International Journal of Civil Engineering Research.
ISSN 2278-3652 Volume 5, Number 4 (2014), pp. 307-314

http://www.ripublication.com/ijcer.htm

Bending Analysis of Paraboloid of Revolution Shell

Nilophar Tamboli[1] and A.B. Kulkarni[2]

[1]Former Post-graduate student of Department of Civil Engineering, Walchand College of Engineering Sangli, Maharashtra, 415416, INDIA.
[2]Associate Professor of Department of Civil Engineering, Walchand College of Engineering Sangli, Maharashtra, 415416, INDIA.
E-mail: nilophartambolii@gmail.com

Abstract

Doubly curvature shells are most efficient shell which allows a multiplicity of alternative stress paths and gives the optimum form for transmission of many different load types. They are widely used as roofing unit for building and industrial structures. Due to wide application of shells as roofing element it is necessary to analyze a shell element more precisely using sophisticated technique like finite element method. In this study the attempt has been made to analyze paraboloid of revolution for bending using finite element method. A vlasov bending theory is used to solve the problem theorotically. In addition to this a model of paraboloid of revolution is developed using staad pro software. These theorotical results are compared with software results. The comparison of therotical and software results is made by plotting the results in graphical form. It is concluded that the results of software and therotical analysis for paraboloid of revolution have shown more or less good agreement in the stresses.

Keywords: Bending analysis, paraboloid of revolution, finite element method.

1. Introduction

Shells are the structures which consist of curved elements and are assembled to form large structures. When it is acted by external loads or by their own weight; it resists the action of these loads both by thrusts and the bending of the surface. Conventional shell theories were well established by researchers particularly for membrane analysis under unsymmetrical loads. Bandyopadhya and A. K. Aditya (1989) studied one paraboloid of revolution and one hyperbolic paraboloid problem with opposite Gaussian curvature using actual & modified boundary conditions by finite element method. They conclude that this technique gives quick and accurate results of analysis of shells all over the surface except at edge strips. Moreover, they reported that, Negative Gaussian curvature does not produce good results. However, doubly curved shells with unsymmetrical loading and possible inducement of moments have not been attempted. Hence, study has been carried out to analyze doubly curved shell for bending by using finite element method. The solved problem of paraboloid of revolution has been taken from the book named design and construction of concrete shell roofs by Ramaswamy and the same problem is analyzed by the stadd pro software for validation.

2. Theoretical Solution and Comparision with Software Solution and Their Discussions

2.1 Bending Analysis of Doubly Curved Shell by Vlasov Bending Theory

Vlasov (1958) has independently developed bending theories which is applicable to shallow shell. The vlasov theory is most easily derived if we regard the shallow doubly curved shell as a combination of disk, plate, and doubly curved membrane. This means that in the equation of equilibrium and the stress strain relation all terms not figuring in the corresponding relation for the disk, plate, and membrane will be symmetrically dropped. To elaborate the plate and curvature effect problem of paraboloid of revolution has analyzed below. In term of $\emptyset$ and w, expressions for the stresses, moments, and shear may be written as follows,

$$N_x = \frac{\partial^2 \emptyset}{\partial y^2} = -A_{mn}\beta_n^2 \sin\alpha_m x \sin\beta_n y \tag{1}$$

$$N_y = \frac{\partial^2 \emptyset}{\partial x^2} = -A_{mn}\alpha_n^2 \sin\alpha_m x \sin\beta_n y \ldots\ldots \tag{2}$$

$$N_{xy} = -\frac{\partial^2 \emptyset}{\partial x\,\partial y} = -A_{mn}\alpha_m \cos\alpha_m x \cos\beta_n y \tag{3}$$

$$M_x = -D\frac{\partial^2 w}{\partial x^2} = DB_{mn}\alpha_m^2 \sin\alpha_m x \sin\beta_n y \ldots \tag{4}$$

$$M_y = -D\frac{\partial^2 w}{\partial y^2} = DB_{mn}\beta_n^2 \sin\alpha_m x \sin\beta_n y \tag{5}$$

$$M_{xy=} - M_{yx} = H = DB_{mn}\alpha_m\beta_n \cos\alpha_m x \cos\beta_n y \ldots. \tag{6}$$

By using these expressions the problem of paraboloid of revolution is solved and a final stress at each coordinate is determined. The analyisis is carried out by assuming separation of plate and curvature effect. The plate effect is calculated using the Levy's solution. The final stresses and moments at each coordinate in the shell are obtained by combining the plate and curvature effect and are tabulated in **Table.1.** Shell dimension are 13.5* 18 m and whose thickness is 0.075m. The rise at the crown above the corners is 2.4m, Poisson's ratio μ = 0 , Imposed load = 3.024 kN/m^2 of shell area.

