolis, Nigeria, were compared. Utilizing questionnaires, data was gathered and analysed using the chi-square statistical method.

Renuka and Paulraj (2014) in their research work stated that there is a strong and positive correlation between customer's satisfaction and ATM usage. A study also done on the deposite system, customer satisfaction at the moment of withdrawal and customer's satisfaction over a 24-hour period. Sisat and Barbuddhe (2014) utilizing the ATM, there are three different kinds of risks: physical damage, logical attacks and currency fraud. Jegede (2014) in Nigeria, ATM's has risen in popularity as banks aim to improve service delivery and reach out to both urban and rural customers. With Nigeria's enormous geographical expanse and various degrees of banking infrastructure, ATM's plays an important role in bridging service gaps, especially for those in underserved or distant locations. In order to perform a purposive study Mahmud *et al.* (2015) choose eight ATM's locations from four banks in Dhaka, Bangladesh. Samples were used for the survey and in-depth interviews. The study came to the conclusion that in order to gain from ATM's, the problems and difficulties must be appropriately addressed. Baliyan and Mittal (2015) solely examined white label ATM's in India. Their work discusses the idea that led to the creation of these ATM's in India as well as the present trend. The writers have also covered the drawbacks and restrictions associated with these particular kinds of ATM's. Meena (2015) conducted research on the various kinds of ATM's, the benefits and drawbacks of utilizing them. The author examined the advantages, drawbacks and issues encountered when utilizing these devices through a survey study. In order to maintain calm and order in customer service, queuing strategies must be used in this circumstance.

Belay and Kindie (2017) derived results from a paper that studied the effect of ATM service quality on customer satisfaction in Ethiopian commercial Banks indicated that with the exception of assurance, factors such as tangibility, reliability, responsiveness and empathy are positive. The significant effect on customer satisfaction and the customers were mostly satisfied with the responsiveness dimensions of *ATM*'s service quality. Maqsood and Hina (2018) examined service delays and customer service ratings at family dining establishments using questionnaires. A queueing or waiting-line issue typically occurs when a collection of clearly defined service facilities is unable to fully meet the demand for client service. In other words, there is a greater need for services than there are facilities to meet them. This could be attributed to a lack of services or restrictions on the scope of services that can be offered. In order to determine the relationship between waiting time and customer satisfaction, Mohammad (2018) used queueing theory on randomly chosen patrons of a business. He found that there is a significant positive correlation between perceived waiting time and service quality on waiting time satisfaction. Ali and Bisht (2018) assess the incongruity in satisfaction levels between customers of private and public sector banks, taking into account multiple factors of satisfaction. However, there was a greater incongruity between their perception and expectation in private sector banks. The customers of public sector banks expressed dissatisfaction with the irregular services of *ATM*'s in rural and distant regions, which were consistently overcrowded. The study recommended that clients expressed higher satisfaction with private sector banks compared to public sector banks. Tadesse (2018) focused on customer satisfaction with *ATM* banking is from an evaluation of the *ATM* banking used throughout the experience whether the *ATM* banking performed relatively well or poorly against expectations.

In a study, Ram (2018) investigated the various types of automated teller machines in use in India. An analysis of *ATM* uses in the Indian banking sector, it was determined that the public awareness of cashless transactions. *ATM*'s usage is both fast increasing and it is the duty of both government agencies and the general population to guide the country toward a cashless economy.

In order to measure the effects before and after implementing service recovery, Chung-Te Ting (2019) explored the significance of pre-processing services in the context of potential restaurant service crises and developed a restaurant service recovery model for willingness to pay using the contingent valuation method. Osahenvemwen and Iroh (2021) stated the *ATM*'s reduces the need for customers to travel great distances to bank offices by providing instant, on-demand access to cash and other banking services, hence increasing financial inclusion. Kulkarni (2022) has highlighted in her article that the Reserve Bank of India has recommended to provide interoperability in cardless cash withdrawal transactions at all banks and *ATM*'s utilizing the UPI capability, which would prevent fraud such as card skimming, card cloning etc., as highlighted by Kulkarni in her essay.

Pal and Kumar (2023) analyses that public sector banks exhibit greater uniformity and decrease variability in the installation of *ATM* machines in comparison to private sector banks. When analysing the compound annual growth rate by sector, private sector banks demonstrate more favourable outcomes than public sector banks. Kene and Mulkalwar (2023) stated an upsurge in crimes involving *ATM*'s; there is a need to improve *ATM* security. In order to comprehend security concerns and future research prospects, they also explore the different security techniques that are accessible for banks and ATM's. Mehta (2024) includes the ATM safety protocol, which is built on and enhanced by biometric identification techniques like facial recognition. By preventing unauthorized access, one of the basic biometrics processes ensures that the consumer must be present in order to use the ATM. Encho *et al*. (2025) examined the ideal number of ATM's required to cut down on customer's wait times in line.

