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# Reliability Modelling and Analysis of 1-out-of-2 unit Repairable System Performing under Different Weather Conditions

**Dr. Savita Deswal**

*Associate Professor Mathematics*

*Govt. P. G. College for Women, Rohtak, Haryana, India*

*Email: sdeswal2981@gmail.com*

**Abstract**

The primary objective of this paper is to do the reliability modelling and analysis of a 1-out-of-2 unit repairable system having two non-identical units (main and duplicate units) with the concept of change in weather conditions. Initially, the main unit is operative whereas the duplicate unit is at cold standby. Single server visits the failed unit immediately for conducting repair in normal weather. The main and duplicate unit ceases to operate in the abnormal weather conditions. The expressions for some important reliability measures such as Transition probabilities, mean-sojourn time, MTSF, Availability and Profit of the system model have been derived in graphical form.

***Index Terms*** — Performance measures, Non-identical Units, Repairable system, Different Environmental conditions.

## I. INTRODUCTION

The performance measures of a repairable system are essential metrics used to evaluate and quantify the reliability, availability, and effectiveness of systems that can be repaired or maintained when they fail. These measures play a crucial role in industries where the continuous operation of complex systems is critical, such as manufacturing, transportation, telecommunications, and healthcare. By assessing the performance of repairable systems, organizations can make informed decisions regarding maintenance schedules, resource allocation, and system improvements. In this context, the two dissimilar Unit Standby System have been investigated stochastically by the scholars

Mokkadis, G.S. et. al. (1989). Sometimes it becomes very difficult to keep the environmental conditions under control which may fluctuate due changing climate and other natural catastrophic. While considering this fact in mind, Malik, S.C. and Barak, M. (2009) examined the reliability and economic analysis of a system operating under different weather conditions. But in case of high cost of identical units, the non-identical unit (may be a substandard unit) might be kept as spare in cold standby not only to improve the reliability of the system but also to maintain performance of the system in emergency, as referred by Malik, S. C. et. al. (2013-2015). Deswal, S. assessed the effect of weather on priority based redundant system in 2018. Further, the economic analysis of system reliability model under operation in changing weather is evaluated by Deswal, S. in 2021.

In the present Paper, a repairable system of two non-identical units – one is original (called main unit) and other is a substandard unit (called duplicate unit) has been modeled and analyzed in detail under two weather conditions – normal and abnormal. The environmental conditions when satisfied to the system correspond to normal weather; otherwise, it is supposed that the system is in abnormal weather. Initially, the system is operative with main unit and duplicate unit is kept a spare in cold standby. Both units have direct complete failure from normal mode. Each unit is capable of performing the same set of functions with different degree of reliability and desirability. There is a single server who visits the system immediately whenever needed to do repair of the failed unit in normal weather only. The operation and repair of the units are not allowed in abnormal weather as a precautionary measure to avoid excessive damage to the system. However, operation and repair of the units are as usual in normal weather. The units work as new after repair.

The distributions of failure time of the units and change of weather conditions follow negative exponential while that of repair times of the units are taken as arbitrary. All random variables are statistically independent. The switch devices and repairs are perfect. The expressions for various measures of system effectiveness such as transition probabilities, mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server and profit function in steady state are derived using semi-Markov process and regenerative point technique. The numerical results giving particular values to the parameters and various costs are obtained for mean time to system failure (MTSF), availability and profit to depict their tabular behavior. The application of the present work can be visualized in a software industry where application software is run through two different databases-one is initially operative and other is kept in cold standby.

## II. SYSTEM DESCRIPTION

The system is described as below –

| | |
|---|---|
| System | Repairable System with two non-identical units as main unit and duplicate unit. Each unit is capable of performing same set of |

| | |
|---|---|
| | activities but with different efficiency |
| Operation | Initially main unit is operative and duplicate unit is at standby |
| Weather Conditions | Normal and Abnormal weather |
| Redundancy | Cold Standby |
| Mode | Unit is either operative in normal mode or failed |
| Server | Single server with immediate arrival in normal weather |
| Switches and Repair | Perfect and Instantaneous |

