

Steady-State Behaviour of $M^x/M/G_\eta/1$ Queueing System with Catastrophe

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Abstract

The present paper describes a $M^x/M/G_\eta/1$ queueing system in which the units arrive in batches of variable size and the arriving units are served one by one in order of their arrival. The service times of the server follows exponential distribution. The service may be interrupted due to random breakdowns by the occurrence of catastrophe. When the service channel fails due to catastrophe, it stops providing service to the units and wait for the repairs to be started. Assume that the delay times, due to repair, are distributed according to Gamma distribution with parameter η . Once repair is completed the service channel instantaneously starts service.

A set of differential-difference equations have been framed for this model through probability generating function and Laplace transform and steady-state probabilities have been derived. The expression for average number of units in the system has also been derived.

Keywords: Bulk arrival, Random breakdown, Catastrophe, Delay times, Gamma distribution, Steady-state Probability.

INTRODUCTION

Bailey, (1954) introduced usual bulk service rule and it has been developed in different directions such as general bulk service rule, accessible and non-accessible rules. Along with bulk size rule, many researchers have analysed queueing systems by using the

techniques when

- (i) the server goes on vacation,
- (ii) the customer makes retrial to get service,
- (iii) the customer selects optional service,
- (iv) the service channel is subject to breakdown from time to time or a kind of service interruptions which are beyond the control of the server and
- (v) the occurrence of catastrophe.

Thiruvengadam, (1963) has discussed queueing system with service interruptions with the assumption that, when the service is impaired, it is immediately attended for repairs. However, in real life circumstances it may not be always possible to start the repair work instantly owing to non-availability of the service. Batch arrival Poisson queue with single vacation policy has been studied by Choudhury and Kalita, (2002). Ganesan and Sundar Rajan, (2008) have analysed bulk arrival queue with server breakdown. Queueing systems subject to breakdowns and batch arrival queue with a Bernoulli vacation schedule under restricted admissibility policy has been studied by Ayyappan *et al.*, (2013b). A Two-stage batch arrival queueing system with modified Bernoulli schedule vacation under N-policy and a bulk arrival queue with random breakdowns and Bernoulli's vacation has been discussed by Baba, (1986) and Bagyam and Chandrika, (2013). Vidhya, (2010) has analysed stochastic models in queues.

The occurrence of a catastrophe is random and it makes the system empty instantly and inactivates the service facility. The system must take some time for its restoration after the occurrence of a catastrophe. This time is referred to as 'the restoration time' until the system takes its own time to get ready to accept new customers. The concept of losing customers due to catastrophic failures has been first introduced by Gelenbe, (1989). Notable models with catastrophe have been developed by many researchers, like, Chen and Renshaw, (1997) Kumar and Soundararajan, (2002), Di Crescenzo *et al.*, (2003) and Chang *et al.*, (2007).

Many studies have described the transient solution or behaviour for queueing system with catastrophe, server failure, repair time, feedback, balking and reneging. The solutions have been derived for steady state probability, performance measures using probability generating function, continued fraction technique etc. (Kumar and Soundararajan, (2002), Kumar and Madheswari, (2005), Jain and Kanethia, (2006), Jain and Kumar, (2007), Kumar *et al.*, (2007a), (2007b), Thangaraj and Vanitha, (2009), Sundar Rajan *et al.*, (2011), Chandrasekaran and Saravanarajan, (2012), Boudali and Economou, (2013), Vinodhini and Vidhya, (2014), Balasubramanian *et al.*, (2015) and Sophia, (2016)).

Consider a Markovian queueing system in which the units arrive in batches of variable size. The arriving units are served one by one in order of their arrival. The service times of the server follows exponential distribution. The service may be interrupted due to random breakdowns by the occurrence of catastrophe. When the service channel fails due to catastrophe, it stops providing service to the units and wait for the repairs to be

started. Assume that the delay times, due to repair, are distributed according to Gamma distribution with parameter η . Once repair is completed the service channel instantaneously starts service. This queueing model is abbreviated as $M^x/M/G_\eta/1$.

For the above said model, set of differential-difference equations have been framed and through probability generating function and Laplace transform, steady-state probabilities have been derived. The expression for average number of units in the system has also been derived.