Table 1: Final stresses and moments in the shell.

x	y	Nx	Ny	Nxy	Mx	My	Mxy
0	0	0	0	-0.46377	0	0	0.009528
1.689	0	0	0	-0.25731	0	0	0.002155
3.375	0	0	0	-0.13359	0	0	0.000431
5.064	0	0	0	-0.06258	0	0	-0.00024
6.75	0	0	0	0	0	0	0
0	2.25	0	0	-0.23203	0	0	-9.6E-05
1.689	2.25	-0.1109858	-0.14766	-0.18759	0.00383	0.002011	-0.00057
3.375	2.25	-0.1674364	-0.11635	-0.10945	-0.00014	0.002298	-0.00024
5.064	2.25	-0.1877854	-0.0733	-0.05257	0.000287	0.002298	-9.6E-05
6.75	2.25	-0.1951589	-0.08393	0	-0.00048	0.002298	0
0	4.5	0	0	-0.09294	0	0	0.000575
1.689	4.5	-0.0599458	-0.20129	-0.08614	0.00383	-0.00077	0.000335
3.375	4.5	-0.1016014	-0.1785	-0.06464	-0.00043	-0.0012	9.58E-05
5.064	4.5	-0.1238656	-0.1299	-0.03289	0.000335	-0.00129	4.79E-05
6.75	4.5	-0.1310476	-0.14163	0	-0.00057	-0.00129	0
0	6.75	0	0	-0.0485	0	0	-0.00034
1.689	6.75	-0.0234133	-0.21445	-0.04252	0.00407	0.000958	-0.00024
3.375	6.75	-0.0479758	-0.2045	-0.02973	-0.00029	0.001436	-9.6E-05
5.064	6.75	-0.0656914	-0.16164	-0.0157	0.000383	0.00158	0
6.75	6.75	-0.0714848	-0.17366	0	-0.00053	0.001628	0
0	9	0	0	0	0	0	0
1.689	9	-0.0434272	-0.22465	0	0.003735	-0.0011	0
3.375	9	-0.0730649	-0.2145	0	-0.00043	-0.00158	0
5.064	9	-0.0899665	-0.17112	0	0.000335	-0.00172	0
6.75	9	-0.096143	-0.18477	0	-0.00057	-0.00177	0

2.2 Software Solution of Paraboloid of Revolution

Software analysis for paraboloid of revolution is carried out by using staad-pro software. *Staad* procedure is basically is a finite element method. Here for each shell a four noded rectangular element has been used. Due to the symmetry of shell only one quadrant of the shell has been taken for analysis. The support conditions are assumed to be pinned and combinations of dead load & live load is considered. Rest of the analysis i.e. calculation of element stiffness & load matrices, Coordinate transformation, assembly procedure, solution of the unknown D.O.F. & finally computations of shell characteristics Nx, Ny, Nxy, Mx, My, Mxy has been done by software itself. **Fig. 1** shows the stresses acting on single rectangular element of one quadrant of a shell. The model prepared for paraboloid of revolution and stress distribution diagram for same is shown in **Fig.2**. The bending analysis results are also tabulated in **Table 2** which shows that the results are not much deviating from theoretical results.

2.3 Discussion

The graphs of x co-ordinate verses stresses and bending moments are plotted **(Fig. 5 to Fig. 6)** to validate the theoretical results with software results. X co-ordinate points indicate the different node points on X axis of single rectangular shell element which is considered for analysis

Fig.5 shows the variation of x co-ordinate verses Nx which shows that there is only one node point where software and theoretical stresses are same. For all other points there is some variation. **Fig.6** and **Fig.7** shows the variation of x co-ordinate verses Ny and the variation of x co-ordinate verses Nxy respectively. These figures show that, at only one node point software and theoretical stresses are much varying and at rest of the point results are varying by 5 to 10%.

