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Oscillation Properties of Higher Order Impulsive Delay Differential Equations

Xiaodi Li
School of Mathematical Sciences, Shandong Normal University,
Ji'nan, 250014, P.R. China
E-mail: sodymath@163.com

Abstract

This paper studies the oscillation properties of higher order impulsive delay differential equations, and some sufficient conditions for all bounded solutions of this kind of higher order impulsive delay differential equations to be nonoscillatory are obtained by using a comparison theorem with corresponding nonimpulsive differential equations.

AMS subject classification: 34C10, 34C15.
Keywords: Oscillation, delay differential problem, impulse, bounded solution, fixed times.

1. Introduction and Preliminaries

The first paper on oscillation of impulsive delay differential equations was published in 1989. Recently the oscillatory behavior of impulsive delay differential equations has attracted the attention of many researchers. For some contributions in this area, the reader is referred to [1–4, 8, 9, 12–14]. However, there are only a few papers on higher order impulsive delay differential equations.

In this paper, we consider a kind of higher order impulsive delay differential equation. Some sufficient conditions for all bounded solutions of this kind of higher order impulsive delay differential equation to be nonoscillatory are obtained by using a comparison theorem with a corresponding nonimpulsive differential equation. Our results generalize and improve several known results in [9, 12].

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Consider the impulsive delay differential problem

$$\begin{cases} x^{(m)}(t) + a(t)x^{(m-1)}(t) + \sum_{i=1}^{n} p_i(t)x(g_i(t)) = 0, & t \ge t_0,\ t \ne t_k, \\ x^{(j)}(t_k) - x^{(j)}(t_k^-) = \alpha_k x^{(j)}(t_k^-), & j = 0, 1, 2, \ldots, m-1, \end{cases} \tag{1.1}$$

where

$$x'(t_k^+) = \lim_{h\to 0^+} \frac{x(t_k+h) - x(t_k)}{h}, \quad x'(t_k^-) = x'(t_k) = \lim_{h\to 0^-} \frac{x(t_k+h) - x(t_k)}{h}$$

and the delay differential problem

$$y^{(m)}(t) + a(t)y^{(m-1)}(t) + \sum_{i=1}^{n} p_i(t) \prod_{g_i(t) < t_k \le t} (1+\alpha_k)^{-1} y(g_i(t)) = 0, \quad t \ge t_0. \tag{1.2}$$

We assume the following:

(A$_1$) $0 \le t_0 < t_1 < \cdots < t_k < \cdots$ are fixed points with $\lim_{k\to\infty} t_k = \infty$;

(A$_2$) $a, p_i \in C([0,\infty), \mathbb{R})$, $i = 1, 2, \ldots, n$, are Lebesgue measurable and locally essentially bounded functions, $\mathbb{R}$ is the real axis;

(A$_3$) $g_i \in C([0,\infty), \mathbb{R})$, $i = 1, 2, \ldots, n$, are Lebesgue measurable functions and $g_i(t) \le t$ satisfies $\lim_{t\to\infty} g_i(t) = \infty$;

(A$_4$) $\{\alpha_k\}$ is a sequence of constants and $\alpha_k > -1$.

When $m = 2$, (1.1) reduces to the impulsive delay differential problem

$$\begin{cases} x''(t) + a(t)x'(t) + \sum_{i=1}^{n} p_i(t)x(g_i(t)) = 0, & t \ge t_0, \quad t \ne t_k, \\ x^{(j)}(t_k) - x^{(j)}(t_k^-) = \alpha_k x^{(j)}(t_k^-), & j = 0, 1. \end{cases} \tag{1.3}$$

Oscillation and nonoscillation of (1.3) has been extensively investigated in [12].

When $m = 2$, $g_i(t) = t$, $n = 1$, (1.1) reduces to the impulsive differential problem

$$\begin{cases} x''(t) + a(t)x'(t) + p(t)x(t) = 0, & t \ge t_0, \quad t \ne t_k, \\ x^{(j)}(t_k) - x^{(j)}(t_k^-) = \alpha_k x^{(j)}(t_k^-), & j = 0, 1. \end{cases} \tag{1.4}$$

Oscillation and nonoscillation of (1.4) has been investigated in [9].