## III. SYSTEM TRANSITION STATES

The following are the possible transition states of the system

| **State** | **Notation** | **Definition** |
|---|---|---|
| $S_0$ | (MO, DCs) | Main unit operative and duplicate at cold standby in normal weather |
| $S_1$ | (MFur, DO) | Main unit failed under repair and duplicate operative in normal weather |
| $S_2$ | $(\overline{MWO}, \overline{DCs})$ | Main unit waiting for operation in abnormal weather and duplicate at cold standby in abnormal weather |
| $S_3$ | $(\overline{MFwr}, \overline{DWO})$ | Main unit failed waiting for repair and duplicate waiting to operate in abnormal weather |
| $S_4$ | (MFUR, DFwr) | Main unit failed continuously under repair and duplicate failed waiting for repair in normal weather |
| $S_5$ | (MCs, DO) | Main unit at cold standby and duplicate operative in normal weather |
| $S_6$ | (MO, DFur) | Main unit operative and duplicate failed under repair in normal weather |
| $S_7$ | $(\overline{MFwr}, \overline{DFWR})$ | Main unit and duplicate unit failed waiting for repair continuously in abnormal weather |
| $S_8$ | (MFur, DFWR) | Main unit failed under repair and duplicate failed waiting |

| | | |
|---|---|---|
| | | for repair in normal weather |
| $S_9$ | ( $\overline{MCs}$ , $\overline{DWO}$ ) | Main unit at cold standby and duplicate waiting for operation in abnormal weather |
| $S_{10}$ | ( $\overline{MWO}$ , $\overline{DFwr}$ ) | Main unit waiting for operation and duplicate waiting for repair in abnormal weather |
| $S_{11}$ | (MFwr, DFUR) | Main unit failed waiting for repair and duplicate failed under repair continuously in normal weather |
| $S_{12}$ | ( $\overline{MFWR}$ , $\overline{DFwr}$ ) | Main unit failed waiting for repair continuously and duplicate failed waiting for repair in abnormal weather |
| $S_{13}$ | (MFWR, DFur) | Main unit failed continuously waiting for repair and duplicate failed under repair in normal weather |

The states $S_0$, $S_1$, $S_2$, $S_3$, $S_5$, $S_6$, $S_9$ and $S_{10}$ are regenerative while the states $S_4$, $S_7$, $S_8$, $S_{11}$, $S_{12}$ and $S_{13}$ are non-regenerative

## IV. TRANSITION PROBABLILTIES

The differential transition probabilities are:

$dQ_{01}(t)=\lambda e^{-(\lambda+\beta)t}dt$, $dQ_{02}(t)=\beta e^{-(\lambda+\beta)t}dt$, $dQ_{13}(t)=\beta e^{-(\beta+\lambda_1)t}\,\overline{G(t)}\,dt$, $dQ_{14}(t)=\lambda_1 e^{-(\beta+\lambda_1)t}\,\overline{G(t)}\,dt$,

$dQ_{15}(t)=g(t)\,e^{-(\beta+\lambda_1)t}dt$, $dQ_{20}(t)=\beta_1 e^{-\beta_1 t}dt$, $dQ_{31}(t)=\beta_1 e^{-\beta_1 t}dt$, $dQ_{46}(t)=g(t)e^{-\beta t}dt$,

$dQ_{47}(t)=\beta e^{-\beta t}\,\overline{G(t)}\,dt$, $dQ_{56}(t)=\lambda_1 e^{-(\beta+\lambda_1)t}dt$, $dQ_{59}(t)=\beta e^{-(\beta+\lambda_1)t}dt$, $dQ_{60}(t)=g_1(t)e^{-(\beta+\lambda)t}dt$,

$dQ_{6,10}(t)=\beta e^{-(\beta+\lambda)t}\,\overline{G_1(t)}\,dt$, $dQ_{6,11}(t)=\lambda e^{-(\beta+\lambda)t}\,\overline{G_1(t)}\,dt$, $dQ_{78}(t)=\beta_1 e^{-\beta_1 t}dt$, $dQ_{86}(t)=g(t)\,e^{-\beta_1 t}dt$,

$dQ_{87}(t)=\beta_1 e^{-\beta_1 t}\,\overline{G(t)}\,dt$, $dQ_{95}(t)=\beta_1 e^{-\beta_1 t}dt$, $dQ_{10,6}(t)=\beta_1 e^{-\beta_1 t}\,{}^{t}dt$, $dQ_{11,1}(t)=g_1(t)\,e^{-\beta t}dt$,