ASSUMPTIONS AND NOTATIONS OF THE MATHEMATICAL MODEL

- Units arrive in batches of size j ($j = 1, 2, \dots, n$), under Bailey's rule of variable bulk size, with probability a_j , in a Poisson stream with mean arrival rate λ (> 0) and queue up before the service channel, if it is busy.
- The units are served one by one in First in First Out (FIFO) basis and the service times follow exponential distribution with mean service time $1/\mu$ ($\mu > 0$).
- Catastrophes occur in the system as a Poisson process with rate ξ and it leads to the failure of service channel.
- Let αdt be the first order probability that the service channel will fail during the short interval of time dt .
- When the service channel fails, it stops providing service and waits for the repairs to be started. In this juncture it is assumed that the waiting times of the failed channel (i.e., the time from the instant it fails to the instant its repairs start) follow a general distribution with the distribution function $B(x)$ initially and further treated as Gamma distribution (G_η) with parameter η as a special case. Here the waiting time is referred as delay time.
- Let γdt be the first order probability that the repairs of the service channel will be completed during the short interval of time dt .
- Let $\beta(x) dx$ be the first order probability in which the repairs will happen during the time interval $(x, x + dx)$ conditioned that the same had not started till time x and is related to $B(x)$ by the relation

$$B(x) = \beta(x)e^{-\int_0^x \beta(x)dx} \quad (1)$$

- Once the repairs are completed the service channel instantaneously starts service.

Now, define the relevant probabilities as follows:

$W_n(t)$: The probability that at time t there are n (≥ 0) units in the system (including one in service, if any) and the service channel is in working order.

$D_n(x, t)$: The probability that at time t there are n (≥ 0) units in the system and the service channel is down and waiting for repair to start with lapsed waiting time lying between x and $x+dx$.

$R_n(t)$: The probability that at time t the service channel is being repaired

EQUATIONS GOVERNING THE SYSTEM

The set of differential-difference equations for the present model are as follows:

$$\frac{d}{dt}W_n(t) + (\lambda + \mu + \alpha + \xi)W_n(t) = \lambda \sum_{j=1}^n a_j W_{n-j}(t) + \mu W_{n+1}(t) + \lambda R_n(t), \quad n \geq 1 \tag{2}$$

$$\frac{d}{dt}W_0(t) + (\lambda + \alpha + \xi)W_0(t) = \mu W_1(t) + \gamma R_0(t), \quad n = 0 \tag{3}$$

$$\frac{\partial}{\partial x}D_n(x, t) + \frac{\partial}{\partial t}D_n(x, t) + (\lambda + \beta(x) + \alpha + \xi)D_n(x, t) = \lambda \sum_{j=1}^n a_j D_{n-j}(x, t), \quad n > 0 \tag{4}$$

$$\frac{\partial}{\partial x}D_0(x, t) + \frac{\partial}{\partial t}D_0(x, t) + (\lambda + \beta(x) + \alpha + \xi)D_0(x, t) = 0 \tag{5}$$

$$\frac{d}{dt}R_n(t) + (\lambda + \gamma)R_n(t) = \lambda \sum_{j=1}^n a_j R_{n-j}(t) + \int_0^\infty D_n(x, t)\beta(x)dx, \quad n > 0 \tag{6}$$

$$\frac{d}{dt}R_0(t) + (\lambda + \gamma)R_0(t) = \int_0^\infty D_0(x, t)\beta(x)dx \tag{7}$$

$$D_n(0, t) = \alpha W_n(t), \quad n \geq 0 \tag{8}$$

Apply Laplace transform to the equations from (2) to (8) and get,

$$(s + \lambda + \mu + \alpha + \xi)W_n(s) = \delta_{ni+n} \lambda \sum_{j=1}^n a_j W_{n-j}(s) + \mu W_{n+1}(s) + \gamma R_n(s), \quad n \geq 1 \tag{9}$$

$$(s + \lambda + \alpha + \xi)W_0(s) = \delta_{oi} + \mu W_1(s) + \gamma R_0(s), \quad n = 0 \tag{10}$$

$$\frac{\partial}{\partial x}D_n(x, s) + (s + \lambda + \beta(x) + \alpha + \xi)D_n(x, s) = \lambda \sum_{j=1}^n a_j D_{n-j}(x, s) \tag{11}$$

$$\frac{\partial}{\partial x}D_0(x, s) + (s + \lambda + \beta(x) + \alpha + \xi)D_0(x, s) = 0 \tag{12}$$

$$(s + \lambda + \gamma)R_n(s) = \lambda \sum_{j=1}^n a_j R_{n-j}(s) + \int_0^\infty D_n(x, s)\beta(x)dx \tag{13}$$

$$(s + \lambda + \gamma)R_0(s) = \int_0^\infty D_0(x, s)\beta(x)dx \tag{14}$$

$$D_n(0, s) = \alpha W_n(s), \quad n \geq 0 \tag{15}$$

For further simplification, define the probability generating function in terms of Laplace transform.

