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Application of Laplace Decomposition Method to Boundary Value Equation in a Semi-Infinite Domain

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ABSTRACT

In this research work, Laplace Decomposition Method (LDM) is applied to solve some nonlinear boundary value equations involving boundary condition at infinity. Adomian polynomial is used to decompose nonlinear functions that exist in a given equation. Semi-analytical solutions were obtained for each of the problem considered. The results obtained showed that the present method is efficient and the solution procedure is very suitable for the type of infinite domain problem considered and the results are in close agreement with other method in the literature.

Keywords: Infinite domain, Laplace Decomposition Method, Adomian polynomial.

1. INTRODUCTION

Most problems in natural and engineering sciences are modeled in linear and non linear differential equations, such as: fluid flow and heat flow in an infinite region. These set of problems are usually subjected to boundary conditions at infinity. Many authors have developed numerical methods for approximating boundary value equations in an infinite domain, among which are: Odejide and Aregbesola [15], where the method of weighted residual was used to solve problems in semi infinite domain. Oderinu and Aregbesola [16], used the Laguerres Quadrature in weighted Residual method for problem with semi-infinite domain, natural roots of Laguerre polynomials were use for both Partition and Collocation points in the weighted residual method for solving boundary value problem involving semi-infinite domain. The decomposition method has been shown to solve [1-11] efficiently, easily and accurately a large class of linear and nonlinear ordinary, partial, deterministic or stochastic differential equations. Laplace Decomposition Method (LDM) was first introduced by Khuri [12], and has been successfully used to find solutions to several ordinary, partial differential equations. Yusufoglu [19] applied this method successfully to find the exact solution to Bratu and Duffing equations. This numerical technique basically illustrates how the Laplace transform can be used to approximate the solutions of the nonlinear differential equations by manipulating the decomposition method which was first introduced by Adomian [18]. In this paper Laplace Decomposition Method (LDM) will be used to solve certain fluid dynamics problems with boundary value at infinity. The non-linear terms in the problems were decomposed using Adomian polynomials and respective iterative algorithms were developed for the determination of the successive terms of series solution in a recursive manner. The resulting systems of equations were then solved based on the boundary conditions using Maple 18.0 software. Pade approximation was used to express the solution in a rational form so that the limit at ∞ could be obtained.

2. THE METHOD OF LAPLACE DECOMPOSITION

Considering the initial Value Problem

$$y'' + a(x)y' + b(x)y = f(y), \tag{1}$$

$$y(0) = \alpha, \; y'(0) = \beta. \tag{2}$$

Here $f(y)$ is a nonlinear operator and $a(x)$ and $b(x)$ are known functions in the underlying function space. The technique involves applying Laplace transformation (denoted throughout this write up by ($\mathcal{L}$) to both sides of equation (1), That is

$$\mathcal{L}[y''] + \mathcal{L}[a(x)y'] + \mathcal{L}[b(x)y] = \mathcal{L}[f(y)] \tag{3}$$

Introducing the formulas of Laplace transform to equation (3), this gives

$$s^2\,\mathcal{L}[y] - y(0)s - y'(0) + \mathcal{L}[a(x)y] + \mathcal{L}[b(x)y] = \mathcal{L}[f(y)] \tag{4}$$

Substituting the initial conditions in equation (2) to equation (4), this gives

$$\mathcal{L}[y] = \frac{\propto}{s} + \frac{\beta}{s^2} - \frac{1}{s^2}\mathcal{L}\,[a(x)y'] - \frac{1}{s^2}\mathcal{L}[b(x)y] + \frac{1}{s^2}\mathcal{L}[f(y)] \tag{5}$$

The Laplace transform decomposition technique involves representing the solution as an infinite series,

$$y = \sum_{n=0}^{\infty} y_n, \tag{6}$$

Where the terms y_n are to be recursively computed. Also the nonlinear operator $f(y)$

is decomposed as follows:

$$f(y) = \sum_{n=0}^{\infty} A_n \tag{7}$$

Where $A_n = (y_1, y_2, y_3, \dots, y_n)$ is the so-called Adomian polynomial.