$dQ_{11,12}(t)=\beta e^{-\beta t}\,\overline{G_1(t)}\,dt$, $dQ_{12,13}(t)=\beta_1 e^{-\beta_1 t}dt$, $dQ_{13,1}(t)=g_1(t)\,e^{-\beta t}dt$, $dQ_{13,12}(t)=\beta e^{-\beta t}\,\overline{G_1(t)}\,dt$

…(1)

Simple probabilistic considerations yield the following expressions for the non-zero elements

$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt$ , we have

$p_{01}=\dfrac{\lambda}{\beta+\lambda}$, $p_{02}=\dfrac{\beta}{\beta+\lambda}$, $p_{13}=\dfrac{\beta}{\beta+\lambda_1}(1-g^*(\beta+\lambda_1))$, $p_{14}=\dfrac{\lambda_1}{\beta+\lambda_1}(1-g^*(\beta+\lambda_1))$, $p_{15}=g^*(\beta+\lambda_1)$,

$p_{20}=1$, $p_{31}=1$, $p_{46}=g^*(\beta)$, $p_{47}=1-g^*(\beta)$, $p_{56}=\frac{\lambda_1}{\beta+\lambda_1}$ $p_{59}=\frac{\beta}{\beta+\lambda_1}$, $p_{60}=g^*_1(\beta+\lambda)$,

$p_{6,10}=\frac{\beta}{\beta+\lambda}(1-g_1^*(\beta+\lambda))$, $p_{6,11}=\frac{\lambda}{\beta+\lambda}(1-g_1^*(\beta+\lambda))$, $p_{78}=1$, $p_{86}=g^*(\beta)$, $p_{87}=1-g^*(\beta)$, $p_{95}=1$,

$p_{10,6}=1$, $p_{11,1}=g_1^*(\beta)$, $p_{11,12}=g_1^*(\beta)p_{12,13}=1$, $p_{13,1}=g^*_1(\beta)$, $p_{13,12}=1-g^*_1(\beta)$,

$p_{16.4}=\frac{\lambda_1}{\beta+\lambda_1}(1-g^*(\beta+\lambda_1))g^*(\beta)$, $p_{16.4(7,8)}{}^n=\frac{\lambda_1}{\beta+\lambda_1}(1-g^*(\beta+\lambda_1))(1-g^*(\beta))$,

$p_{61.11}=\frac{\lambda}{\beta+\lambda}(1-g_1^*(\beta+\lambda))g_1^*(\beta)$, $p_{61.11,(12,13)}{}^n=\frac{\lambda}{\beta+\lambda}(1-g_1^*(\beta+\lambda))(1-g_1^*(\beta))$

…(2)

## V. MEAN SOJOURN TIMES

The mean sojourn times ($\mu_i$) in the state $S_i$ are

$\mu_0=m_{01}+m_{02}=\frac{1}{\beta+\lambda}$, $\mu_1=m_{13}+m_{14}+m_{15}=\frac{1}{\beta+\lambda_1}(1-g^*(\beta+\lambda_1))$, $\mu_2=m_{20}=\frac{1}{\beta_1}$, $\mu_3=m_{31}=\frac{1}{\beta_1}$,

$\mu_4=m_{46}+m_{47}=\frac{1}{\beta}(1-g^*(\beta)$, $\mu_5=m_{56}+m_{59}=\frac{1}{\beta+\lambda_1}$, $\mu_6=m_{60}+m_{6,10}+m_{6,11}=\frac{1}{\beta+\lambda}(1-g_1^*(\beta+\lambda))$,

$\mu_7=m_{78}=\frac{1}{\beta_1}$, $\mu_8=m_{86}+m_{87}=\frac{1}{\beta}(1-g^*(\beta))$, $\mu_9=m_{95}=\frac{1}{\beta_1}$, $\mu_{10}=m_{10,6}=\frac{1}{\beta_1}$,

$\mu_{11}=m_{11,1}+m_{11,12}=\frac{1}{\beta}(1-g_1^*(\beta))$, $\mu_{12}=m_{12,13}=\frac{1}{\beta_1}$, $\mu_{13}=m_{13,1}+m_{13,12}=\frac{1}{\beta}(1-g^*_1(\beta))$,

$$\mu'_1=m_{13}+m_{15}+m_{16.4}+m_{16.4,(7,8)}{}^n=\frac{(1-g^*(\beta+\lambda_1))(\beta\beta_1 g^*(\beta)+\lambda_1(\beta+\beta_1)(1-g^*(\beta)))}{\beta\beta_1(\beta+\lambda_1)g^*(\beta)},$$

$$\mu'_6=m_{60}+m_{61.11}+m_{61.11,(12,13)}{}^n+m_{6,10}=\frac{(1-g_1^*(\beta+\lambda))(\beta\beta_1 g_1^*(\beta)+\lambda(\beta+\beta_1)(1-g_1^*(\beta)))}{\beta\beta_1(\beta+\lambda)g_1^*(\beta)}$$

