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STUDY ON ASYMPTOTIC STABILIZATION OF NONLINEAR DELAY DIFFERENCE EQUATIONS

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ABSTRACT

In this paper, we discuss the stability of deterministic type delay differential equations through obtaining the stability condition for the respective deterministic difference equation involving delay. The system formulation is done by considering the differential equation that describes the dynamics of a single isolated neuron involving delay. Here the discretization of the deterministic differential equation is done through the Euler- Maruyama Method.

Keyword: Delay Differential equation, Difference equation, Lyapunov-krasovkii functional, Neural networks.

Subject Code: 39A10 , 37B25, 92B20.

INTRODCTION

Difference equations have attracted much attention as a result of their devotions in many areas of real world problems. Specifically, difference equation in neural networks involving delays light on numerous applications in particular fields such as optimization problems, pattern recognition, signal processing and associative memory. These applications strongly depend on the stability of the equilibrium point of the system designed. Consequently, the stability analysis is fundamental for the design and applications in digital signal processing. In hardware implementation of recurrent neural networks, time delays occur due to finite switching speed of the amplifiers and communication time. In recent years, considerable number of research works has been made to study the global asymptotic or exponential stability for the neural networks with time delays via Lyapunov function method. In particular, there is an increasing interest in the study of differential equations with both discrete and distributed delays, see [5]. A designed system could be stabilized or destabilized by adding certain inputs. However, besides these effects, impulsive effects occur in real world systems. In the case of non-linear differential equations the mean-square asymptotic stability of the numerical methods has been studied by several authors e.g., [4,3]. But the almost sure asymptotic stability of numerical methods has been less studied.

Motivated by the above discussions, the main objective of this paper is to study the global asymptotic stability of system of difference equation describing the dynamics of a neuron. Here we apply Euler–Maruyama method to the nonlinear differential equation. We establish new stability conditions for the stochastic difference equation with the help of Lyapunov-Krasovskii functional method and some well-known inequalities.We prove results on global a.s. asymptotic stability of the trivial solution Y_k of equation (2). We show that (2) is a good discrete model for a corresponding deterministic delay differential equation, since under the same conditions on the function ϕ, their solutions have similar asymptotic behavior.

The paper is organized as follows. In section 2, gives the preliminaries and we recall the necessary definitions and lemmas that will be used to prove our results. In section 3, deals with system formulation, conversion of differential equation system into difference equation system and some assumptions. In section 4, the desired stability condition is formulated in terms of lemmas and theorems. In section 5, gives the conclusion of the paper and discusses our future work.

PRELIMINARIES:

An equilibrium point is said to be asymptotically stable if it is both stable and convergent. If y^* is an equilibrium point of a system then it is stable if for given $\epsilon > 0$ and initial non-negative value n_0 there exists a $\delta(\epsilon, n_0)$ such that $|y_0 - y^*| < \delta \Rightarrow$ $|y(n) - y^*| < \epsilon$ for all $n \geq n_0$. And the system is said to be convergent to the equilibrium point if $\lim_{n\to\infty} Y_n = y^*$

A function $V: D \to R$ is said to be positive semi definite in if it satisfies the following two conditions: (1)$V(0) = 0$ and (2)$V(k) \geq 0$

Lyapunov Stability Theorem: Let $V(k)$ be a positive definite function. Let $y = 0$ be an equilibrium point of the system, such that

(i)$V(k)$ is a positive definite function (ii)$\Delta V(k) < 0$

Then the system is said to be asymptotically stable. i.e.) $\lim_{k\to\infty} Y_k = 0$

SYSTEM FORMULATION

The following represents the system of deterministic differential equation which describes the dynamics of an isolated neuron described in terms of delay differential equations

$$dy(t) = [-y(t) + \alpha\varphi(y(t) - \beta y(t-\tau)]dt \qquad t \geq 0 \qquad (1)$$

where $y(t) -$ is the activation level of a neuron at a time t

α −is constant describing the range of the variable $y(t)$

β −is the mesure that describes the influence of past history

τ −represents the delay

φ −activation function of the neuron

and the constants $\alpha \in \mathbb{R}^+, \beta \geq 0$ and $\tau \in [0, \infty)$. And the above system forms a model of neural network.

The stability of the above system is obtained by considered the related discretized form of equation. Hence the discretization of the above model through Euler-Maruyama is given by

$$Y_{k+1} = (1-d)Y_k + \alpha d\phi(Y_k - \beta Y_{k-\tau}) \qquad , k \in \mathbb{N}_0 \qquad (2)$$

with $Y_0 \in \mathbb{R}$ -as arbitrary nonrandom initial value. $d \in (0,1]$ is the mesh size. ϕ −is a nonrandom continuous real valued function

MAIN RESULTS

Theorem 4.1: Let $Y_k$be the solution of the equation (2) with the conditions

$$|\psi_{k,Y_k}| \leq \gamma_k |Y_k|^2 + {\eta_k}^2, \qquad \psi_{k,0} = 0 \qquad (5)$$

Where $\sum_{i=1}^{\infty} {\eta_i}^2 < \infty$ and

$$\alpha^2(1 + |\beta|)^2 + \gamma_k < 1 \qquad (6)$$

Satisfied by equation(4). Then $\lim_{k\to\infty} Y_k = 0$ almost everywhere.