The first few polynomials are given by

$$A_0 = f(y_0)$$

$$A_1 = y_1 f^{(1)}(y_0)$$

$$A_2 = y_2 f^{(1)}(y_0) + \frac{1}{2!} {y_1}^2 f^{(2)}(y_0),$$

$$A_3 = y_3 f^{(1)}(y_0) + y_1 y_2 f^{(2)}(y_0) + \frac{1}{3!} {y_1}^3 f^{(3)}(y_0), \tag{8}$$

We obtained the above equation (8) from the Adomian polynomial formula which is

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[\mathrm{N} \left(\sum_{n=0}^{\infty} \lambda^j y_j \right) \right]_{\lambda=0} \qquad n\text{=0, 1, 2, 3, - - -}$$

Substituting equation (6) and equation (7) into equation (5) results to

$$\mathcal{L}\left[\sum_{n=0}^{\infty} y_n\right] = \frac{\propto}{s} + \frac{\beta}{s^2} - \frac{1}{s^2}\mathcal{L}\left[a(x)\sum_{n=0}^{\infty} y_n\right] - \frac{1}{s^2}\mathcal{L}\left[b(x)\sum_{n=0}^{\infty} y_n\right] + \frac{1}{s^2}\mathcal{L}\left[\sum_{n=0}^{\infty} A_n\right] \tag{9}$$

Using the linearity of Laplace transform it follows that

$$\sum_{n=0}^{\infty} \mathcal{L}[y_n] = \frac{\propto}{s} + \frac{\beta}{s^2} - \frac{1}{s^2}\sum_{n=0}^{\infty} \mathcal{L}[a(x)y'_n] - \frac{1}{s^2}\sum_{n=0}^{\infty} \mathcal{L}[b(x)y_n] - \frac{1}{s^2}\sum_{n=0}^{\infty} \mathcal{L}[A_n]. \tag{10}$$

Equating both sides of equation (10) yields the following iterative algorithm:

$$\mathcal{L}[y_0] = \frac{\propto}{s} + \frac{\beta}{s^2}, \tag{11}$$

$$\mathcal{L}[y_1] = \frac{1}{s^2}\mathcal{L}[a(x)y'_0] - \frac{1}{s^2}\mathcal{L}[b(x)y_0] + \frac{1}{s^2}\mathcal{L}[A_0] \tag{12}$$

$$\mathcal{L}[y_2] = \frac{1}{s^2}\mathcal{L}[a(x)y'_1] - \frac{1}{s^2}\mathcal{L}[b(x)y_1] + \frac{1}{s^2}\mathcal{L}[A_1] \tag{13}$$

In general,

$$\mathcal{L}[y_{n+1}] = -\frac{1}{s^2}\mathcal{L}[a(x)y'_n] - \frac{1}{s^2}\mathcal{L}[b(x)y_n] + \frac{1}{s^2}\mathcal{L}[A_n] \qquad (14)$$

Applying the inverse Laplace transform to equation (11) gives

$$y_0 = \alpha + \beta x. \qquad (15)$$

Substituting the value of y_0 in equation (15) into equation (12), gives

$$\mathcal{L}[y_1] = -\frac{1}{s^2}\mathcal{L}[\beta a(x)] - \frac{1}{s^2}\mathcal{L}[b(x)(\alpha + \beta x)] + \frac{1}{s^2}\mathcal{L}[A_0] \qquad (16)$$

Evaluating the Laplace transform of the quantities on the right-hand side of equation (16) then applying the inverse Laplace transform, then $y_1$is obtained. The other iterative terms y_2, y_3,. . . can be obtained recursively in a similar scheme using equation (14).

3. APPLICATIONS

Example 1

Consider the differential equation arising from the fluid dynamics [3], given by

$$f''' + ff'' - (D + Re)f' - (1 - \alpha)f'^2 = 0, \qquad (17)$$

Subject to $f(0) = 0,\ f'(0) = 1,\ f'(\infty) = 0$ assume $f''(0) = a$ (18)

Where D=1, Re=0, $\alpha = 0$

With analytical solution $f = \frac{1 - e^{-\eta}\sqrt{1+D+\mathcal{R}}}{\sqrt{1+D+\mathcal{R}}}$ when $\alpha = 0$

