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STABILITY ANALYSIS OF COMPLEX VALUED NEURAL NETWORK WITH LEAKAGE TIME VARYING DELAY

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Abstract:

In this paper, we examine the global asymptotic stability of a discrete type of complex valued neural network with leakage time varying case of delay. By retaining Lyapunov functional method and introducing matrix inequality procedure of linear case, quite a few adequate points in complex valued linear matrix inequality forms are obtained to ensure the existence, uniqueness and global asymptotic stability of equilibrium point for the considered neural networks. In addition, the asymptotic convergence rate index is approximated, which depends on the system values. The planned stability results are lesser amount of conservative, which is confirmed through Numerical examples.

Keywords: Complex valued neural networks, leakage delay, discrete time system, asymptotic stability, complex valued inequality, Lypunov function.

Introduction:

Neural networks are the functional tools to several area such as sales forecasting, industrial process control, customer research, data validation, risk management, target marketing and recognition of speakers in communications. The complex values are frequently applied via the Fourier transformation. It point out that the complex valued neural networks (CNN) are helpful. Besides, the brain of human an action sequence may be different pulse model, and the distance among the pulses should be different. This explains that the constructing complex values for the phase and amplitude in an appropriate manner. In these years the complex valued neural networks are applied in the field of computer vision, image processing, computer vision, optoelectronic imaging, and communication and so on [1]. The possible applicability give new features of theories necessary for novel or more effectual mechanisms. There are many conditions of physical and biological model, the rate of variation in the system state depends on previous states. It suggested the characteristic is called a time delay or delay. If the system with a time delay then the system is called a time-delay system. Time delay were first discovered in biological systems and were later originate in many engineering systems, such as transmission of fluid, transmissions of mechanical, metallurgical processes, and networked control systems. They are usually basis of instability and poor control execution. So it is essential to examine the stability of the neural network systems.

In current scenario, diverse types stability of a complex valued neural networks are established in many places [1-7]. In several reviews, while forming a neural network with delay were considered in their systems [6]. In many papers, we are come across with a continuous case. This work consist of the discrete case of complex valued neural network with time delay. The important work of this paper is to discover the existence and uniqueness of the main method which certify the strong asymptotic stability of the proposed model.

This paper prepared as follows, In section 1, we provide concise introduction for the complex valued neural network in discrete case. Section 2, we expound few definitions and lemmas which are helpful for our results. Section 3, we deliberate the mathematical formulation of complex valued neural network with the delay and initial conditions. In section 4, we verify the main theorem and essential requirement which are guarantee to the existence, uniqueness and global asymptotic stability of equilibrium point of complex valued linear matrix inequalities. In section 5, we furnish numerical examples for the main results and ultimately we exhibit the success of the anticipated model of complex valued neural network by numerical simulation by means of MATLAB.

2. Preliminaries:

Lemma 1:

Let $a, b \in C^n$ and $P \in C^{n*n}$be a positive definite Hermitian Matrix, then

$$a^* b + a b^* \leq a^* P^{-1} a + b^T P b. \quad (1)$$

Lemma 2:

The subsequent conditions are satisfies for the continuous map $H(y): C^n \to C^n$,

(1) $H(y)$ is injective on C^n,

(2) $\lim_{\|y\| \to \infty} \|H(y)\| = \infty.$

such that *H(y)* is a homeomorphism of C^n onto C^n.

Lemma 3: [Schur Complements]

The considered Linear Matrix Inequality,

$$\begin{bmatrix} Q(x) & S(x) \\ * & R(x) \end{bmatrix} < 0$$

where, $Q(x) = Q^T(x), R(x) = R^T(x)$ and $S(x)$depends on x, is equivalent to either of the two conditions given below,

$$R(x) < 0, Q(x) - S(x)R(x)^{-1}S^T(x) < 0$$

$$Q(x) < 0, R(x) - S(x)Q(x)^{-1}S^T(x) < 0$$

3. Problem Formulation:

The following are the complex valued neural network with leakage time varying delays for the discrete time,

$$y(k+1) = Ay(k) + Bf\big(y(k)\big) + Cf\Big(y\big(k - d(k)\big)\Big) + L \quad (2)$$