Proof:

Consider equation(2),

$$Y_{k+1} = (1-d)Y_k + \alpha d\phi(Y_k - \beta Y_{k-\tau}) \qquad , k \in \mathbb{N}_0$$

Squaring on both sides we get

$$Y_{k+1}{}^2 = [(1-d)Y_k + \alpha d\phi(Y_k - \beta Y_{k-\tau})]^2$$

$$= [(1-d)Y_k + \alpha d\phi(Y_k - \beta Y_{k-\tau})]^2$$

$$= [(1-d)Y_k + \alpha d\phi(Y_k) - \alpha\beta d\phi(Y_{k-\tau})]^2 \tag{7}$$

Consider

$$\sum_{i=1}^{n} x_i y_i \leq \sum_{i=1}^{n} |x_i y_i| \leq \sum_{i=1}^{n} \sqrt{|x_i|}(\sqrt{|x_i|}\,|y_i|)$$

Now from Holder's inequality we have

$$\sum_{i=1}^{n} |x_i y_i| \leq \sum_{i=1}^{n} \sqrt{|x_i|}(\sqrt{|x_i|}\,|y_i|) \leq \sqrt{\sum_{i=1}^{n} |x_i|}\sqrt{\sum_{i=1}^{n} |x_i|\,|y_i|^2}$$

Now consider,

$$|(1-d)Y_k + \alpha d\phi(Y_k - \beta Y_{k-\tau})| \leq |(1-d)Y_k| + |\alpha|d|\phi(Y_k - \beta Y_{k-\tau})|$$

$$\leq |(1-d)Y_k| + |\alpha|d|(Y_k - \beta Y_{k-\tau})|$$

$$\leq (1-d)|Y_k| + |\alpha|d[|Y_k| + |\beta||Y_{k-\tau}|]$$

Here let $n = 3$ $x_1 = 1-d,$ $y_1 = |Y_k|,$ $x_2 = |\alpha|d,$ $y_2 = |Y_k|,$ $x_3 = |\alpha|d|\beta|,$ $y_3 = |Y_{k-\tau}|,$

$$\leq [1-d+|\alpha|d+|\alpha|d|\beta|]\,[(1-d+|\alpha|d)|Y_k|^2 + |\alpha|d|\beta||Y_{k-\tau}|^2] \tag{9}$$

Substituting (9) in (7) we have

$$Y_{k+1}{}^2 \leq [1-d+|\alpha|d+|\alpha|d|\beta|]\,[(1-d+|\alpha|d)|Y_k|^2 + |\alpha|d|\beta||Y_{k-\tau}|^2]$$

Let $a = d|\alpha||\beta|(1-d+|\alpha|d+|\alpha|d|\beta|)$

Let

$$V(k)^{(1)} = a\sum_{s=k-\tau}^{k-1} Y_s{}^2$$

$$V(k) = Y_k{}^2 + V(k)^{(1)}$$

Consider,

$$\Delta V(k)^{(1)} = V(k+1)^{(1)} - V(k)^{(1)}$$

$$= a\sum_{s=k+1-\tau}^{k} {Y_s}^2 - a\sum_{s=k-\tau}^{k-1} {Y_s}^2$$

$$= a{Y_k}^2 - a{Y_{k-\tau}}^2$$

Now consider

$$\Delta V(k) = {Y_{k+1}}^2 - {Y_k}^2 + \Delta V(k)^{(1)}$$

$$= {Y_{k+1}}^2 - {Y_k}^2 + a{Y_k}^2 - a{Y_{k-\tau}}^2$$

$$\leq [1 - d + |\alpha|d + |\alpha|d|\beta|]\,[(1 - d + |\alpha|d)|Y_k|^2 + |\alpha|d|\beta||Y_{k-\tau}|^2] - {Y_k}^2 + a{Y_k}^2 - a{Y_{k-\tau}}^2$$

$$\leq [a - 1 + (1 - d + |\alpha|d + |\alpha|d|\beta|)(1 - d + |\alpha|d)]|Y_k|^2 + [1 - d + |\alpha|d + |\alpha|d|\beta|]|\alpha|d|\beta||Y_{k-\tau}|^2 - a{Y_{k-\tau}}^2$$

$$\leq [d|\alpha||\beta|(1 - d + |\alpha|d + |\alpha|d|\beta|) - 1 + (1 - d + |\alpha|d + |\alpha|d|\beta|)(1 - d + |\alpha|d)]|Y_k|^2 + [1 - d + |\alpha|d + |\alpha|d|\beta|]|\alpha|d|\beta||Y_{k-\tau}|^2 - d|\alpha||\beta|(1 - d + |\alpha|d + |\alpha|d|\beta|){Y_{k-\tau}}^2$$

$$\leq ([1 - d - d|\alpha|(1 + |\beta|)]^2 - 1)|Y_k|^2 \quad (10)$$

From (6) and for all $d \in (0,1]$ we get,

$0 < 1 - |\alpha|(1 + |\beta|) < 1$,$0 < d[1 - |\alpha|(1 + |\beta|)] < 1$

$\Rightarrow 0 < 1 - d[1 - |\alpha|(1 + |\beta|)] < |\alpha|(1 + |\beta|)$

Hence,

$$(1 - d[1 - |\alpha|(1 + |\beta|)])^2 \leq \alpha^2(1 + |\beta|)^2 < 1.$$

And,

$$(1 - d[1 - |\alpha|(1 + |\beta|)])^2 - 1 < 0$$

Therefore from (10), we have

$$\Delta V(k) < 0 \quad (11)$$

Therefore we have,

$$\lim_{k\to\infty} Y_k = 0$$

Hence the theorem is proved.

CONCLUSION

In this paper we established new stability conditions for the deterministic delay difference equation with the help of Lyapunov functional method and some well-known inequalities. And the desired stability is obtained by applying suitable assumptions and through the help of theorems. And the future work is to extend the paper to stochastic system and to obtain the stability results.

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