The study will address the following research quarries in an effort to address the aforementioned issue:

a) In the banking sector, how can queueing theory be used to improve customer satisfaction and wait times?
b) What are the main determinants of bank line customer satisfaction?
c) What are the main elements affecting bank line patron satisfaction?

Study aims to investigate how queueing theory can be applied as a model to increase service efficiency and decrease wait times in order to improve customer satisfaction in the banking sector. This goal will be accomplished by considering the following objectives:

a) Utilise queueing theory in a bank visibility study to maximize customer wait times.
b) Analyse how queueing theory affects customer satisfaction levels.
c) Identify suggestions for improving bank queueing systems.

2. QUEUES STRUCTURE

Queueing model structure is divided into input and output queueing system that includes queue that must follow a queueing rule and service mechanism as shown in Figure 1 (Hiller and Libernan, 2005):

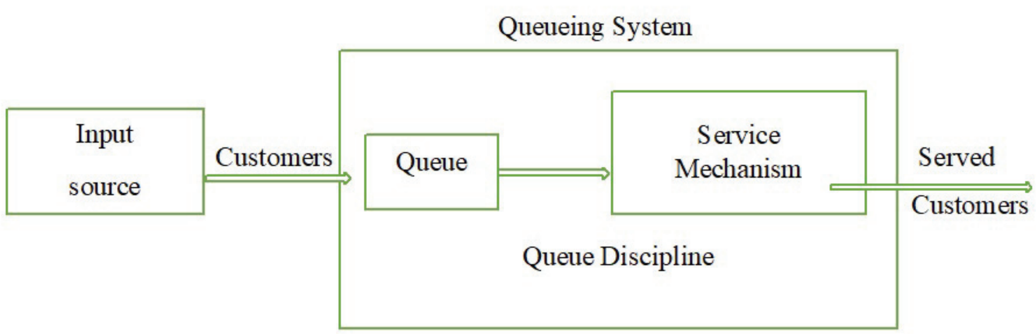


Figure 1: Structure of queueing model

2.1. Input process

In queueing theory, the pattern of customers or entities entering the system is referred to as the input process. It explains how customers get into the line, whether it follows a set pattern or is random and how quickly they join the system.

2.2. Output process

The output process, as defined by queueing theory, is the pattern by which customers or other entities leave the system. It explains how customers exit the line after being served, whether they are present in groups or individually, how quickly they are served and move through the system.

Place	Input Process	Output Process
Canara bank	Arrival of customers at the ATM	Discussion and Release

Table 1: Input and Output Process

3. METHODOLOGY

A successful fair queueing model was developed, which can reduce congestion by allocating resources on the network between contending users. The proposed model prioritizes real time in order to guarantee dependable performance. The study will evaluate the performance of the service mechanism and ascertain whether customers are satisfied with the banks services based on the arrival and service pattern of the system. $(M/M/1)(\infty/FCFS)$. will be used as the queueing system to analyse the data. This is a situation where there is only one person or a team working as a unit providing the service at the facility. Also, the service is completed in one stage.

3.1. Mathematical notation

m : Number of customers in the system (waiting time +service facility) at time t

$\lambda(t)$: Mean arrival rate (number of arrivals per unit of time)

$\mu(t)$: Mean service rate per busy serve (number of customers served per unit of time)

$P_m(t)$: Steady state probability of exactly n customers in the system

$L_q(t)$: Expected number of customers waiting in the queue

$L_s(t)$: Expected number of customers in the system (waiting + being served)

$W_q(t)$: Expected waiting time per customers in the queue (expected time a customer keeps waiting in queue)

$W_s(t)$: Expected time a customer spends in the system (in waiting +being served)

ρ : System utilization

$P_0(t)$: The probability that there are zero customers in the system

$P_w(t)$: The probability that a customer has to wait

To getting the steady state equations, we assumed that the probability that there will be m units $(m > 0)$ in the system at time $(t + \Delta t)$ may be expressed as the sum of 3 independent compound probabilities by using the fundamental properties of probability, Poisson arrivals and exponential service as in table 2.