$$\left. \begin{aligned} W(s, z) &= \sum_{n=0}^\infty z^n W_n(s) \\ D(x, s, z) &= \sum_{n=0}^\infty z^n D_n(x, s) \\ R(s, z) &= \sum_{n=0}^\infty z^n R_n(s) \end{aligned} \right\} \text{ and } \left. \begin{aligned} a(z) &= \sum_{i=1}^n a_i z^n \end{aligned} \right\} \tag{16}$$

$D_n(s)$ is the Laplace transform of the probability that at time t there are n units in the system and the service channel is down and waiting for repairs to start irrespective of the values of x so that

$$D_n(s) = \int_0^\infty D_n(x, s) dx \quad (17)$$

and

$$D(s, z) = \int_0^\infty D(x, s, z) dx \quad (18)$$

Multiply the equation (9) by z^{n+1} and summing over $n = 1, 2, \dots, \infty$ and adding the equation (10) after multiplied by z , the resultant equation is

$$[(s + \lambda - \lambda a(z) + \mu + \alpha + \xi)z - \mu]W(s, z) = Z^{i+1} + \mu(z - 1)W_0(s) + \gamma z R(s, z) \quad (19)$$

Multiply the equation (11) by z^n and summing over $n = 1, 2, \dots, \infty$ and adding the equation (12), one gets,

$$\frac{\partial}{\partial x} D(x, s, z) + \{s + \lambda - \lambda a(z) + \beta(x) + \alpha + \xi\} D(x, s, z) = 0 \quad (20)$$

Multiply the equation (13) by z^n and summing over $n = 1, 2, \dots, \infty$ and adding the equation (14), one gets,

$$(s + \lambda - \lambda a(z) + \gamma)R(s, z) = \int_0^\infty D(x, s, z)\beta(x) dx \quad (21)$$

Multiply the equation (15) by z^n and summing over $n = 1, 2, \dots, \infty$, we get,

$$D(0, s, z) = \alpha W(s, z) \quad (22)$$

Now, integrating the equation (20) we get,

$$D(x, s, z) = D(0, s, z)e^{-\{s+\lambda-\lambda a(z)+\alpha+\xi\}x - \int_0^\infty \beta(x) dx} \quad (23)$$

After applying the equation (22) in the equation (23), we get,

$$D(x, s, z) = \alpha W(s, z)e^{-\{s+\lambda-\lambda a(z)+\alpha+\xi\}x - \int_0^\infty \beta(x) dx} \quad (24)$$

Integrating the expression (24) with respect to x and using the equations (11) and (18), we get,

$$D(s, z) = \alpha W(s, z) \left[\frac{(1 - B(s + \lambda - \lambda a(z) + \alpha + \xi))}{(s + \lambda - \lambda a(z) + \alpha + \xi)} \right] \quad (25)$$

Substituting the equation (24) in the equation (21), we get,

$$R(s, z) = \frac{\alpha B(s + \lambda - \lambda a(z) + \alpha + \xi)}{(s + \lambda - \lambda a(z) + \alpha + \xi)} W(s, z) \quad (26)$$

The equation (19) is re-written, after using the equation (26), as

$$W(s, z) = \frac{1}{\phi(s, z)} [z^{i+1} + \mu(z - 1)W_0(s)](s + \lambda - \lambda a(z) + \gamma) \quad (27)$$

Comparing the equations (25) and (27), we get,

$$D(s, z) = \frac{1}{\phi(s, z)} [\alpha \{1 - B(s + \lambda - \lambda a(z) + \alpha + \xi)\} (s + \lambda - \lambda a(z) + \alpha + \xi)^{-1} \{ (s + \lambda - \lambda a(z) + \gamma) \{ z^{i+1} + \mu(z - 1)W_0(s) \} \}] \quad (28)$$

Also, apply the equation (27) in the equation (26) and get,

$$R(s, z) = \frac{1}{\phi(s, z)} [\alpha B(s + \lambda - \lambda a(z) + \alpha + \xi) \{z^{i+1} + \mu(z - 1)W_0(s)\}] \quad (29)$$

where,

$$\phi(s, z) = \{(s + \lambda - \lambda a(z) + \mu + \alpha + \xi)z - \mu\}(s + \lambda - \lambda a(z) + \gamma) - \alpha \gamma z B(s + \lambda - \lambda a(z) + \alpha + \xi)$$

The above three expressions are derived under the condition that the distribution of delay time for repairs to start is general distribution.