$$y(k) = \theta(k), k \in [-d, 0]$$

where k is a nonnegative integer and y(k) is a vector described as $y(k) = \big(y_1(k), y_2(k), \ldots, y_N(k)\big)^T$, where $y_i(k)$ indicate the activity of the i^{th} neuron. Furthermore, f is a complex valued function named as,

$$f(y) = \max\{0, Re(y)\} + i \max\{0, Im(y)\} \quad (3)$$

$f(y(k)) = \left(f(y_1(k)), f(y_2(k)), \ldots, f(y_N(k))\right)^T$ and $f(y(k-d)) = \left(f(y_1(k-d)), f(y_2(k-d)), \ldots, f(y_N(k-d))\right)^T$ are the neuron activation functions. equation (2), L, A, B and C are declared as the input vector $(l_1, l_2, \ldots, l_N)^T$, the connection weight matrix $(a_{ij})_{NXN}$ and the delayed connection weight matrix $(b_{ij})_{NXN}$, respectively, and d describes time delay and its is a positive integer. The initial condition associated with construct mathematical model of neural network (2) is given by

$$y(k) = \theta(k), k \in [-d, 0] \tag{4}$$

Assumption 1:

For the existence positive diagonal matrix $F = diag\{F_1, F_2, \ldots, F_n\}$ for all $u_1, u_2 \in C, i = 1,2, \ldots n$, we have

$$|f_i(u_1) - f_i(u_2)| \leq F_i |u_1 - u_2| \tag{5}$$

4. Main Results:

From the assumption 1,the following theorem states that there exist a equilibrium point of the model (2) is unique and globally asymptotically stable.

Theorem 4.1:

There exist a positive definite Hermitian matrix P and two real valued positive diagonal matrices L and R, such that the equation of the system given in (2) is unique if the following inequality is true,

$$M = \begin{bmatrix} P - PC - C^*P + FLF & PA & PB \\ A^*P & -L & 0 \\ B^*P & 0 & -R \end{bmatrix} < 0 \tag{6}$$

$$FRF < P$$

Proof:

To prove the existence and uniqueness of the model. The equilibrium point is the solution of the proposed model (2) so we obtain,

$$Ay(k) + Bf(y(k)) + C\left(f(y(k))\right) + L = y(k) \tag{7}$$

$$(A-1)y(k) + (B+C)f(y(k)) + L = 0$$

For $H(y) = (A-1)y + (B+C)f(y) + L$ the following, we have establish that $H(y)$ is a homeomorphism of C^n. in fact, if there exist $x \in C^n \; and \; y \in C^n \; and \; x \neq y$ such that $H(x) = H(y)$, then

$$(A-1)(x-y) + (B+C)\big(f(x) - f(y)\big) = 0 \tag{8}$$

$$(x-y)^* P(A-1)(x-y) + (x-y)^* P(B+C)\big(f(x) - f(y)\big) = 0 \tag{9}$$

$$(x-y)^* P(A-1)^T(x-y) + \big(f(x) - f(y)\big)^* P(B+C)^*(x-y) = 0 \tag{10}$$

From (9) and (10), we get

$$(x-y)^*(P(A-1) + (A-1)^* P)(x-y) + (x-y)^* P(B+C)\big(f(x) - f(y)\big) + \big(f(x) - f(y)\big)^*(B+C)^* P(x-y) = 0 \tag{11}$$

In this we follows from Lemma 1 and (12) we obtain,

$$0 \leq (x-y)^*(P(A-1) + (A-1)^* P)(x-y) + (x-y)^* P(B+C)(L+R)^{-1}(B+C)^* P(x-y) + \big(f(x) - f(y)\big)^*(L+R)\big(f(x) - f(y)\big). \tag{12}$$

L and R are real valued positive diagonal matrices and we have obtain from assumption 2 that

$$\big(f(x) - f(y)\big)^*(L+R)\big(f(x) - f(y)\big) \leq (x-y)^* F(L+R)F(x-y). \tag{13}$$

From the above equation by $\begin{bmatrix} I & 0 & 0 \\ 0 & I & I \end{bmatrix}$ and its Hermitian matrix we arrive,

$$\begin{bmatrix} P + P(A-1) + (A-1)^* P + FLF & P(B+C) \\ (B+C)^* P & -(L+R) \end{bmatrix} < 0 \tag{14}$$

Using Schur Complements lemma we get,

$$P + P(A-1) + (A-1)^* P + FLF + P(B+C)(L+R)^{-1}(B+C)^* P < 0 \tag{15}$$

From (13) and (15), we identify that $x = y$, which disagree the given Proof. So $H(y)$is an infective map on the complex plan. Second we prove that $\lim_{\|y\| \to \infty} \|H(y)\| = \infty$.