Time t number of units	Arrival	Service	Time $(t+\Delta t)$
M	0	0	M
$m-1$	1	0	M
$m+1$	0	1	M

Table 2: Poisson arrivals and exponential service time

From above 3 independent compound probabilities can be added as to obtain the probability of m units in the system at time $(t+\ \Delta t)$.

$$P_m(t\ +\ \Delta t) =\ P_m\{1-(\lambda+\mu)\Delta t\} + P_{m-1}(t)\lambda\Delta t + P_{m+1}(t)\mu\Delta t + O(\Delta t)$$

$$\frac{P_m(t\ +\ \Delta t) - P_m(t)}{\Delta t} = -(\lambda+\mu)P_m(t) + \lambda P_{m-1}(t) + \mu P_{m+1}(t) + \frac{O(\Delta t)}{\Delta t}$$

$$\lim_{\Delta t\to 0}\frac{P_m(t\ +\ \Delta t) - P_m(t)}{\Delta t}$$

$$= \lim_{\Delta t}\left\{-(\lambda+\mu)P_m(t) + \lambda P_{m-1}(t) + \mu P_{m+1}(t) + \frac{O(\Delta t)}{\Delta t}\right\}$$

$$\frac{dP_m(t)}{dt} - -(\lambda+\mu)P_m(t) + \lambda P_{m-1}(t) + \mu P_{m+1}(t)$$

where, $\lim_{\Delta t}\frac{O(\Delta t)}{\Delta t} = 0$, for $m > 0$.

In a steady state,
$P_m(t)$ approaches to 0, $P_m(t) = P_m$,

Therefore, $\lambda P_{m-1}(t) + \mu P_{m+1}(t) = (\lambda+\mu)P_m(t)$. (1)

Also, the probability that there will be m units in the system at a time $(t+\Delta t)$ will be the sum of the following independent probabilities.

i. Probability that there is no unit in the system at a time t and no arrival in time $\Delta t = P_0(t)(1-\lambda\Delta t)$.
ii. Probability that there is one unit in the system at a time t, one unit serviced in Δt in no arrival in $\Delta t = P_1(t)\mu\Delta t(1-\lambda\Delta t) = P_1(t)\mu\Delta t + 0(\Delta t)$.

On summing these two probabilities, we get

$$P_0(t)(1-\lambda\Delta t) = P_0(t)(1-\lambda\Delta t)P_1(t)\Delta t + 0(\Delta t)$$

$$\frac{P_0(t\ +\ \Delta t) - P_0(t)}{\Delta t} = -\lambda P_0(t) + \mu P_1(t), \text{for } m = 0$$

$$\frac{dP_m(t)}{dt} = -\lambda P_0(t) + \mu P_1(t)$$

Under steady state, we have

$$\lambda P_0(t) + \mu P_1(t) = 0 \qquad (2)$$

Therefore, equations (1) and (2) are called steady state difference equations for model $(M/M/1)(\infty/FCFS)$.
Now, from equation (2), we have

$P_1(t) = \frac{\lambda}{\mu} P_0(t)$ and from equation (1), we get

$$P_2(t) = \frac{\lambda}{\mu} P_1(t) = \frac{\lambda}{\mu} * \frac{\lambda}{\mu} P_0(t) = \left(\frac{\lambda}{\mu}\right)^2 P_0(t).$$

In generally, $P_n(t) = \left(\frac{\lambda}{\mu}\right)^m P_0(t)$.

Therefore,

$$\sum_{m=0}^{\infty} P_m(t) = 1 \Rightarrow P_0 + \frac{\lambda}{\mu} P_0 + \left(\frac{\lambda}{\mu}\right)^2 P_0 + \cdots = 1,$$

$$P_0 \left[1 + \frac{\lambda}{\mu} + \left(\frac{\lambda}{\mu}\right)^2 + \cdots.\right] = 1.$$

Implies, $P_0 \left(\frac{1}{1-\frac{\lambda}{\mu}}\right) = 1.$ (3)

Since, it is a geometric series with first term 1 and common ratio $r = \left(\frac{\lambda}{\mu}\right)^2$.

Also, $\lambda < \mu$, therefore the sum of infinite GP is valid, $P_0 = 1 - \frac{\lambda}{\mu} = 1 - \rho$.