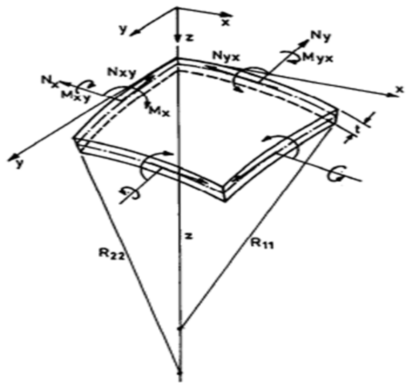


Fig. 1: Stresses and moments acting on shell element.

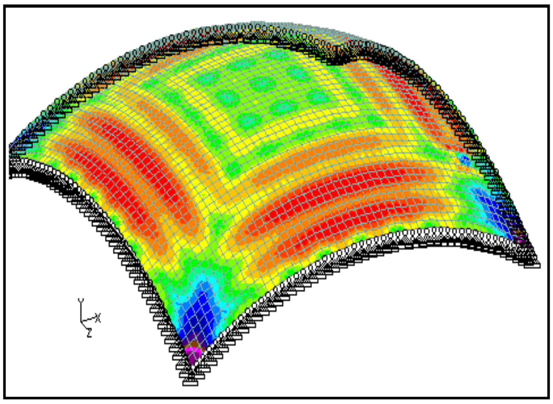


Fig. 2 Stress distribution diagram for paraboloid of revolution (for D.L+L.L)

Table 2: Final stresses and moments in the paraboloid of revolution by software.

x	y	Nx	Ny	Nxy	Mx	My	Mxy
0	0	0.1	0	-0.456	0	0	0.009315
1.689	0	0.125	0	-0.324	0	0	0.003214
3.375	0	0.169	0	-0.11254	0	0	0.000321
5.064	0	0.23	0	-0.0542	0	0	-0.0001454
6.75	0	0.11	0	0	0	0	0
0	2.25	0	0	-0.22240	0	0	-9.6E-06
1.689	2.25	-0.20	-0.1124	-0.16750	0.00283	0.001989	-0.00012
3.375	2.25	-0.27	-0.12547	-0.10945	-0.00121	0.002152	-0.00022
5.064	2.25	-0.17	-0.0698	-0.05257	0.000287	0.002209	-9.6E-04
6.75	2.25	-0.151	-0.0899	0	-0.00348	0.002254	0
0	4.5	0.126	0	-0.0794	0	0	0.000425
1.689	4.5	-0.099	-0.2897	-0.0814	0.00283	-0.00025	0.000258
3.375	4.5	-0.101	-0.1589	-0.06464	-0.000124	-0.0008	9.58E-05
5.064	4.5	-0.1012	-0.1355	-0.0225	0.000214	-0.00130	4.79E-05

6.75	4.5	-0.1125	-0.14258	0	-0.000321	-0.00135	0
0	6.75	0.01	0	-0.0475	0	0	-0.00031
1.689	6.75	-0.01256	-0.21550	-0.0358	0.00401	0.000967	-0.00020
3.375	6.75	-0.0398	-0.21523	-0.01985	-0.00321	0.00154	-9.6E-05
5.064	6.75	-0.05981	-0.14259	-0.0124	0.000383	0.00214	0
6.75	6.75	-0.0677	-0.18259	0	-0.00025	0.00121	0
0	9	0	0	0	0	0	0
1.689	9	-0.04124	-0.21589	0	0.002535	-0.00101	0
3.375	9	-0.0521	-0.2857	0	-0.00041	-0.001214	0
5.064	9	-0.0945	-0.1689	0	0.000289	-0.00101	0
6.75	9	-0.07214	-0.1785	0	-0.00024	-0.00127	0

Fig. 8 shows the variation of x co-ordinate verses bending moment Mx which shows that there is only one node point where software and theoretical results are same but for all other points variation of stresses nearly about 5 to 10%.