…(3)

## VI. RELIABILITY AND MEAN TIME TO SYSTEM FAILURE (MTSF)

Let $\emptyset_i(t)$ be the cdf of first passage time from regenerative state $S_i$ to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\emptyset_i(t)$:

$$\phi_i(t)=\sum_j Q_{i,j}(t)ⓢ\phi_j(t)+\sum_j Q_{i,j}(t) \quad …(4)$$

Where Sj is an un-failed regenerative state to which the given regenerative state $S_i$ can transit and $S_k$ is a failed state to which the state $S_i$ can transit directly.

Taking LST of the above relations (4) and solving

$$\phi_0^{**}(s) = \frac{|N_{11}(s)|}{|D_{11}(s)|}$$

$$D_{11}(s) = \begin{pmatrix} 1 & -Q_{01}^{**}(s) & -Q_{02}^{**}(s) & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -Q_{13}^{**}(s) & -Q_{15}^{**}(s) & 0 & 0 & 0 \\ -Q_{20}^{**}(s) & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -Q_{31}^{**}(s) & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -Q_{56}^{**}(s) & -Q_{59}^{**}(s) & 0 \\ -Q_{60}^{**}(s) & 0 & 0 & 0 & 0 & 1 & 0 & -Q_{6,10}^{**}(s) \\ 0 & 0 & 0 & 0 & -Q_{95}^{**}(s) & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -Q_{10,6}^{**}(s) & 0 & 1 \end{pmatrix}$$

$$N_{11}(s) = \begin{pmatrix} 0 & -Q_{01}^{**}(s) & -Q_{02}^{**}(s) & 0 & 0 & 0 & 0 & 0 \\ Q_{14}^{**}(s) & 1 & 0 & -Q_{13}^{**}(s) & -Q_{15}^{**}(s) & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -Q_{31}^{**}(s) & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -Q_{56}^{**}(s) & -Q_{59}^{**}(s) & 0 \\ Q_{6,11}^{**}(s) & 0 & 0 & 0 & 0 & 1 & 0 & -Q_{6,10}^{**}(s) \\ 0 & 0 & 0 & 0 & -Q_{95}^{**}(s) & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -Q_{10,6}^{**}(s) & 0 & 1 \end{pmatrix}$$

we have $R^*(s) = \frac{1-\phi_S^{**}(s)}{s}$ …(5)

The reliability of the system can be obtained by taking inverse Laplace transform of (5).

The mean time to system failure (MTSF) is given by

$$MTSF = \lim_{s \to 0} \frac{1-\phi_0^{**}(s)}{s} = \frac{N_1}{D_1}$$

where

$N_1 = p_{56}(1-p_{6,10})((1-p_{13})(\mu_0+p_{02}\mu_2) + p_{01}(\mu_1+p_{13}\mu_3))+p_{01}p_{15}((1-p_{6,10})(\mu_5+p_{59}\mu_9)$

$+ p_{56}(\mu_6+p_{6,10}\mu_{10}))$

$D_1 = p_{01}p_{56}((1-p_{6,10})(1-p_{13})-p_{15}p_{60})$

## VII. STEADY STATE AVAILABILITY

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state $S_i$ at t=0.

The recursive relations for $A_i(t)$ are given as:

$A_i(t) = M_i(t) + \sum_j q_{i,j}^{(n)}(t) © A_j(t)$ …(6)

Where $S_j$ is any successive regenerative state to which the regenerative state $S_i$ can transit through n transitions $M_i(t)$ is the probability that the system is in up state initially in the state $S_i \in E$ up at time t without visiting to any other regenerative state, we have

$M_0(t)=e^{-(\beta+\lambda)t}$ ,$M_1(t)=e^{-(\beta+\lambda_1)t}\overline{G(t)}$ ,$M_5(t)= e^{-(\beta+\lambda_1)t}$ ,$M_6(t)= e^{-(\beta+\lambda)t}\overline{G_1(t)}$