$$\text{As, } P_0(t) = \left(\frac{\lambda}{\mu}\right)^n P_0 = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right),$$

$$P_m(t) = \rho^n (1 - \rho);$$

a) The expected number of units in the system

$$L_s(t) = \sum_{n=1}^{\infty} nP_n = \sum_{n=1}^{\infty} n\left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right) = \left(1 - \frac{\lambda}{\mu}\right)\frac{\lambda}{\mu} \sum_{n=1}^{\infty} n\left(\frac{\lambda}{\mu}\right)^{n-1}$$

$$L_s(t) = \left(1 - \frac{\lambda}{\mu}\right)\frac{\lambda}{\mu}\left[1 + 2\left(\frac{\lambda}{\mu}\right) + 3\left(\frac{\lambda}{\mu}\right)^2 + \cdots\right] = \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)\left(1 - \frac{\lambda}{\mu}\right)^2$$

$$L_s(t) = \frac{\frac{\lambda}{\mu}}{1 - \frac{\lambda}{\mu}} = \frac{\rho}{1 - \rho}, \rho = \frac{\lambda}{\mu} < 1$$

$$L_s(t) = \frac{\rho}{1-\rho}.$$

b) Expected queue length $L_q(t) = L_s - \frac{\lambda}{\mu} = \frac{\lambda^2}{\mu(\mu-\lambda)}$

c) Expected waiting line in the queue $W_q(t) = \frac{\lambda}{\mu(\mu-\lambda)}$

d) Expected waiting line in the system $W_s(t) = W_q + \frac{1}{\mu} = \frac{\lambda}{\mu(\mu-\lambda)} + \frac{1}{\mu} = \frac{1}{\mu-\lambda}$

e) Expected waiting time of a customers who has to wait $(w/W > 0) = \frac{1}{\mu-\lambda} = \frac{1}{\mu(1-\rho)}$

Expected length of a non-empty $(l/L > 0) = \frac{\mu}{\mu-\lambda} = \frac{1}{(1-\rho)}$

The probability of queue size $\geq N = \rho^N = \int_1^{\infty} \rho(\mu-\lambda)e^{(\mu-\lambda)w}dw$

Probability of waiting time in the queue ≥ 1

f) Traffic intensity $\rho = \frac{\lambda}{\mu}$

g) Inter-relationship between $L_s(t), L_q(t), W_s(t), W_q(t)$, we obtain

$L_s(t) = \frac{\frac{\lambda}{\mu}}{1-\frac{\lambda}{\mu}}$, $W_s(t) = \frac{1}{\mu-\lambda}$, $L_s = \lambda W$.

In the similar way, $L_q(t) - \lambda W_q(t)$ holds.

$$W_q(t) = \frac{\lambda}{\mu(\mu-\lambda)}$$

$$W_s(t) = \frac{1}{\mu-\lambda}$$

$$W_s(t) = \frac{1}{\mu} = \frac{1}{\mu-\lambda} - \frac{1}{\mu} = \frac{\mu-(\mu-\lambda)}{\mu(\mu-\lambda)}.$$

Thus, $W_s(t) = W_q(t) - \frac{1}{\mu}$

On both sides multiply by λ, one can get

$$\lambda W_q(t) = \lambda\left(W_s(t) - \frac{1}{\mu}\right)$$

$$L_q(t) = \lambda W_s(t) - \frac{\lambda}{\mu} = L_s(t) - \frac{\lambda}{\mu}$$

Therefore, $L_q(t) = L_s(t) - \frac{\lambda}{\mu}$.

If the value of λ does not remain fixed, it means some arrivals interested in joining the queue may not join due to long queue and since μ also depends on equal length, the service rate may be affected. In this observation the $P_0(t) = e^{-\rho}$, where $\rho = \frac{\lambda}{\mu}$

$$P_n(t) = \frac{\rho^n}{n!}e^{-\rho}$$

$$L_s(t) = \rho$$

$W_s(t) = \frac{L_s(t)}{\lambda}$.

3.2. Arrival poisson process

Steps of the process as follows:

1) The Process $\{X(t), t > 0\}$ has independent increment, $t_0 < t_1 < t_2 < t_3 \ldots$ increments as $X(t_1) - X(t_0), X(t_2) - X(t_1) \ldots$. These increments are independent i.e., the distribution of $X(t-h) - X(t) = P(\hbar)$ depends only on $\hbar$.
2) Consider a very small interval of time $(t, t+\hbar)$
$P\left[X(t, t+\hbar) - X(\hbar) = \frac{1}{X(t)}\right] = \lambda\hbar + O(\hbar)$, where $\lim_{\hbar \to 0} \frac{O(\hbar)}{\hbar} = 0$.
3) $P\left[X(t+\hbar) = \frac{0}{X(t)} = 0\right] = 1 - \lambda\hbar + O(\hbar)$.

Since, the value of $O(\hbar)$ is very much small, therefore $O(\hbar) = 0$.

Now, to achieve the distribution of the process, define $P_k(t) = P[X(t) = k]$ and consider the event that $X(t+\hbar) = k$

a) $X(t) = k$ and $X(t+\hbar) - X(t) = 0$
b) $X(t) = k - 1$ and $X(t+\hbar) - X(t) = 1$
c) $X(t) = k - i$ and $X(t+\hbar) - X(t) = i$, for $i \geq 2$.