Now, consider a special case in which delay times for repairs to start be Gamma distribution with parameters η and its Laplace transform is stated as

$$B(s + \lambda - \lambda a(z) + \alpha + \xi) = \frac{1}{(s + \lambda - \lambda a(z) + \alpha + \xi + 1)^\eta} \quad (30)$$

Now, the equations (27), (28) and (29) are re-written, after applying the equation (30), as

$$W(s, z) = \frac{1}{\phi(s, z)} [(s + \lambda - \lambda a(z) + \alpha + \xi + 1)^\eta (s + \lambda - \lambda a(z) + \gamma) \{z^{i+1} + \mu(z - 1)W_0(s)\}] \quad (31)$$

$$D(s, z) = \frac{1}{\phi(s, z)} [\alpha \{(s + \lambda - \lambda a(z) + \alpha + \xi + 1)^\eta - 1\} (s + \lambda - \lambda a(z) + \alpha + \xi)^{-1} (s + \lambda - \lambda a(z) + \gamma) \{z^{i+1} + \mu(z - 1)W_0(s)\}] \quad (32)$$

$$R(s, z) = \frac{1}{\phi(s, z)} [\alpha \{z^{i+1} + \mu(z - 1)W_0(s)\}] \quad (33)$$

where,

$$\phi(s, z) = \{(s + \lambda - \lambda a(z) + \mu + \alpha + \xi)z - \mu\}(s + \lambda - \lambda a(z) + \gamma) (s + \lambda - \lambda a(z) + \alpha + \xi + 1)^\eta - \alpha \gamma z$$

The next stage is to derive the steady-state probability generating functions by using Tauberian property

$$\lim_{s \rightarrow 0} s f(s) = \lim_{t \rightarrow \infty} f(t)$$

And the expressions (31), (32) and (33) become:

$$W(z) = \frac{1}{K(z)} [(\lambda - \lambda a(z) + \alpha + \xi + 1)^\eta (\lambda - \lambda a(z) + \gamma) \{z^{i+1} + \mu(z - 1)W_0\}] \quad (34)$$

$$D(z) = \frac{1}{K(z)} [\alpha \{(\lambda - \lambda a(z) + \alpha + \xi + 1)^\eta - 1\} (\lambda - \lambda a(z) + \gamma) \{z^{i+1} + \mu(z - 1)W_0\} (\lambda - \lambda a(z) + \alpha + \xi)^{-1}] \quad (35)$$

$$R(z) = \frac{1}{K(z)} [\alpha \{z^{i+1} + \mu(z - 1)W_0\}] \quad (36)$$

Where

$$K(z) = \{(\lambda - \lambda a(z) + \mu + \alpha + \xi)z - \mu\} (\lambda - \lambda a(z) + \gamma) (\lambda - \lambda a(z) + \alpha + \xi + 1)^\eta - \alpha \gamma z$$

Now, the steady state probability generating function for overall system $P(z)$ is obtained by adding the expressions (34), (35) and (36).

$$P(z) = \frac{1}{K(z)} \left[\{z^{i+1} + \mu(z-1)W_0\} \{(\lambda - \lambda a(z) + \gamma)(\lambda - \lambda a(z) + \alpha + \xi + 1)^n + \alpha\{(\lambda - \lambda a(z) + \alpha + \xi + 1)^n - 1\}(\lambda - \lambda a(z) + \alpha + \xi)^{-1} + \alpha\} \right] \quad (37)$$