Taking LT of above relations (6),

$$D_{21}(s) = \begin{pmatrix} 1 & -q_{01}^*(s) & -q_{02}^*(s) & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -q_{13}^*(s) & -q_{15}^*(s) & -(q_{16.4}^*(s) + q_{16.4((7,8)^n}^*(s)) & 0 & 0 \\ -q_{20}^*(s) & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -q_{31}^*(s) & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -q_{56}^*(s) & -q_{59}^*(s) & 0 \\ -q_{60}^*(s) & -(q_{61.11}^*(s) + q_{61.11((12,13)^n}^*(s)) & 0 & 0 & 0 & 1 & 0 & -q_{6,10}^*(s) \\ 0 & 0 & 0 & 0 & -q_{95}^*(s) & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -q_{10,6}^*(s) & 0 & 1 \end{pmatrix}$$

$$N_{21}(s) = \begin{pmatrix} M_0^*(s) & -q_{01}^*(s) & -q_{02}^*(s) & 0 & 0 & 0 & 0 & 0 \\ M_1^*(s) & 1 & 0 & -q_{13}^*(s) & -q_{15}^*(s) & -(q_{16.4}^*(s) + q_{16.4((7,8)^n}^*(s)) & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -q_{31}^*(s) & 0 & 1 & 0 & 0 & 0 & 0 \\ M_5^*(s) & 0 & 0 & 0 & 1 & -q_{56}^*(s) & -q_{59}^*(s) & 0 \\ M_6^*(s) & -(q_{61.11}^*(s) + q_{61.11((12,13)^n}^*(s)) & 0 & 0 & 0 & 1 & 0 & -q_{6,10}^*(s) \\ 0 & 0 & 0 & 0 & -q_{95}^*(s) & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -q_{10,6}^*(s) & 0 & 1 \end{pmatrix}$$

We have $A_0^*(s) = \frac{|N_{21}(s)|}{|D_{21}(s)|}$

The steady state availability is given by $A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2}$

$N_2 = p_{56}(1-p_{13})(p_{60}\mu_0+p_{01}\mu_6)+p_{01}(1-p_{6,10})(p_{56}\mu_1+p_{15}\mu_5)$

$D_2 = p_{56}(1-p_{13})(p_{60}\mu_0+p_{01}\mu_6'))+p_{01}(1-p_{6,10})(p_{56}\mu_1'+p_{15}\mu_5)$

## VIII. BUSY PERIOD ANALYSIS OF THE SERVER

Let $B_i(t)$ be the probability that the server is busy in repairing the unit at an instant 't' given that the system entered regenerative state $S_i$ at t=0.The recursive relations for $B_i(t)$ are as follows:

$B_i(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) © B_j(t)$ …(7)

where $W_i(t)$ be the probability that the server is busy in state $S_i$ due to failure up to time t without making any transition to any other regenerative state or returning to the same via one or more non regenerative states

so, $W_1(t) = e^{-(\beta+\lambda_1)t}\,\overline{G(t)} + (\lambda_1 e^{-(\beta+\lambda_1)t} ©1)\,\overline{G(t)}$

$W_6(t) = e^{-(\beta+\lambda)t}\,\overline{G_1(t)} + (\lambda e^{-(\beta+\lambda)t} ©1)\,\overline{G_1(t)}$

Taking L.T, and solving $B_0^*(s) = \frac{|B_{31}(s)|}{|D_{21}(s)|}$

$$B_{31}(s) = \begin{pmatrix} 0 & -q_{01}^*(s) & -q_{02}^*(s) & 0 & 0 & 0 & 0 & 0 \\ W_1^*(s) & 1 & 0 & -q_{13}^*(s) & -q_{15}^*(s) & -(q_{16.4}^*(s) + q_{16.4((7,8)^n}^*(s)) & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -q_{31}^*(s) & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -q_{56}^*(s) & -q_{59}^*(s) & 0 \\ W_6^*(s) & -(q_{61.11}^*(s) + q_{61.11((12,13)^n}^*(s)) & 0 & 0 & 0 & 1 & 0 & -q_{6,10}^*(s) \\ 0 & 0 & 0 & 0 & -q_{95}^*(s) & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -q_{10,6}^*(s) & 0 & 1 \end{pmatrix}$$

The time for which server is busy due to repair is given by $B_0^*(\infty) = \lim_{s\to 0} sB_0^*(s) = \frac{N_3}{D_2}$

where

$N_3 = p_{01}p_{56}(W_1^*(0)(1-p_{6,10}) + W_6^*(0)(1-p_{13}))$ and $D_2$ is already defined