Implies that,

$$P_k(t+\hbar) - P_k(\hbar)P_0(\hbar) + P_{k+1}(t)P_0(\hbar) + \sum_{i=2}^{k} P_{k-i}(t)P_i(\hbar)$$

$$\text{i.e.,}\ P_k(t)[1 - \lambda\hbar + O(\hbar)] + P_{k+1}(t)[\lambda\hbar + O(\hbar)] + \sum_{i=2}^{k} P_{k-i}(t)O(\hbar),$$

where O$(\hbar)$ multiply by any quantity remains an $O(\hbar)$

$$P_k(t+\hbar) = P_k(t) - \lambda\hbar P_k(t) + O(\hbar) + \lambda\hbar P_{k-1}(\hbar) + O(\hbar). \quad (4)$$

On rearrange these and dividing by $\hbar$ along with as $\hbar \to 0$, we get

$$P'_k(t) = -\lambda P_k(t) + \lambda P_{k-1}(t), k = 1,2, \ldots \quad (5)$$

On assuming the solution from equation (5), whenever k is zero,

$P'_0(t) = -\lambda P_0(t)$,

$$\text{i.e.,}\ -\lambda = \frac{P'_0(t)}{P_0(t)}.$$

Imply that, $\log P_0(t) = -\lambda t + c$.

As, $\int \frac{dt}{P_0(t)} = -\int \lambda dt$

$P_0(t) = e^{-\lambda t + c}$

Recall, $P_0(0) = P\{X(0) = 0\} = 1, 1 = e^c$ that is $c = 0$ (6)

$$P_0(t) = e^{-\lambda t},$$

when $k = 1$, one can obtain

$$P_1'(t) = -\lambda P_1(t) + \lambda P_0(t)$$
$$-\lambda P_1(t) + \lambda e^{-\lambda t}$$

$P_1(t) - \lambda t e^{-\lambda t}$ (7)

In general,

$P_k'(t) = \frac{\lambda t^k e^{-\lambda t}}{k!}, k = 0,1,2,3 \ldots$ (8)

Therefore, it shows that the arrival process has Poisson distribution with
$E[t] = \lambda t$ and $V[t] = \lambda t$.

Now, assume that X be a customer's arrival process then X has a Poisson distribution with parameter (λ).

$f(x) = \frac{e^{-\lambda}}{x!} \lambda^x, x = 0,1,2 \ldots$
$E[x] = \lambda(t)$ and $V[x] = \lambda(t)$.

3.3. Service process
Consider that μ be the service rate, then we have
$\mathrm{P}(T > t) = e^{-\mu t}$,
$\mathrm{P}(T < t) = 1 - e^{-\mu t}$
To obtain the probability density function (pdf),
$f(t) = \frac{d}{dt}[F(t)] = \mu e^{-\mu t}$, for $t > 0$.

The service process has exponential distribution with parameter μ:
$E[t] = \frac{1}{\mu}$ and $V[t] = \frac{1}{\mu^2}$.

Conditions Determining the effectiveness of the system: λ is the arrival rate, $\frac{1}{\lambda}$ is the inter-arrival rate and $\frac{1}{\mu}$ is the service rate. We have the following conditions:

[1] If $\frac{1}{\lambda} = \frac{1}{\mu}$, there is no queue and the server will not rest.

[2] If $\frac{1}{\lambda} > \frac{1}{\mu}$, implies there will be no queue and the server will sometime relex.

[3] If $\frac{1}{\lambda} < \frac{1}{\mu}$ implies there will be queue.

4. DATA PRESENTATION AND ANALYSIS

This section considers data presentation, analysis and discussions of the result. The arrival and service process of the customers arrive to the bank were observed and recorded in Tables 1 and 2 respectively. In Table 1, X indicate the arrival per minute and f be the number of arrivals, while in Table 2, y indicate the service process and F be the number of patients served within the intervals $\{0,-2\},\{2-4\},\ldots\ldots,\{48-50\}$ minutes. In appendix, we also describe the respective tables values of our secondary data.