For any $\tau_0 \ge 0$, let $\tau_0^- = \min_{1\le i\le n} \inf_{t\ge\tau_0} g_i(t)$. Let Ψ denote the set of functions $\phi : [\tau_0^-, t_0] \to \mathbb{R}$, which are bounded and Lebesgue measurable on $[\tau_0^-, \tau_0]$.

Definition 1.1. For any $\tau_0 \geq 0$ and $\phi \in \Psi$, a function $x : [\tau_0^-, \infty) \to \mathbb{R}$ is said to be a solution of (1.1) on $[\tau_0^-, \infty)$ satisfying the initial value condition

$$x(t) = \phi(t), \quad \phi(\tau_0) > 0, \quad t \in [\tau_0^-, \tau_0], \tag{1.5}$$

if the following conditions are satisfied:

(i) x satisfies (1.5);

(ii) x is absolutely continuous in each interval (τ_0, t_{k_0}), (t_k, t_{k+1}), $k \geq k_0$, $k_0 = \min\{k \mid t_k > \tau_0\}$, $x(t_k^+)$, $x(t_k^-)$ exist and $x(t_k^-) = x(t_k)$, the second condition in (1.1) holds;

(iii) x satisfies the first equation in (1.1) almost everywhere in (τ_0^-, ∞).

Definition 1.2. The solution x of system (1.1) is said to be nonoscillatory if it is eventually negative or eventually positive. Otherwise, it is said to be oscillatory.

By a solution y of (1.2) on $[\tau_0^-, \infty)$ we mean a function which has an absolutely continuous derivative y' on $[\tau_0^-, \infty)$, satisfies (1.2) a.e. on $[\tau_0^-, \infty)$ and satisfies (1.5) on $[\tau_0^-, t_0]$. In this paper, we always suppose $\tau_0^- = t_0^-$, $\tau_0 = t_0$.

2. Main Results

In this section we shall establish theorems which enable us to reduce the oscillation and nonoscillation of (1.1) to the corresponding problem (1.2).

Theorem 2.1. Assume that (A_1)–(A_4) hold.

(i) If y is a solution of (1.2) on $[t_0^-, \infty)$, then $x(t) = \prod_{t_0 < t_k \leq t} (1 + \alpha_k) y(t)$ is a solution of (1.1) on $[t_0^-, \infty)$.

(ii) If x is a solution of (1.1) on $[t_0^-, \infty)$, then $y(t) = \prod_{t_0 < t_k \leq t} (1 + \alpha_k)^{-1} x(t)$ is a solution of (1.2) on $[t_0^-, \infty)$.

Proof. First we shall prove (i). Let y be a solution of (1.2) on $[t_0^-, \infty)$. Then $x(t) = \prod_{t_0 < t_k \leq t} (1 + \alpha_k) y(t)$ has an absolutely continuous derivative x' on (t_0^-, t_0), $[t_k, t_{k+1})$,

$k \geq 0$. For any $t \neq t_k$, $t > t_0^-$, it is easy to prove that

$$\begin{aligned}
x^{(m)}(t) &= \prod_{t_0 < t_k \leq t} (1+\alpha_k) y^{(m)}(t) \\
&= \prod_{t_0 < t_k \leq t} (1+\alpha_k) \left\{ -a(t) y^{(m-1)}(t) - \sum_{i=1}^{n} p_i(t) \prod_{g_i(t) < t_k \leq t} (1+\alpha_k)^{-1} y(g_i(t)) \right\} \\
&= -a(t) \prod_{t_0 < t_k \leq t} (1+\alpha_k) y^{(m-1)}(t) - \sum_{i=1}^{n} p_i(t) \prod_{t_0 < t_k \leq g_i(t)} (1+\alpha_k) y(g_i(t)) \\
&= -a(t) x^{(m-1)}(t) - \sum_{i=1}^{n} p_i(t) x(g_i(t)).
\end{aligned}$$

So we have

$$x^{(m)}(t) + a(t) x^{(m-1)}(t) + \sum_{i=1}^{n} p_i(t) x(g_i(t)) = 0, \quad t \geq t_0, \quad t \neq t_k,$$

which implies that x solves (1.1). On the other hand, we note

$$x^{(j)}(t_m) = \prod_{t_0 < t_k \leq t_m} (1+\alpha_k) y^{(j)}(t_m)$$

and

$$x^{(j)}(t_m^-) = \prod_{t_0 < t_k \leq t_{m-1}} (1+\alpha_k) y^{(j)}(t_m),$$

that is

$$x^{(j)}(t_m) = (1+\alpha_m) x^{(j)}(t_m^-),$$

which implies that x solves the second condition in (1.1). Hence $x(t) = \prod_{t_0 < t_k \leq t} (1 + \alpha_k) y(t)$ is a solution of (1.1) on $[t_0^-, \infty)$.