AVERAGE QUEUE LENGTH

The average number of units in the system when the server is in working condition, the occurrence of catastrophe and the system down and the system under repair are derived by differentiating the expressions (34), (35) and (36) with respect to z and letting $z=1$ and respectively obtained as,

$$L_W = \frac{[(\alpha + \xi)(\alpha + \xi + 1)^n \gamma - \alpha \gamma][(\alpha + \xi + 1)^n \{\gamma(i + 1 + \mu w_0) - \lambda a'(1)\} - \lambda \gamma \eta (\alpha + \xi + 1)^{n-1} a'(1)] - \gamma (\alpha + \xi + 1)^n [\gamma (\alpha + \xi + 1)^n (\mu + \alpha + \xi - \lambda a'(1) - \alpha \gamma - \lambda (\gamma \eta + \alpha + \xi + 1) (\alpha + \xi) (\alpha + \xi + 1)^{n-1} a'(1)]}{[(\alpha + \xi)(\alpha + \xi + 1)^n \gamma - \alpha \gamma]^2} \quad (38)$$

$$L_D = \frac{[(\alpha + \xi)(\alpha + \xi + 1)^n \gamma - \alpha \gamma] [\alpha \{(\alpha + \xi + 1)^n - 1\} \{\lambda a'(1) (\alpha + \xi)^{-2} (\gamma - (\alpha + \xi)) + \gamma (\alpha + \xi)^{-1} (i + 1 + \mu w_0)\} - \lambda \alpha \gamma \eta (\alpha + \xi)^{-1} (\alpha + \xi + 1)^{n-1} a'(1)] - \alpha \gamma \{(\alpha + \xi + 1)^n - 1\} (\alpha + \xi)^{-1} [\gamma (\alpha + \xi + 1)^n (\mu + \alpha + \xi - \lambda a'(1)) - \alpha \gamma - \lambda (\gamma \eta + \alpha + \xi + 1) (\alpha + \xi) (\alpha + \xi + 1)^{n-1} a'(1)]}{[(\alpha + \xi)(\alpha + \xi + 1)^n \gamma - \alpha \gamma]^2} \quad (39)$$

$$L_R = \frac{[(\alpha + \xi)(\alpha + \xi + 1)^n \gamma - \alpha \gamma] \alpha \{(i + 1 + \mu w_0)\} - \alpha [\gamma (\alpha + \xi + 1)^n (\mu + \alpha + \xi - \lambda a'(1)) - \alpha \gamma - \lambda (\gamma \eta + \alpha + \xi + 1) (\alpha + \xi) (\alpha + \xi + 1)^{n-1} a'(1)]}{[(\alpha + \xi)(\alpha + \xi + 1)^n \gamma - \alpha \gamma]^2} \quad (40)$$

The average number of units in the overall system is obtained by adding the results (38), (39) and (40).

$$L = L_W + L_D + L_R$$

$$L = \frac{K(1)[A'(1)+B'(1)+C'(1)]-K'(1)[A(1)+B(1)+C(1)]}{[K(1)]^2} \quad (41)$$

Where,

$$K(1) = (\alpha + \xi)(\alpha + \xi + 1)^n \gamma - \alpha \gamma$$

$$A'(1) = (\alpha + \xi + 1)^n \{\gamma(i + 1 + \mu w_0) - \lambda a'(1)\} - \lambda \gamma \eta (\alpha + \xi + 1)^{n-1} a'(1)$$

$$B'(1) = \alpha\{(\alpha + \xi + 1)^\eta - 1\}\{\lambda a'(1)(\alpha + \xi)^{-2}(\gamma - (\alpha + \xi)) \\ + \gamma(\alpha + \xi)^{-1}(i + 1 + \mu w_0)\} \\ - \lambda \alpha \gamma \eta (\alpha + \xi)^{-1} (\alpha + \xi + 1)^{\eta-1} a'(1)$$

$$C'(1) = \alpha\{i + 1 + \mu w_0\}$$

$$K'(1) = \gamma(\alpha + \xi + 1)^\eta (\mu + \alpha + \xi - \lambda a'(1)) - \alpha \gamma - \lambda(\gamma \eta + \alpha + \xi + 1) \\ (\alpha + \xi)(\alpha + \xi + 1)^{\eta-1} a'(1)$$

$$A(1) = \gamma(\alpha + \xi + 1)^\eta$$

$$B(1) = \alpha \gamma \{(\alpha + \xi + 1)^\eta - 1\} (\alpha + \xi)^{-1}$$

$$C(1) = \alpha$$

CONCLUSION

A bulk arrival Markovian queueing system with random breakdown by occurrences of catastrophe is considered. Explicit expressions for the steady-state probabilities and expected number of units in the system have been derived. This model may be appropriately used for system network and communications.

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