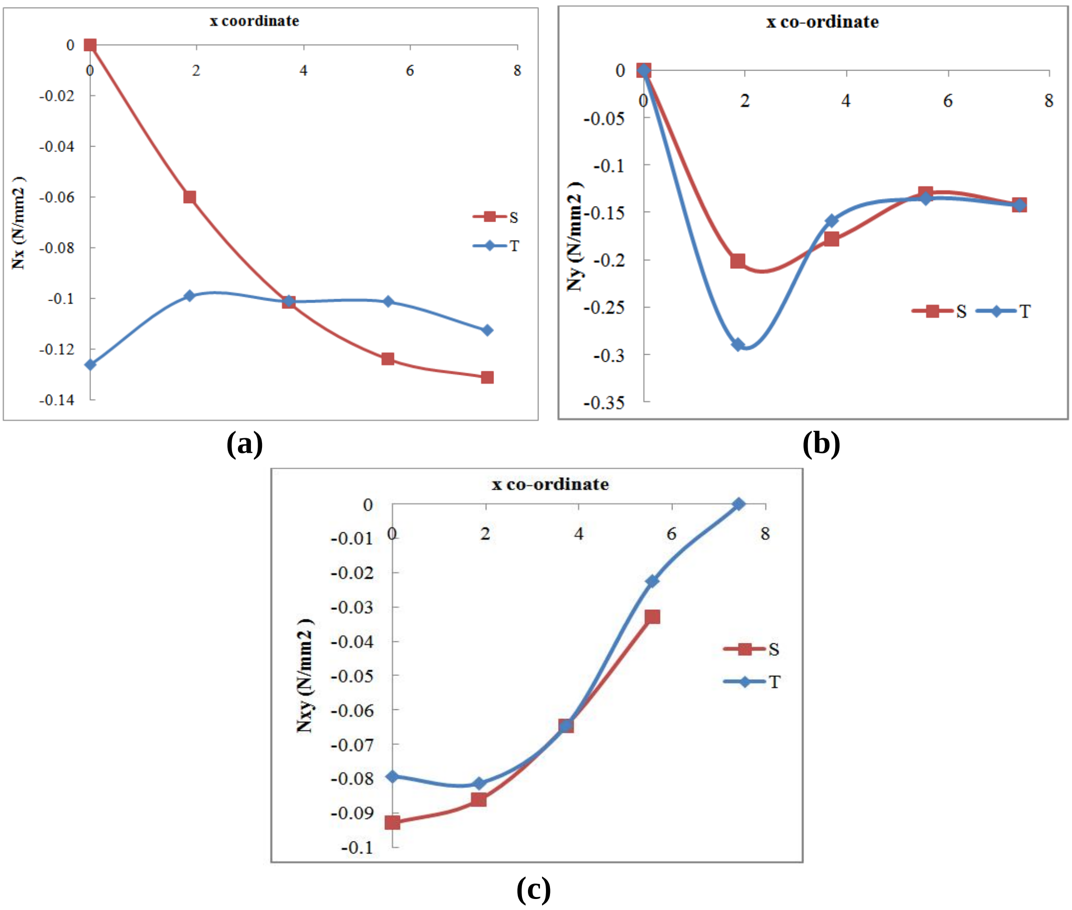
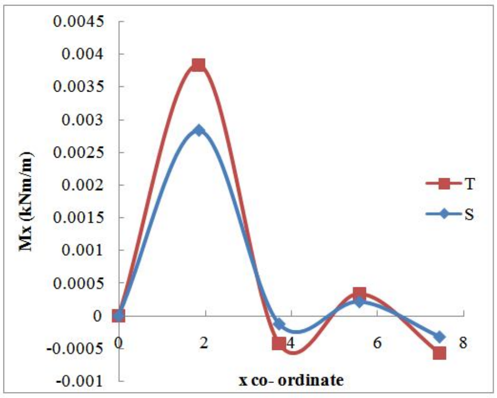
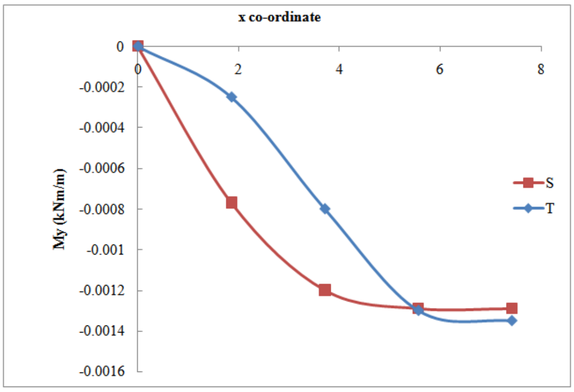