Next we prove (ii). Let x be a solution of (1.1). We prove that $y(t) = \prod_{t_0 < t_k \leq t} (1 + \alpha_k)^{-1} x(t)$ is a solution of (1.2) on $[t_0^-, \infty)$. For any $t \neq t_k$, $t > t_0^-$,

$$
\begin{aligned}
y^{(m)}(t) &= \prod_{t_0<t_k\le t} (1+\alpha_k)^{-1} x^{(m)}(t) \\
&= \prod_{t_0<t_k\le t} (1+\alpha_k)^{-1} \left\{ -a(t)x^{(m-1)}(t) - \sum_{i=1}^{n} p_i(t)x(g_i(t)) \right\} \\
&= -a(t) \prod_{t_0<t_k\le t} (1+\alpha_k)^{-1} x^{(m-1)}(t) - \sum_{i=1}^{n} p_i(t) \prod_{t_0<t_k\le t} (1+\alpha_k)^{-1} x(g_i(t)) \\
&= -a(t)y^{(m-1)}(t) - \sum_{i=1}^{n} p_i(t) \prod_{g_i(t)<t_k\le t} (1+\alpha_k)^{-1} y(g_i(t))
\end{aligned}
$$

and

$$
\begin{aligned}
y^{(j)}(t_m) &= \prod_{t_0<t_k\le t_m} (1+\alpha_k)^{-1} x^{(j)}(t_m), \\
y^{(j)}(t_m^-) &= \prod_{t_0<t_k\le t_{m-1}} (1+\alpha_k)^{-1} x^{(j)}(t_m^-) \\
&= \prod_{t_0<t_k\le t_{m-1}} (1+\alpha_k)^{-1}(1+\alpha_m)^{-1} x^{(j)}(t_m) \\
&= \prod_{t_0<t_k\le t_m} (1+\alpha_k)^{-1} x^{(j)}(t_m) = y^{(j)}(t_m).
\end{aligned}
$$

So $y(t) = \prod_{t_0<t_k\le t} (1+\alpha_k)^{-1} x(t)$ is a solution of (1.2) on $[t_0^-, \infty)$. The proof is therefore complete. ■

Using Theorem 2.1, we obtain the following results.

Theorem 2.2. Assume that (A_1)–(A_4) hold. Then all solutions of (1.1) are oscillatory (nonoscillatory) if and only if all solutions of (1.2) are oscillatory (nonoscillatory).

Theorem 2.3. Assume that (A_1)–(A_4) hold. Then all solutions of (1.1) asymptotically approach to zero if and only if all solutions of (1.2) asymptotically approach to zero.

Let $r(t) = \exp\left(\int_0^t a(s)ds\right)$. Then (1.2) reduces to the problem

$$
(ry^{(m-1)})'(t) + r(t)\sum_{i=1}^{n} p_i(t) \prod_{g_i(t)<t_k\le t} (1+\alpha_k)^{-1} y(g_i(t)) = 0, \ t > 0. \tag{2.1}
$$

Theorem 2.4. Assume that (A_1)–(A_4) hold. Moreover, suppose that

(A$_5$) $\prod_{t_0<t_k\le t}(1+\alpha_k)$ is bounded and $\liminf_{t\to\infty}\prod_{t_0<t_k\le t}(1+\alpha_k)>0$;

(A$_6$) $p_i(t)\ge 0, i=1,2,\ldots$;

(A$_7$) $$\int_{t_0}^{t}\int_{t_0}^{\sigma_{m-2}}\cdots\int_{t_0}^{\sigma_1}\frac{1}{r(s)}\int_s^\infty r(u)\sum_{i=1}^n p_i(u)\prod_{g_i(u)<t_k\le u}(1+\alpha_k)^{-1}du\cdots \cdots d\sigma_{m-3}d\sigma_{m-2}<\infty.$$

Then (1.1) has a bounded nonoscillatory solution x with $\liminf_{t\to\infty}\mid x(t)\mid>0$.