Consider the case where D=1, Re=0, $\alpha = 0$

Therefore, equation (17) becomes

$$f''' + ff'' - f' - f'^2 = 0 \qquad (19)$$

And operating the Laplace transform to both sides of equation (19) this gives

$$s^3\mathcal{L}[f] - s^2 f(0) - sf'(0) - f''(0) + \mathcal{L}[ff'] - s\mathcal{L}[f] - f(0) - \mathcal{L}\left[f'^2\right] = 0 \qquad (20)$$

Simplifying equation (20) and applying the boundary conditions in equation (18) this yields

$$\mathcal{L}[f] = \frac{1}{(s^2-1)} + \frac{a}{(s^3-s)} + \frac{1}{(s^3-s)}\left[-\mathcal{L}[ff'] + \mathcal{L}\left[f'^2\right]\right] \qquad (21)$$

If we assume an infinite series solution of the form of equation (6) yields

$$\mathcal{L}\left[\sum_{n=0}^{\infty} f_n\right] = \frac{1}{(s^2-1)} + \frac{a}{(s^3-s)} - \frac{1}{(s^3-s)}\mathcal{L}\left[\sum_{n=0}^{\infty} A_n\right] + \frac{1}{(s^3-s)}\mathcal{L}\left[\sum_{n=0}^{\infty} B_n\right] \tag{22}$$

Where the nonlinear term operator $f(y) = ff''$ and ${f'}^2$ are decomposed from Adomian polynomials, for the first nonlinear operator

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\left(\sum_{n=0}^{\infty} \lambda^j f_j\right)\right)\left(\left(\sum_{n=0}^{\infty} \lambda^j {f''}_j\right)\right)\right]_{\lambda=0} \qquad n=0,\ 1,\ 2,\ 3,\ ---$$

And the second nonlinear operator

$$B_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[\mathrm{N}\left(\sum_{n=0}^{\infty} \lambda^j {f'}_j\right)^2\right]_{\lambda=0} \qquad n=0,\ 1,\ 2,\ 3,\ ---$$

The first few Adomian polynomials for $f(y) = ff''$ is given by

$$A_0 = f_0{f_0}''$$

$$A_1 = f_0{f_1}'' + f_1{f_0}''$$

$$A_2 = f_0{f_2}'' + f_1{f_1}'' + f_2{f_0}''$$

$$A_3 = f_0{f_3}'' + f_1{f_2}'' + f_2{f_1}'' + f_3{f_0}'' \tag{23}$$

And the second Adomian polynomials $f(y) = {f'}^2$ are decomposed as

$$B_0 = {f_0}'^2$$

$$B_1 = 2{f_0}'{f'}_1$$

$$B_2 = 2{f_0}'{f'}_2 + {f_1}'^2$$

$$B_3 = 2{f_0}'{f'}_3 + 2{f_1}'{f_2}'$$

$$\ldots.. \tag{24}$$

Upon using the linearity of Laplace transform, then equating both sides of equation (22), results in the iterative scheme

$$\mathcal{L}[f_0] = \frac{1}{(s^2-1)} + \frac{a}{(s^3-s)} \tag{25}$$

$$\mathcal{L}[f_1] = \frac{1}{(s^3-s)}\mathcal{L}[A_0] - \frac{1}{(s^3-s)}\mathcal{L}[B_0] \tag{26}$$

$$\mathcal{L}[f_2] = \frac{1}{(s^3-s)}\mathcal{L}[A_1] - \frac{1}{(s^3-s)}\mathcal{L}[B_1] \tag{27}$$

In general,

$$\mathcal{L}[y_{n+1}] = \frac{1}{(s^3-s)}\mathcal{L}[A_n] - \frac{1}{(s^3-s)}\mathcal{L}[B_n] \tag{28}$$

Operating the inverse Laplace transform to equation (25), this gives

$$f_0 = \sin(\eta) + a(-1 + \cos(\eta)) \tag{29}$$

Substituting equation (30) and $A_0 = f_0 f_0{}'$ and $B_0 = f_0{}'^2$ into equation (26) yields

$$\mathcal{L}[f_1] = \frac{1}{(s^3-s)}\mathcal{L}[f_0 f_0{}'] + \frac{1}{(s^3-s)}\mathcal{L}\left[f_0{}'^2\right] \tag{30}$$