Fig. 5: (a) Variation of x co-ordinate vs .Nx (b) Variation of x co-ordinate vs. Ny (c) Variation of x co-ordinate vs.Nxy.

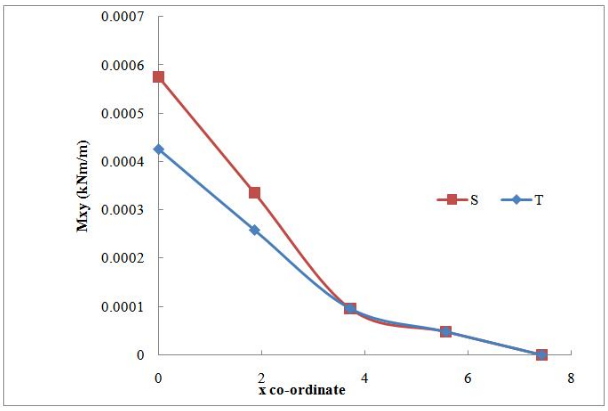
Fig.9 shows the variation of x co-ordinate verses bending moment My which shows that there are three node points where software and theoretical results are matching but for all other points there is large difference nearly about 10 to 15%. **Fig.10** shows the graph of x co-ordinate verses bending moment Mxy which shows that there are three node points where software and theoretical results have good resemblance but for all other points there is some variation nearly about 5 to 10%. The variation in results may be due to software limitation as the location of coordinate points is slightly differ than the exact location which has been used in therotical calculation.



(a)



(b)



(c)

Fig. 6: (a) Variation of x co-ordinate vs.Mx (b) Variation of x co-ordinate vs. My (c)Variation of x co-ordinate vs.Mxy

3. Conclusions

From all these studies we conclude that in all cases of graphs of X co-ordinate verses stresses (Nx, Ny, Nxy) and X co ordinate verses bending moments (Mx, My, Mxy), at some node points theoretical and software results shows a very good resemblance and at rest of the node points result differ by 10 to 15%. This is due because, at node points where results are exactly matching, at those points we found an exact magnitudes of the X co ordinate nodal points during software analysis and at node points where results show some variations, this is because, magnitudes of the X co ordinate nodal points of theoretical and software analysis have slight difference. According to IS code 2210-1988 minimum M20 grade concrete has been used for concrete shells. As per the IS code 456-2000, permissible stresses in compression for bending and direct stresses are 7 N/mm^2 and 5 N/mm^2 respectively. Hence it can be concluded that the analysis which is carried out in the present study are within permissible limits and safe.

References

[1] J. N. Bandopadhya and A. K. Aditya (1989), Simplified Bending analysis of doubly curved shell structures, *Computers & Structures*, 33, 3, 781-784.

[2] Vlasov,V. Z. (1958), “Allgemeine Schalentheorie and Ihre Anwendung in der Technik” , Akademie –Verlag GmbH, Berlin.

[3] Vlasov,V.Z.(1944), The Basic Differential Equations in the General Theory of Elastic Shells, Prikladnaya Matematika Mekhanika, Vol.8. (Translated from Russian into English by TNACA, Technical Memorandum 1241, February, 1951).

[4] “Indian Standard Code of Practice for Design of Loads (Other than Earthquake)for Buildings and Structures”, PART 3,1987.

[5] “Indian Standard Criteria for Design of Reinforced Concrete Shell Structures and Folded Plates”,IS 2210-1988, Bureau of Indian Standards, New Delhi,1992.

[6] G. S. Ramaswamy, *Design and construction of Concrete Shell Roofs.* McGraw-Hill, New Delhi