Proof. We only need to prove that (2.1) has a bounded nonoscillatory solution y. From (A$_7$), there exists $T>0$ such that for all $t\ge T$, $g_i(t)\ge T_0>0$, $i=1,2,\ldots$, and for all $t>T$,

$$\int_{T}^{t}\int_{t_0}^{\sigma_{m-2}}\cdots\int_{t_0}^{\sigma_1}\frac{1}{r(s)}\int_s^\infty r(u)\sum_{i=1}^n p_i(u)\prod_{g_i(u)<t_k\le u}(1+\alpha_k)^{-1}du\cdots \cdots d\sigma_{m-3}d\sigma_{m-2}<\frac{1}{4}. \tag{2.2}$$

Let Y denote the locally convex space of all continuous functions $y\in C([T_0,\infty),\mathbb{R})$ with the topology of uniform convergence on compact subintervals of $[T_0,\infty)$. Let $\Gamma=\left\{y\in Y:\frac{\gamma}{2}\le y(t)\le\frac{2\gamma}{3},\ t\ge T_0\right\}$, where $\gamma>0$ is an arbitrary given constant. We note that Γ is a closed and convex subset of Y and it is nonempty.

Now, we define a map $V:\Gamma\to Y$ by

$$(Vy)(t)=\begin{cases}\frac{\gamma}{2}+(\Omega y)(t), & t>T,\\ \frac{\gamma}{2}, & T_0\le t<T,\end{cases}$$

where

$$(\Omega y)(t)=\int_T^t\int_0^{\sigma_{m-2}}\cdots\int_0^{\sigma_1}\frac{1}{r(s)}\int_s^\infty r(u)\sum_{i=1}^n p_i(u)\times \times\prod_{g_i(u)<t_k\le u}(1+\alpha_k)^{-1}y(g_i(t))du\cdots d\sigma_{m-2}.$$

First we verify $V\Gamma\subset\Gamma$. For all $y\in\Gamma$, it is obvious that $Vy\subset\Gamma$ for $T_0\le t<T$.

When $t > T$, combining (2.2) we get

$$\begin{aligned} Vy &\le \frac{\gamma}{2} + \frac{2\gamma}{3} \int_T^t \int_0^{\sigma_{m-2}} \cdots \int_0^{\sigma_1} \frac{1}{r(s)} \int_s^\infty r(u) \sum_{i=1}^n p_i(u) \times \\ &\quad \times \prod_{g_i(u) < t_k \le u} (1+\alpha_k)^{-1} du \cdots d\sigma_{m-2} \\ &\le \frac{\gamma}{2} + \frac{2\gamma}{3} \cdot \frac{1}{4} = \frac{2\gamma}{3}. \end{aligned}$$

So V maps Γ into Γ. On the other hand, $\{Vy\}$ is uniformly bounded. The continuity of $V\Gamma \to \Gamma$ is verified as follows: Let $y_n \in \Gamma$, $y \in \Gamma$ with $\lim_{n\to\infty} y_n = y$. For any $\varepsilon > 0$, there exists a positive integer N_ε such that $| y_n - y | < 4\varepsilon$ for any $n > N_\varepsilon$. In particular,

$$| y_n(g_i(t)) - y(g_i(t)) | < 4\varepsilon, \quad n > N_\varepsilon, \quad t > T_0.$$

Hence

$$\begin{aligned} | (Vy_n)(t) - (Vy)(t) | &\le \int_T^t \int_0^{\sigma_{m-2}} \cdots \int_0^{\sigma_1} \frac{1}{r(s)} \int_s^\infty r(u) \sum_{i=1}^n p_i(u) \times \\ &\quad \times \prod_{g_i(u) < t_k \le u} (1+\alpha_k)^{-1} | y_n(g_i(t)) - y(g_i(t)) | \, du \cdots \\ &\quad \cdots d\sigma_{m-3} d\sigma_{m-2} \\ &\le 4\varepsilon \int_T^t \int_0^{\sigma_{m-2}} \cdots \int_0^{\sigma_1} \frac{1}{r(s)} \int_s^\infty r(u) \sum_{i=1}^n p_i(u) \times \\ &\quad \times \prod_{g_i(u) < t_k \le u} (1+\alpha_k)^{-1} du \cdots d\sigma_{m-3} d\sigma_{m-2} \\ &\le 4\varepsilon \cdot \frac{1}{4} = \varepsilon. \end{aligned}$$