4.1. Arrival process

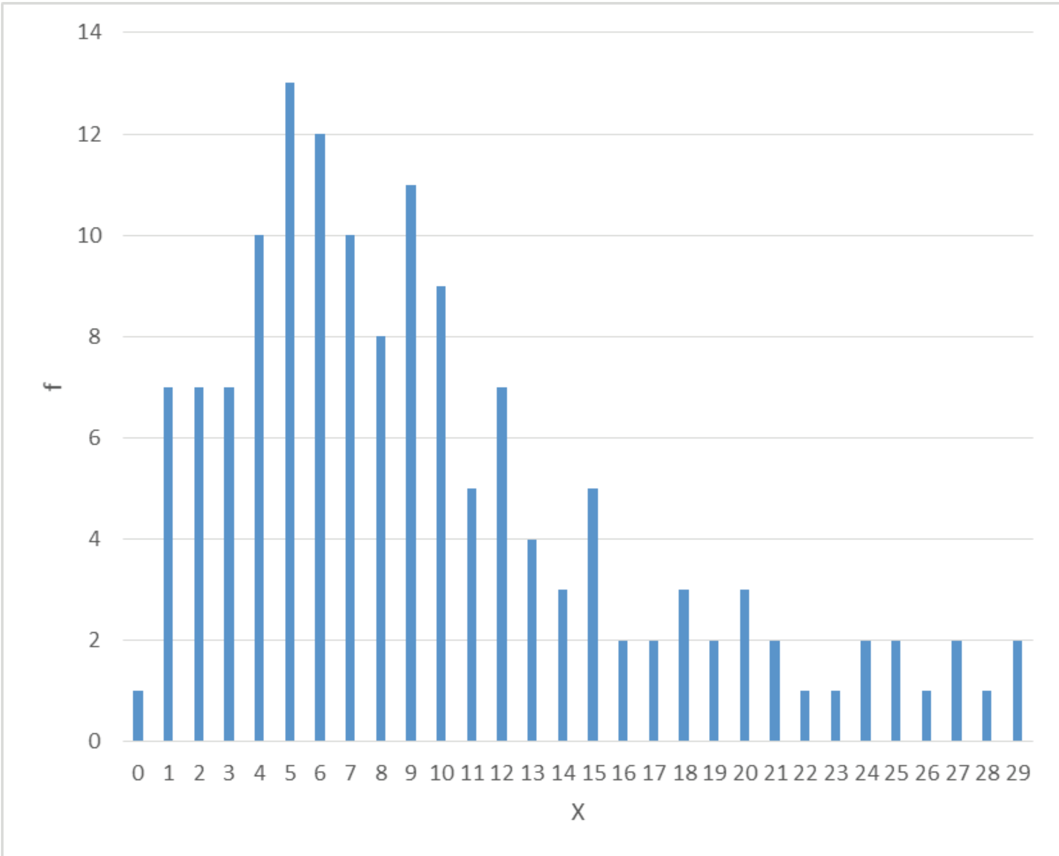


Figure 2: Bar chart represents the arrivals process (discrete data)

Figure 2. shows 1 customer arrive at 1 minute, 7 customers at 2 minutes and the highest number of arrivals are at 6 minutes.

4.2. Service process

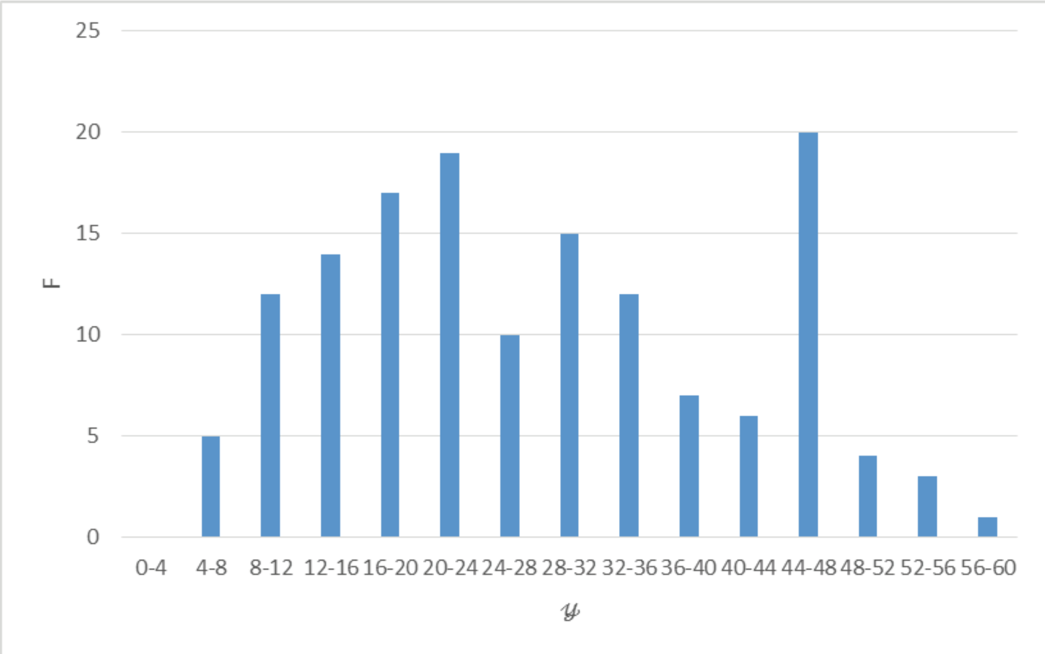


Figure 3: A histogram represents the service process (continuous data)

From above figure 3, we observe that at $0-4$ minutes no customer is served and the highest number of customers are served at $44-48$ minutes.