Operating Laplace inverse on both sides of equation (30), this gives

$$\begin{aligned} f_0 = {} & \frac{1}{2} - \frac{1}{8}\eta + \frac{a^2}{8(e^\eta)^2} - \frac{a}{(e^\eta)^2} + \frac{1}{8(e^\eta)^2} + \frac{3a^2\eta}{4e^\eta} + \frac{13a^2}{12e^\eta} - \frac{3a\eta}{4e^\eta} - \frac{a}{3e^\eta} - \frac{2}{3e^\eta} \\ & + \frac{1}{24}(e^\eta)^2 a^2 + \frac{1}{24}(e^\eta)^2 a + \frac{1}{2}a^2\eta - \frac{1}{2}a^2\eta - \frac{1}{2}a^2 - \frac{1}{4}e^\eta a^2\eta \\ & + \frac{1}{4}e^\eta a^2 - \frac{1}{4}e^\eta a\eta + \frac{1}{2}a \quad (31) \end{aligned}$$

and consequently, other $f_i, i = 2{,}3 \ldots$ were obtained in a similar manner.

Therefore the semi analytical solution is

$$f = f_0 + f_1 + f_2 + f_3$$

$$f = \sum_{i=0}^{n} f_i \;, n = 3. \tag{32}$$

Now, in order to find the value of a, apply the condition

$$\lim_{\eta\to\infty}(f'(\eta)) = 0 \tag{33}$$

which is obtained from boundary condition at infinity,

$f'(\infty) = 0$. Apply Pade approximation [2,2] on the derivative of the solution, that is on f'.

$$\lim_{\eta\to\infty} \frac{4a - 3 + \left(a^2 - \frac{5}{2}a + \frac{1}{2}a^3 + 2\right)\eta + \left(\frac{11}{2}a - \frac{37}{12}3a^3 + \frac{1}{4}a^2 + \frac{1}{2}a^4\right)\eta^2}{4a - 3 + \left(-3a^2 + \frac{1}{2}a + \frac{1}{2}a^3 + 2\right)\eta + \left(-\frac{1}{4}a^2 + \frac{3}{2}a - \frac{19}{12}\right)\eta^2}$$

$$= \frac{\frac{11}{2}a - \frac{37}{12} - 3a^3 + \frac{1}{4}a^2 + \frac{1}{2}a^4}{-\frac{1}{4}a^2 + \frac{3}{2}a - \frac{19}{12}} \tag{34}$$

solving equation (34) implies that

$$\frac{\frac{11}{2}a-\frac{37}{12}-3a^3+\frac{1}{4}a^2+\frac{1}{2}a^4}{-\frac{1}{4}a^2+\frac{3}{2}a-\frac{19}{12}}=0,$$

gives $a=-1.407608388$

Table 1 Show the results of analytical solution and semi-analytical solution of fluid problem using Laplace Decomposition Method

X	Exact	LDM	Error
0.1	0.0932508061	0.09270959264	0.00054121346
0.2	0.1742040171	0.1715821539	0.0026218632
0.3	0.2444813976	0.2377984044	0.0066829932
0.4	0.3054908392	0.2925589374	0.0129319018
0.5	0.3584545659	0.3370102857	0.0214442802
0.6	0.4044336189	0.3721829761	0.0322506428
0.7	0.4443491127	0.3989422344	0.0454068783
0.8	0.4790006888	0.4179521392	0.0610485496
0.9	0.5090825343	0.4296541053	0.0794284290
1.0	0.5351972898	0.4342604945	0.1009367953

Table 1 above shows the results obtained through the use of LDM and Adomian decomposition method to decompose the nonlinear terms, the result is found to be more efficient than other methods for the type of problem considered and converge rapidly to the exact solution. Laplace Decomposition method is simple and expresses its solution in a series form which allows the solution at any point of interest to be obtained. This method produces solutions in series form which is a series analytic form of solutions that can be evaluated at any point in the domain of the problem.

Example 2

$f'''+\sigma\left(ff''-\beta f'^{2}\right)=0,\qquad \beta=\frac{2m}{(m+1)},\ \ \sigma=\pm$ (Magyari, Ali and Keller [13]) (35)

With boundary conditions

$f(0)=-5,\ \ f'(0)=1, f'(\infty)=0\ assume\ f''(0)=\alpha$ (36)

Equation (35) was modelled by Magyari *et al* [13] and in the article shooting method was used to solve the equation. Here the method of Laplace Decomposition will be used to solve the same problem.