So we know that V maps Γ continuously into a compact subset of Γ. Therefore, by Schauder–Tychonov's fixed point theorem, V has a fixed point y in Γ. It is easy to check that the fixed point y is a solution of (2.1). So (1.2) has a bounded nonoscillatory solution y. By Theorem 2.1, $x(t) = \prod_{t_0 < t_k \le t} (1+\alpha_k) y(t)$ is a bounded nonoscillatory solution of (1.1). Using condition (A_5), we eventually get $\liminf_{t\to\infty} | x(t) | > 0$. The proof is therefore complete. ■

Next we shall give an oscillation criterion for (1.1). Suppose m is a given even number. First we give some lemmas whose proofs are omitted, because their proofs are similar to [14] but without impulses.

Lemma 2.5. Let y be a given solution of (2.1). Suppose that there exists $T > 0$ such that $x^{(i)}(t) > 0\ (< 0)$, $x^{(i+1)}(t) > 0\ (< 0)$ for $t \geq T$. Moreover suppose that (A_1)–(A_4) and (A_6) hold and

$$(A_8)\quad \int_{t_0}^{\infty} \frac{1}{r(s)} ds = \infty.$$

Then there exists $T_1 > 0$ such that $x^{(i-1)}(t) \geq 0\ (< 0)$, $t > T_1$.

Lemma 2.6. Let y be a given solution of (2.1). Suppose that there exists $T > 0$ such that $x(t) > 0$, $x^{(i)}(t) \leq 0$ for $t \geq T$. Moreover suppose that (A_1)–(A_4), (A_6) and (A_8) hold. Then there exists $T_2 > 0$ such that $x^{(i-1)}(t) > 0$, $t > T_2$.

Lemma 2.7. Let y be a given solution of (2.1). Suppose that there exists $T > 0$ such that $x(t) > 0$ for $t \geq T$. Moreover suppose that (A_1)–(A_4), (A_6) and (A_8) hold. Then there exists $T_3 > T$ and $N \in \{1, 3, \ldots, m-1\}$ such that for $t > T_3$

$$\begin{cases} x^{(i)}(t) > 0, & i = 0, 1, 2, \ldots, N; \\ (-1)^{(i-1)} x^{(i)}(t) > 0, & i = N+1, N+2, \ldots, m-2; \\ x^{(m-1)}(t) > 0. \end{cases} \tag{2.3}$$

Let

$$g(t) = \min_{1 \leq i \leq n} g_i(t).$$

Theorem 2.8. Assume that (A_1)–(A_4), (A_6) and (A_8) hold, and

(A_9) g_i has an absolutely continuous derivative g_i' on $(t_0^{-}\infty)$, and $g_i' \geq 0$;

$$(A_{10})\quad \int_{t_0}^{\infty} s^{m-1} r(s) \sum_{i=1}^{n} p_i(t) \prod_{g_i(s) < t_k \leq s} (1+\alpha_k)^{-1} ds = \infty;$$

(A_{11}) there exists $G > 0$ such that $r(t) < G$.

Then all bounded solutions of (1.1) are oscillatory.

Proof. We only need to prove that all bounded solutions of (2.1) are oscillatory. Suppose that the assertion is not true. Without loss of generality, we may suppose that there exists $T > 0$ such that $y(t) > 0$ for $t \geq T$.

First we consider the case when $N = 1$. From Lemma 2.7, we get $y'(t) > 0$, $y''(t) < 0$, $y'''(t) > 0, \ldots, y^{(m-1)}(t) > 0$, $t > T'$. So $(y(g_i(t)))' = y'(g_i(t)) g_i'(t) > 0$, which implies $y(g_i(t))$ is increasing in t for $t > T'$. Therefore, for $t > T'$

$$\begin{aligned}(ry^{(m-1)})'(t) &= -r(t)\sum_{i=1}^{n} p_i(t) \prod_{g_i(t)<t_k\le t}(1+\alpha_k)^{-1}y(g_i(t)) \\ &\le -r(t)\sum_{i=1}^{n} p_i(t) \prod_{g_i(t)<t_k\le t}(1+\alpha_k)^{-1}y(g_i(T')) \\ &\le -R\cdot r(t)\sum_{i=1}^{n} p_i(t) \prod_{g_i(t)<t_k\le t}(1+\alpha_k)^{-1}, \qquad (2.4)\end{aligned}$$

where $R = y(g(T')) > 0$. We multiply (2.4) by t^{m-1} and integrate on $[T', t)$ to find

$$\int_{T'}^{t} s^{m-1}(ry^{(m-1)})'(s)ds \le -R\int_{T'}^{t} t^{m-1}r(s)\sum_{i=1}^{n} p_i(s) \prod_{g_i(s)<t_k\le s}(1+\alpha_k)^{-1}. \qquad (2.5)$$