5. DATA ANALYSIS

Consider the arrival process denoted as X, then X have a Poisson distribution with parameter (λ), such as

$$f(x) = \begin{Bmatrix} \frac{e^{-\lambda}}{x!}\lambda^x, x = 0,1,2, \dots, \lambda > 0 \\ 0, \qquad \text{otherwise} \end{Bmatrix},$$

with $E[x] = \lambda$, where λ is the mean arrival rate and $V[x] = \lambda$

$$E[x] = \lambda = \frac{\sum fx}{\sum f} = \frac{1419}{145} = 9.7062.$$

To get the daily average, divide λ by 6, we obtain $\frac{9.78}{6} = 1.63$. Which is approximately two arrivals per minute.

Now, assume $\frac{1}{\lambda}$ be the inter arrival rate, then $\frac{1}{\lambda} = \frac{1}{1.63} = 0.61$.

$\frac{1}{\lambda}$ is the time between successive arrivals of the costumers. Also, since y be the service process, then y obey exponential distribution with parameter μ,

$$F(y) = \begin{Bmatrix} \mu e^{-\mu y}, \qquad 0 < y < \infty, \mu > 0 \\ 0, \qquad \text{otherwise} \end{Bmatrix},$$

with $E[y] = \frac{1}{\mu}$ and $V[y] = \frac{1}{\mu^2}$.

$\text{i.e.}, E[y] = \frac{1}{\mu} = \frac{\sum Fy}{\sum F} = \frac{4046}{145} = 27.90.$

To get daily average divide this by 6, we get $\frac{27.90}{6} = 4.65$, which is the expected service time.

We observe that, $\frac{1}{\lambda} = 0.61 < \frac{1}{\mu} = 3.49$.

Therefore, we can imply that for the queue exist, should be eliminated.

5.1. Optimum number of ATM needed to eliminate the queue

Consider C denotes the number of service point ATM's needed to reduce the queue. Then an additional $(C - 1)$ ATM's, will be needed to reduce the queue.

As, $C = \frac{\lambda}{\mu}$, $\lambda = 1.63$ and $\mu = \frac{1}{4.65} = 0.2150$.

Thus, $C = \frac{1.63}{0.21} = 7.761 \approx 7$.

Therefore, $C - 1 = 6$, 6 ATM's to be added for an effective service delivery

5.2. Distribution of the developed queue

As queue already exist, we needed to calculate distribution of the developed queue. Consider W be the waiting time of customers in the queue, then W have a Poisson distribution with parameter (λ) and

$$f(w) = \begin{Bmatrix} \frac{e^{-\lambda}}{w!}\lambda^{w}, w = 0,1,2\ldots \\ 0, \quad \text{otherewise} \end{Bmatrix},$$

along with $E[w] = \lambda$, where λ is the mean waiting time and $V[w] = \lambda$.

As, $P(W = w) = \frac{e^{-1.63}}{w!}(1.63)^{w}, w = 0,1,2, \ldots$

Therefore, we deduce that the probability of waiting at $w = 0,1,2, \ldots$

Table for probability of waiting time for W as follows:

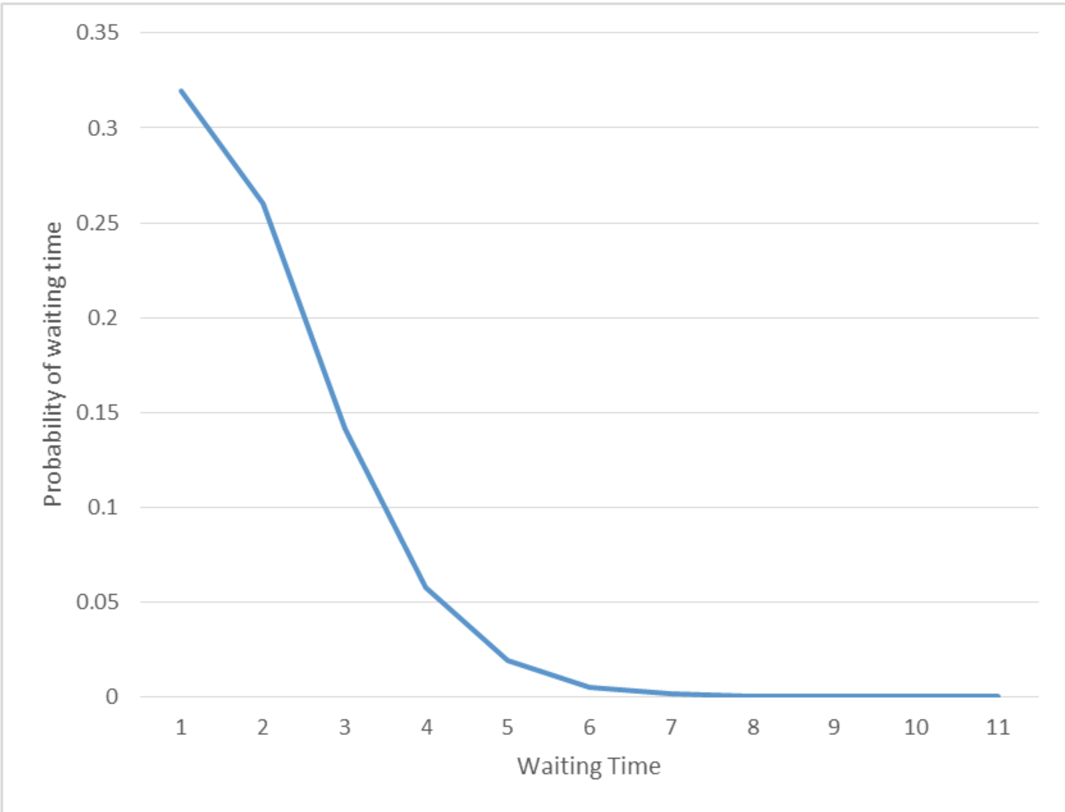
W	$P(W = w)$
1	0.3193652
2	0.260233
3	0.1413988
4	0.0576200
5	0.0187841
6	0.0051030
7	0.0011882
8	0.0002421

9	0.0000438
10	0.0000071
11	1.073231E-7

Table 3: Probability of waiting time

As form Table 3, we determined probability values are decreasing exponentially with an increasing the waiting time. Therefore, we imply that the inter arrival time between customers has an exponential distribution.

Now, a graphically view of Table 3 can also be represented as follows:



Figures 4: Probability of waiting time against waiting time

6. DISCUSSION

According to the calculation and observation of our work, we conclude that the inter arrival rate $\frac{1}{\lambda} = 0.61$ is less than the expected service time $\frac{1}{\mu} = 4.65$, indicating the existence of a queue. Based on the customers arrival patterns, the traffic intensity C or the number of ATM's required for optimal service is determined to be 6. This means that one more ATM is needed for effective service delivery. It was discovered that the interval between customer arrivals had an exponential distribution.

CONCLUSION

Queueing theory is a statistical method that can be applied to solve specific problems and is also essential in the many workplaces of research as well. The queueing characteristic at Canara bank was observed to get the ATM needed to reduce the waiting time of customers. By the service capability level of the Bank's ATM's is raised to 7, overuse of the machines may be decreased. Time spent on line declines as service levels rise. The study can be applied to similar organization's queueing situations. One ATM cannot handle the volume of customers that visit the ATM service point, according to the present study, so the management of the Canara bank needs 6 more ATM to boost the service rate.

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APPENDIX

Table 1: Arrival process

S/N	Arrival per minute (X)	Number of arrival (f)	(Xf)
1	0	1	0
2	1	7	7
3	2	7	14
4	3	7	21
5	4	10	40
6	5	13	65
7	6	12	72
8	7	10	70
9	8	8	64
10	9	11	99
11	10	9	90
12	11	5	55
13	12	7	84
14	13	4	52
15	14	3	42
16	15	5	75
17	16	2	32
18	17	2	34
19	18	3	54
20	19	2	38
21	20	3	60
22	21	2	42
23	22	1	22
24	23	1	23
25	24	2	48
26	25	2	50
27	26	1	26
28	27	2	54
29	28	1	28
30	29	2	58
	TOTAL	**145**	**1419**

Table 2: Service process

y	**F**	y**F**
0-4	0	0
4-8	5	30
8-12	12	120
12-16	14	196
16-20	17	306
20-24	19	418
24-28	10	260
28-32	15	450
32-36	12	408
36-40	7	266
40-44	6	252
44-48	20	920
48-52	4	200
52-56	3	162
56-60	1	58
TOTAL	**145**	**4046**