From the definition of $H(y)$, we found that,

$$H(y) - H(0) = (A-1)y + (B+C)\big(f(y) - f(0)\big) \tag{16}$$

$$y^* P\big(H(y) - H(0)\big) = y^* PCy + y^* P(A+B)\big(f(y) - f(0)\big)$$

$$\big(H(y) - H(0)\big)P^* y = y^*(A-1)^* Py + \big(f(y) - f(0)\big)^*(B+C)^* Py.$$

From the above equations and a similar discussion procedure in left of (9), we obtain that

$$y^*P\big(H(y)-H(0)\big)+\big(H(y)-H(0)\big)^*Py$$
$$\le y^*(P(A-1)+P(A+B)(L+R)^{-1}(A+B)^*P+F(L+R)F)y$$

Let $N=P(A-1)+(A-1)^*P-P(B+C)(L+R)^{-1}(B+C)^*P-F(L+R)F$.

From equation (15) we recognize that $N>0$.Therefore,

$$y^*P\big(H(y)-H(0)\big)+\big(H(y)-H(0)\big)^*Py\le -y^*Ny\le -\delta\_\min\ N\|y\|^2.$$
$$\le 2\|y\|.\|P\|(\|H(y)\|+\|H(0)\|)$$

So, $\lim_{\|y\|\to\infty}\|H(y)\|=\infty$. Hence, we have $H(y)$ is a homeomorphism of complex plan onto complex plan. That is the obtained system has a unique Equilibrium point of the proposed model.

Theorem 4.2

If there exists a MXM Hermitian positive definite matrices P,R and S, two MXM positive diagonal matrices D and F such that the matrix

$$\omega=\begin{pmatrix} A^*PA-P+R+\rho(A-1)^*S(A-1)-\frac{S}{\rho} & \frac{S}{\rho} & A^*PB+\rho(A-1)^*SB & A^*PC+\rho(A-1)^*SC \\ \frac{S}{\rho} & -R-\frac{S}{\rho} & 0 & 0 \\ B^*PA+\rho B^*S(A-1) & 0 & B^*PB+\rho B^*SB & B^*PC+\rho B^*SC \\ C^*PA+\rho C^*S(A-1) & 0 & C^*PB+\rho C^*SB & C^*PC+\rho C^*SC \end{pmatrix}<0$$

That is the given above matrix is negative definite matrix then the network of the proposed model is globally asymptotically stable.

Proof:

Consider the following Lypunov Krosovkii functions,

$$V(k)=V_1(k)+V_2(k)+V_3(k) \tag{17}$$

where,

$$V_1(k) = x^*(k)Px(k)$$

$$V_2(k) = \sum_{i=k-\rho}^{k-1} x^*(i)Rx(i)$$

$$V_3(k) = \sum_{i=-\rho}^{-1}\sum_{l=k+i}^{k-1} \mu^*(l)S\mu(l)$$

where, $\mu(k) = x(k+1) - x(k)$

Take the derivative of the Lypunov Krosavskii function we have obtain,

$$\Delta V_1(k) = x^*(k+1)Px(k+1) - x^*(k)Px(k)$$

$$= \Big(Ax(k) + Bf\big(x(k)\big) + Cf\big(x(k-\rho)\big)\Big)^* P \Big(Ax(k) + Bf\big(x(k)\big) + Cf\big(x(k-\rho)\big)\Big) - x^*(k)Px(k)$$

$$\Delta V_2(k) = x^*(k)Rx(k)x^*(k-\rho)Rx(k-\rho)$$

$$\Delta V_3(k) = \rho\mu^*(k)S\mu(k) - \sum_{i=-\rho}^{-1} \mu^*(k+i)S\mu(k+i)$$

$$= \rho[x(k+1) - x(k)]^*S[x(k+1) - x(k)] - \sum_{i=-\rho}^{-1} \mu^*(k+i)S\mu(k+i)$$