## IX. EXPECTED NUMBER OF VISITS BY THE SERVER

Let $N_i(t)$ be the expected number of visits by the server in (0,t] given that the system entered the regenerative state $S_i$ at t=0. The recursive relations for $N_i(t)$ are given as

$$N_i(t) = \sum_j Q_{i,j}^{(n)}(t)\,Ⓢ[\delta_j + N_j(t)] \quad \text{…(8)}$$

Where $S_j$ is any regenerative state to which the given regenerative state $S_j$ transits and $\delta_j = 1$, if $S_j$ is the regenerative state where the server does the job afresh, otherwise $\delta_j$=0.

Taking LST and solving $N_0^{**}(s) = \frac{|N_{41}(s)|}{|D_{41}(s)|}$

$$N_{41}(s) = \begin{pmatrix} Q_{01}^{**}(s) & -Q_{01}^{**}(s) & -Q_{02}^{**}(s) & 0 & 0 & 0 & 0 & 0 \\ (Q_{16.4((7,8)^n}^{**}(s)) & 1 & 0 & -Q_{13}^{**}(s) & -Q_{15}^{**}(s) & -(Q_{16.4}^{**}(s) + Q_{16.4((7,8)^n}^{**}(s)) & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ Q_{31}^{**}(s) & -Q_{31}^{**}(s) & 0 & 1 & 0 & 0 & 0 & 0 \\ Q_{56}^{**}(s) & 0 & 0 & 0 & 1 & -Q_{56}^{**}(s) & -Q_{59}^{**}(s) & 0 \\ Q_{61.11,((12,13)^n}^{**}(s) & -(Q_{61.114}^{**}(s) + Q_{61.11,((12,13)^n}^{**}(s)) & 0 & 0 & 0 & 1 & 0 & -Q_{6,10}^{**}(s) \\ 0 & 0 & 0 & 0 & -Q_{95}^{**}(s) & 0 & 1 & 0 \\ Q_{10,6}^{**}(s) & 0 & 0 & 0 & 0 & -Q_{10,6}^{**}(s) & 0 & 1 \end{pmatrix}$$

$$D_{41}(s) = \begin{pmatrix} 1 & -Q_{01}^{**}(s) & -Q_{02}^{**}(s) & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -Q_{13}^{**}(s) & -Q_{15}^{**}(s) & -(Q_{16.4}^{**}(s) + Q_{16.4((7,8)^n}^{**}(s)) & 0 & 0 \\ -Q_{20}^{**}(s) & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -Q_{31}^{**}(s) & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -Q_{56}^{**}(s) & -Q_{59}^{**}(s) & 0 \\ -Q_{6,0}^{**}(s) & -(Q_{61.114}^{**}(s) + Q_{61.11,((12,13)^n}^{**}(s)) & 0 & 0 & 0 & 1 & 0 & -Q_{6,10}^{**}(s) \\ 0 & 0 & 0 & 0 & -Q_{95}^{**}(s) & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -Q_{10,6}^{**}(s) & 0 & 1 \end{pmatrix}$$

The expected numbers of visits per unit time by the server are given by

$$N_0(\infty) = \lim_{s \to 0} sN_0^{**}(s) = \frac{N_4}{D_2}$$

Where

$N_4 = p_{01}p_{56}((1-p_{6,10})(1+p_{15}-p_{14}p_{47})-(1-p_{13})p_{6,11}p_{11,1})$ and $D_2$ is already specified.