First, in the ranges $m>-1$ of the stretching exponents. The stretching velocities may be classified as “slowly decreasing” for $-1 < m < 0$, “slowly increasing” for $0 < m < 1$, “linearly increasing” for $m = 1$ and as “rapidly increasing” ones for $m > 1$. The value $m = 0$ corresponds obviously to the uniform moving rigid.

First, in this paper let $m < -1$. Where m is called stretching exponent.

Case one, let $m = -2$, therefore $\beta = 4$, hence the equation (35) becomes

$$f''' + \left(ff'' - 4f'^2\right) = 0 \tag{37}$$

Now, taking the Laplace transform of both sides of equation (37) yields

$$s^3\mathcal{L}[f] - s^2 f(0) - sf'(0) - f''(0) = 4\mathcal{L}\left[f'^2\right] - \mathcal{L}[ff''] \tag{38}$$

If we assume an infinite series solution, yields

$$\mathcal{L}\left[\sum_{n=0}^{\infty} f_n\right] = \frac{-5}{s} + \frac{1}{s^2} + \frac{\propto}{s^3} - \frac{1}{s^3}\mathcal{L}\left[\sum_{n=0}^{\infty} A_n\right] + \frac{4}{s^3}\mathcal{L}\left[\sum_{n=0}^{\infty} B_n\right] \tag{39}$$

The first few components A_n and B_n in equation (39) are in equation (22) and equation (23) respectively.

Upon using the linearity of Laplace transform then equating both sides of equation (39), results in the iterative scheme

$$\mathcal{L}[f_0] = \frac{-5}{s} + \frac{1}{s^2} + \frac{\propto}{s^3} \tag{40}$$

$$\mathcal{L}[f_1] = -\frac{1}{s^3}\mathcal{L}[A_0] + \frac{4}{s^3}\mathcal{L}[B_0] \tag{41}$$

In general

$$\mathcal{L}[f_{n+1}] = -\frac{1}{s^3}\mathcal{L}[A_n] + \frac{4}{s^3}\mathcal{L}[B_n] \tag{42}$$

Now applying the Laplace inverse on both sides of equation (40), this gives

$$f_0 = -5 + x + \frac{1}{2}\alpha x^2 \tag{43}$$

and consequently, we have

$$\mathcal{L}[f_1] = -\frac{5\alpha}{s^4} + \frac{\alpha^2}{s^6} + \frac{41\alpha}{s^5} + \frac{40}{s^4} - \frac{24\alpha}{s^6} - \frac{8}{s^5} - \frac{24\alpha^2}{s^7} \tag{44}$$

Taking the inverse Laplace of equation (44), yield

$$f_1 = -\frac{1}{30}\alpha^2 x^6 + \frac{1}{120}\alpha^2 x^5 - \frac{1}{5}\alpha x^5 + \frac{41}{24}\alpha x^4 - \frac{5}{6}\alpha x^3 - \frac{1}{3}x^4 + \frac{20}{3}x^3 \tag{45}$$

$$\begin{aligned} f_2 = {} & \frac{2}{945}\alpha^3 x^9 - \frac{29}{40320}\alpha^3 x^8 + \frac{2}{105}\alpha^2 x^8 - \frac{101}{560}\alpha^2 x^7 + \frac{19}{144}\alpha^2 x^6 \\ & + \frac{37}{630}\alpha x^7 - \frac{59}{48}\alpha x^6 - \frac{35}{24}\alpha x^5 + \frac{1}{18}x^6 + \frac{25}{24}\alpha x^4 - \frac{5}{3}x^5 - \frac{25}{3}x^4 \end{aligned} \tag{46}$$