Adding all the above equations we have,

$$\Delta V(k) = \gamma^*(k)\omega_1\gamma(k) - \sum_{i=-\rho}^{-1} \mu^*(k+i)S\mu(k+i) \tag{18}$$

where,

$$\gamma(k) = \big[x(k)^* \quad x(k-\rho)^* \quad g\big(x(k)\big)^* \quad g\big(x(k-\rho)\big)^*\big]^*$$

$$\omega_1 = \begin{pmatrix} A^*PA - P + R + \rho(A-1)^*S(A-1) - \frac{S}{\rho} & \frac{S}{\rho} & A^*PB + \rho(A-1)^*SB & A^*PC + \rho(A-1)^*SC \\ \frac{S}{\rho} & -R - \frac{S}{\rho} & 0 & 0 \\ B^*PA + \rho B^*S(A-1) & 0 & B^*PB + \rho B^*SB & B^*PC + \rho B^*SC \\ C^*PA + \rho C^*S(A-1) & 0 & C^*PB + \rho C^*SB & C^*PC + \rho C^*SC \end{pmatrix}$$

Using Schur Complements lemma we have$\omega_1 < 0$, Therefore we obtain $\Delta V(k) \leq 0$. From the Stability theory of Lypunov function we conclude the system is Asymptotically Stable.

Corollary:4.3

If there exists a MXM Hermitian positive definite matrices P, R and S, two MXM positive diagonal matrices D and F such that the matrix

$$\omega_1 = \begin{pmatrix} A^*PA - P + R + \rho(A-1)^* S(A-1) - (\rho - \frac{1}{\rho})S & \frac{S}{\rho} & A^*PB + \rho(A-1)^* SB & A^*PC + \rho(A-1)^* SC \\ \frac{S}{\rho} & -R - \frac{S}{\rho} & 0 & 0 \\ B^*PA + \rho B^* S(A-1) & 0 & B^*PB + \rho B^* SB & B^*PC + \rho B^* SC \\ C^*PA + \rho C^* S(A-1) & 0 & C^*PB + \rho C^* SB & C^*PC + \rho C^* SC \end{pmatrix} < 0 \quad (19)$$

That is the given above matrix is negative definite matrix then the network of the proposed model is globally asymptotically stable.

Proof:

In the Proposed theorem 4.2 if we apply the following condition,

$$-\rho \sum_{i=-\rho}^{-1} \mu^*(k+i) S \mu(k+i) \leq$$

$$- \sum_{i=-\rho}^{-1} \mu^*(k+i) S \sum_{i=-\rho}^{-1} \mu(k+i)$$

$$= \begin{pmatrix} x(k) \\ x(k-\rho) \end{pmatrix}^* \begin{pmatrix} -R & R \\ R & -R \end{pmatrix} \begin{pmatrix} x(k) \\ x(k-\rho) \end{pmatrix}$$

And substitute in the equation (18) we have got the Linear Matrix Inequlity given in equation (18).

From the above equation (18) and (19) we attain,

$$\Delta V(k) \leq \gamma^*(k) \omega_1 \gamma(k)$$

From the Same Argument of stability theory we say system is asymptotically stable.

5. Numerical Example

Example 5.1:

Consider a discrete time neural network (1), where

$$A = \begin{pmatrix} 0.3 & -0.1+0.1i \\ -0.1-0.1i & 0.3 \end{pmatrix},\ B = \begin{pmatrix} -0.1-0.5i & -0.1+0.1i \\ -0.1+0.1i & -0.1-0.1i \end{pmatrix}, \text{C}= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$H = (10+i \quad 1+10i)^{*}$$

Further, when $P = 989, R = 451, S = I$ and $D = F = 401$, the weight matrix ω in the Theorem 4.2 is a negative definite. So we have the provide the system is asymptotically stable.

Conclusion:

This paper discussed about the discrete case complex valued neural network with leakage time varying delay and global asymptotic stability of the proposed model. By choosing Lyapunov conditions and applying matrix inequality technique to ensure the existence, uniqueness and global asymptotic stability of equilibrium point for the considered neural networks. The presented stability results are less amount of conservative, which is confirmed using Numerical examples.

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