## X. PROFIT ANALYSIS

The profit incurred to the system model in steady state can be obtained as

$P = K_0A_0 - K_1B_0 - K_2N_0$

where

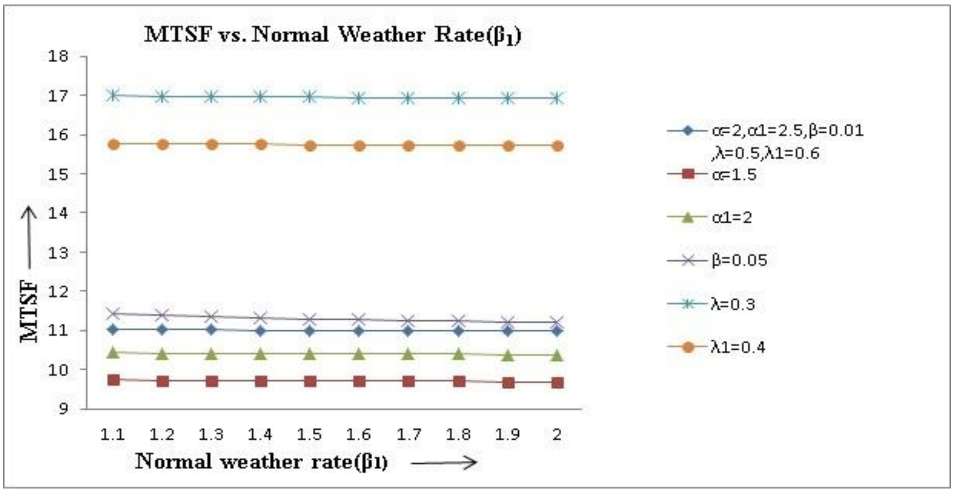
$K_0$=Revenue per unit up-time of the system

$K_1$=Cost per unit for which server is busy

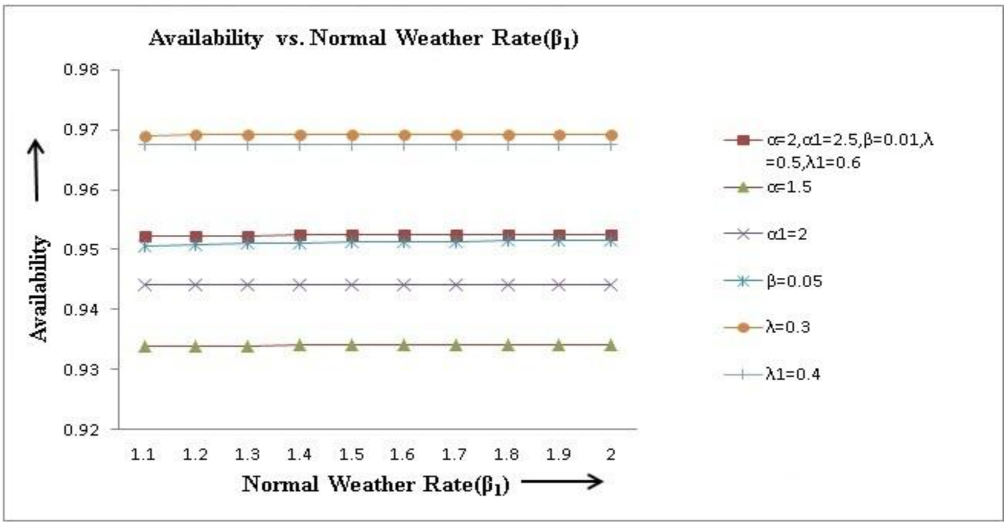
$K_2$= Cost per unit visit by the server and $A_0$, $B_0$, $N_0$ are already defined.

## XI. GRAPHS

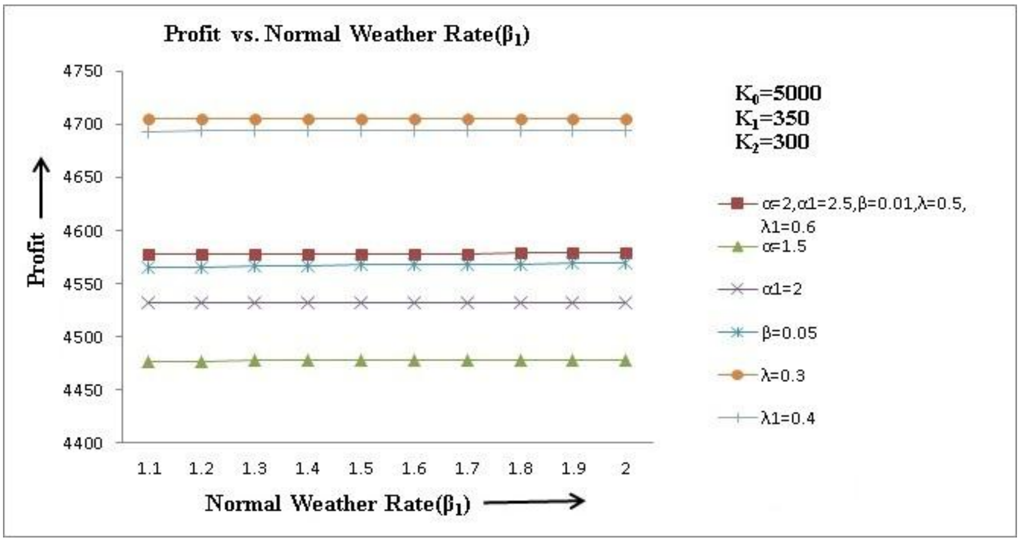
**Graph 1: MTSF vs. Normal Weather Rate ($\beta_1$)**