On the other hand, combining (A_{11}), we have

$$\begin{aligned}\int_{T'}^{t} s^{m-1}(ry^{(m-1)})'(s)ds &= \int_{T'}^{t} s^{m-1}d(r(s)y^{(m-1)}(s)) \\ &\ge s^{m-1}r(s)y^{(m-1)}(s)\mid_{T'}^{t} -G(m-1)\int_{T'}^{t} s^{m-2}y^{(m-1)}(s)ds \\ &= s^{m-1}y^{(m-1)}(s)\mid_{T'}^{t} -G(m-1)\times \\ &\quad \times\left\{s^{m-2}y^{(m-2)}(s)\mid_{T'}^{t} -(m-2)\int_{T'}^{t} s^{m-3}dy^{(m-3)}(s)\right\} \\ &= s^{m-1}y^{(m-1)}(s)\mid_{T'}^{t} -G\{(m-1)s^{m-2}y^{(m-2)}(s)\mid_{T'}^{t} \\ &\quad -(m-1)(m-2)s^{m-3}y^{(m-3)}(s)\mid_{T'}^{t} \\ &\quad +(m-1)(m-2)(m-3)\int_{T'}^{t} s^{m-4}dy^{(m-4)}(s)\} \\ &\quad \cdots\cdots \\ &= s^{m-1}y^{(m-1)}(s)\mid_{T'}^{t} +G\sum_{i=0}^{m-2} t^i y^{(i)}(t)(-1)^{m+i+1}\frac{(m-1)!}{i!} \\ &\quad +G\sum_{i=0}^{m-2}(T')^i y^{(i)}(T')(-1)^{m+i}\frac{(m-1)!}{i!}.\end{aligned}$$

Considering this and the fact that m is an even number, we get

$$\begin{aligned}
&\int_{T'}^{t} s^{m-1}(ry^{(m-1)})'(s)ds \\
\geq\ & t^{m-1}y^{(m-1)}(t) - (T')^{m-1}y^{(m-1)}(T') - Gx(t)(-1)^{m+1}(m-1)! \\
&+G\sum_{i=1}^{m-2} t^i y^{(i)}(t)(-1)^{m+i+1}\frac{(m-1)!}{i!} + G\sum_{i=0}^{m-2}(T')^i y^{(i)}(T')(-1)^{m+i}\frac{(m-1)!}{i!} \\
\geq\ & -(T')^{m-1}y^{(m-1)}(T') - Gy(t)(m-1)! + G\sum_{i=0}^{m-2}(T')^i y^{(i)}(T')(-1)^{m+i}\frac{(m-1)!}{i!}.
\end{aligned}$$

In view of (2.4), (2.5), we obtain

$$\begin{aligned}
&-(T')^{m-1}y^{(m-1)}(T') - Gy(t)(m-1)! + G\sum_{i=0}^{m-2}(T')^i y^{(i)}(T')(-1)^{m+i}\frac{(m-1)!}{i!} \\
&\qquad \leq -R\int_{T'}^{t} t^{m-1}r(s)\sum_{i=1}^{n} p_i(s)\prod_{g_i(s)<t_k\leq s}(1+\alpha_k)^{-1}.
\end{aligned}$$

So using (A_{10}), we obtain $y(t)\to\infty$ as $t\to\infty$, which is a contradiction.

Next we consider the case when $N>1$. Since $y'(t)>0$, $y''(t)>0$, $t>T$, so y' is increasing in t for $t\in[T,\infty)$. We note

$$y(t) = y(T) + \int_T^t y'(\tau)d\tau \geq y(T) + y'(T)(t-T).$$

So $y(t)\to\infty$, as $t\to\infty$, which is a contradiction. The proof is complete. ■

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