Therefore the semi analytical solution is

$$f = f_0 + f_1 + f_2 + \cdots$$

$$\begin{aligned} f = {} & -5 + x - \frac{7821313}{332107776000}\alpha^5 x^{16} + \frac{1844911}{1307674368000}\alpha^5 x^{15} + \frac{16680593}{10897286400}\alpha^4 x^{16} \\ & + \frac{1607}{5811886080}\alpha^5 x^{14} - \frac{9056227}{26153487360}\alpha^4 + \frac{5548120763}{163459296000}\alpha^3 x^{15} \\ & - \frac{825351899}{14529715200}\alpha^3 x^{14} + \frac{17777254301}{5448643200}\alpha^2 x^{14} - \frac{114686783}{177914880}\alpha^2 x^{13} \\ & + \frac{543815603}{389188800}\alpha x^{13} - \frac{1053348463}{479001600}\alpha x^{12} - \frac{53023}{4536}x^9 + \frac{10777}{1008}x^8 + \frac{25}{7}x^7 \\ & - \frac{2257}{362880}\alpha^3 x^9 - \frac{29}{40320}\alpha^3 x^8 + \frac{10889}{20160}\alpha^2 x^8 - \frac{1909}{5040}\alpha^2 x^7 + \frac{1223}{2520}\alpha x^7 \\ & + \frac{49}{72}\alpha x^6 + \frac{20}{3}x^3 - \frac{26}{3}x^4 - \frac{37}{9}x^6 + \frac{20}{3}x^5 + \frac{71}{720}\alpha^2 x^6 + \frac{1}{120}\alpha^2 x^5 - \frac{27}{10}\alpha x^5 \\ & + \frac{11}{4}\alpha x^4 - \frac{5}{6}\alpha x^3 + \frac{1}{2}\alpha x^2 \end{aligned} \tag{47}$$

Now, in order to find the value of α we use the condition

$$\lim_{x\to\infty}(f'(\alpha)) = 0, \tag{48}$$

which is obtained from boundary condition at infinity, namely, $f'(\infty) = 0$. We apply Pade approximation [4,4] on the derivative of the solution, that is on f'. which gives,

$$\begin{aligned} & -\frac{1607476413877657600}{21}\alpha + \frac{4276215232706528}{3}\alpha^4 + \frac{836934762328993}{42}\alpha^5 \\ & + \frac{2241054174058496000}{21} - \frac{640797198845784 00}{7}\alpha^3 \\ & - \frac{1182415720940995}{84}\alpha^6 + \frac{68981549094865}{168}\alpha^7 \\ & + \frac{15121372773953}{840}\alpha^8 - \frac{7884488243}{112}\alpha^9 + \frac{114005}{56}\alpha^{10} \\ & + \frac{2037832652021849 60}{7}\alpha^2 = 0 \end{aligned} \tag{49}$$

solve equation (49), this gives $\alpha = 9.346570329$.

Table 2 Show the results of semi-infinite domain when $m < -1$ that is for rapidly decreasing stretching velocity using Laplace Decomposition Method of fluid problem (Magyari et al 2001)

X	**M= -2 (LDM)**	**M= -4 (LDM)**	**M= -6 (LDM)**	**M= -8 (LDM)**
0.1	1.960716052	1.656273011	1.590183719	1.561283699
0.2	3.149362044	2.404027574	2.251174013	2.185331332
0.3	4.692267022	3.325164699	3.053209210	2.937263989
0.4	6.541000881	6.541000881	4.015153903	3.836498127
0.5	8.515198864	5.694507989	5.112770268	4.864341421
0.6	10.38975868	7.022337004	6.289080492	5.972570881
0.7	11.98476668	8.321999225	7.469341701	7.095586381
0.8	13.21051382	9.504541138	8.578418716	8.164950252
0.9	14.06089538	10.50675532	9.555670007	9.122650738
1.0	14.58134923	11.29764910	10.36328053	9.929525449

Table 2 above the results of the approximate solution [51] using LDM , the main concern in the problem was that of the case of various m, $m < -1$ that is for rapidly decreasing stretching velocity, it was observed that with the Laplace Decomposition Method, solution to problems involving semi-infinite domain can be obtained, to any desired accuracy and these solution is valid for the whole domain and these results is in agreement with the results obtained by Magyari et al (2001) with shooting method in the literature.

4. CONCLUSION

In this paper, Laplace decomposition method was used to solve some problems involving infinite domains, which is proven to be simple and effective. We observed that with the Laplace decomposition method, solutions to problems involving semi-infinite domain can be obtained, to any desired accuracy, and these solutions are valid for the whole domain and are of high accuracy.

Generally speaking, boundary conditions at infinity pose a problem when applying the various numerical methods. So in order to tackle this problem and avoid such a difficulty, pade' approximations with the Laplace Decomposition Method (LDM) present a potential and effective answer to the condition at infinity compare with those in literature.

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