**Graph 2: Availability vs. Normal Weather Rate ($\beta_1$)**




**Graph 3: Profit vs. Normal weather rate ($\beta_1$)**




## XII. CONCLUSION

**Graph 1** indicates that MTSF decreases with failure rates ($\lambda$, $\lambda_1$) of the units for fixed values of the other parameters. It shows downward trend with increase in normal weather rate ($\beta_1$). It indicates upwards trend with increase of abnormal weather rate ($\beta$). Also, with increase in repair rates of the units ($\alpha$, $\alpha_1$), it also increases. of the units.

**Graph 2** indicates that availability function keeps on increasing with normal weather rate ($\beta_1$), while it shows downward trend with increase of abnormal weather rate ($\beta$) for fixed values of the parameters.

**Graph 3** indicates that the numerical results giving particular values to the various parameters and cost are obtained to depict the behavior profit functions reveals that it increasing as normal weather rate ($\beta_1$) and repair rates ($\alpha$, $\alpha_1$) increase while their values decline with the increase of abnormal weather rate ($\beta$) and failure rates ($\lambda$, $\lambda_1$) of the units.

**Main Findings** In conclusion, this research paper has delved into the critical field of Reliability Modelling and Analysis. On the basis of the results obtained for a particular case, it is interpreted that a system of non-identical units which is not allowed to operate in abnormal weather conditions can be made more available and profitable to use either by providing normal weather for operation or by providing better repair facilities like calling server of high repair rates.

**Note** The study of Reliability Modelling and Analysis is an ongoing and evolving endeavor, and this research contributes to the body of knowledge in this field. I encourage further exploration and experimentation to refine existing models, develop new techniques, and adapt to the ever-changing landscape of technology and industry. Through continued collaboration and innovation, we can collectively work towards a more reliable and resilient future.

## XIII. Notations

| | |
|---|---|
| E | The set of regenerative states |
| $\lambda/\ \lambda_1$ | Constant failure rate of Main /Duplicate unit |
| $\beta/\ \beta_1$ | Constant rate of change of weather from normal to abnormal/abnormal to normal weather |
| g(t)/G(t) | pdf/cdf of repair time of Main unit |
| $g_1(t)/\ G_1(t)$ | pdf/cdf of repair time of Duplicate unit |
| $q_{ij}\ (t)/Q_{ij}\ (t)$ | pdf/cdf of passage time from regenerative state i to regenerative state j or to a failed state j without visiting any other regenerative state in (0,t] |
| $q_{ij.kr}\ (t)/\ Q_{ij.kr}\ (t)$ | pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k,r once in (0,t] |
| $q_{ij.k,(r,s)}{}^{n}(t)/Q_{ij.k,(r,s)}{}^{n}(t)$ | pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k once and n times states r and s. |
| $M_i(t)$ | Probability that the system is up initially in regenerative state Si at time t without visiting to any other regenerative state |
| $W_i(t)$ | Probability that the server is busy in state Siupto time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states |

| | |
|---|---|
| $m_{ij}$ | The conditional mean sojourn time in regenerative state Si when system is to make transition in to regenerative state Sj. Mathematically, it can be written as $m_{ij} = E(T_{ij}) = \int_0^\infty td[Q_{ij}(t)] = -q_{ij}^{*'}(0)$, where $T_{ij}$ is the transition time from state Si to Sj; Si, Sj ε E. |
| $\mu_i$ | The mean Sojourn time in state Si this is given by $\mu_i = E(T_i) = \int_0^\infty P(T_i > t)dt = \sum_j m_{ij}$, where $T_i$ is the sojourn time in state Si. |
| Ⓢ/©/©$^n$ | Symbol for Laplace Stieltjes convolution/Laplace convolution/Laplace convolution n times |
| ** / * | Symbol for Laplace Steiltjes Transform (L.S.T.)/ Laplace transform (L.T.) |
| ´(desh) | Used to represent alternative result |

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