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MODEL ON DEFORESTATION DUE TO HUMAN POPULATION AND ITS EFFECT ON FARM FIELDS: A STATE SPACE APPROACH

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ABSTRACT:

In this paper, we construct a model on deforestation using a system of stochastic difference equations. We propose a state space approach for this model. The Kalman filter (KF) works by transforming the nonlinear models at each time step into linearized system of equations. The main advantage of the Kalman filter is its ability to provide the quality of the estimate (i.e., the variance). For this state space model, we have developed procedures for estimating and predicting the extinction of forests through Kalman filter method. In this model, the observation equations are observed from the deforestation statistics in India in 2009. Finally, we provide numerical simulations through MATLAB to prove our theoretical results.

Keywords: Stochastic Difference Equations, Kalman Filter, Estimation.

1. INTRODUCTION:

Human ecology is a study about the interaction between human beings and their environment. "Deforestation, is the elimination of a forest or a set of trees. Deforestation means altering the forestland and using it for various purposes. Due to industrialization, many forests have been ruined and lot of animals have been dying out. Forests envelop almost a third of the earth's land surface. Forests provides many remuneration namely the hydrologic phase, avoidance of climate change and safeguarding of biodiversity."

As a consequence of deforestation, sinks are no longer capable of sustaining and regulating water pour from rivers to streams. Trees are efficient in water mass intake. The forests acts as an envelope against corrosion. Forests carryout an important task by taking in the rainwater and distributing it to rivers that supply water for cities and agriculture. When there are no forests there is no absorption of rainwater by soil and hence it flows causing floods. Crops in the farm fields are irrigated by water that flow from the hills. Due to floods caused by deforestation, the farm fields receive water that are contaminated with mud. This deterioration in water quality affects the production of crops. This in turn affects the human beings by causing many health problems. Therefore, we need to analyze the use of technology in preserving trees and farm fields.

Tan et al. have proposed a stochastic dynamic model which is referred to as multinomial model. The purpose of the discrete-time Kalman filter is to provide the closed form recursive solution for estimation of linear discrete-time dynamic systems, which can be described by equations of the form . State space approach has been done for HIV models by various authors. Modelling the HIV Epidemic: A State-Space Approach was studied by Hulin Wu and Wai-Yuan Tan[1].

In Section 2, we describe the multinomial model for the deforestation by human population and its effect on farm fields. In Section 3, we formulate a state-space model for the deforestation by human population and its effect on farm fields and introduce the Kalman recursive estimation methods in Section 4. In Section 5, we have provided a numerical illustration of deforestation in India and its effect on the growth of rice crops in India in 2009. Conclusion is given in Section 6.

2. STOCHASTIC MODEL FOR DEFORESTATION:

Let $H(t), B(t)$, $F(t)$ and $T(t)$ denote the human population, density of trees in the forests, density of crops in the farm fields and effort of technology applied to conserve forests using technology at time t respectively [1],[2].

$$X(t) = \left[H(t), B(t), F(t), T(t)\right]^T \tag{1}$$

Table 1. Notation for transitions of the Deforestation due to human population and its effect on farm fields

S.No	Transition	Transition Probability	Transition Number
1	$H \to B$	$\beta_1(t) = 0$	$B_1(t) = 0$
2	$B \to H$	$\alpha_1(t)$	$F_1(t)$
3	$B \to F$	$\beta_2(t)$	$B_2(t)$
4	$F \to B$	$\alpha_2(t) = 0$	$F_2(t) = 0$
5	$F \to H$	$\gamma_1(t)$	$G_1(t)$
6	$H \to F$	$\gamma_2(t)$	$G_2(t)$
7	$H \to T$	$\mu_1(t)$	$M_1(t)$
8	$T \to B$	$\mu_2(t)$	$M_2(t)$
9	$H \to Death$	$d_H(t)$	$D_H(t)$
10	$B \to Death$	$d_B(t)$	$D_B(t)$
11	$F \to Death$	$d_F(t)$	$D_F(t)$
12	$T \to Decay$	$d_T(t)$	$D_T(t)$
13	$immigration \to H$	$\mu_H(t)$	$R_H(t)$

By using the Multinomial Model, we obtain the following Stochastic Difference Equations:

$$\begin{aligned} H(t+1) &= H(t) + R_H(t) - D_H(t) + F_1(t) + G_1(t) - G_2(t) - M_1(t) \\ B(t+1) &= B(t) - F_1(t) - D_B(t) - B_2(t) + M_2(t) \\ F(t+1) &= F(t) + B_2(t) + G_2(t) - G_1(t) \\ T(t+1) &= T(t) + M_1(t) - M_2(t) \end{aligned} \quad (2)$$

The distributional properties of the quantities in these equations are given below:

- $R_H(t) \sim$ Binomial$\left[H(t); \mu_H(t)\right]$ independent of $D_H(t), F_1(t), G_1(t)$ and $G_2(t)$.
- $\left[D_H(t), G_2(t), M_1(t)\right] \| X(t) \sim$ Multinomial $\left[H(t); d_H(t), \gamma_2(t), \mu_1(t)\right]$
- $\left[F_1(t), B_2(t), D_B(t)\right] \| X(t) \sim$ Multinomial$\left[B(t); \alpha_1(t), \beta_2(t), d_B(t)\right]$
- $\left[D_F(t), G_1(t)\right] \| X(t) \sim$ Multinomial$\left[F(t); d_F(t), \gamma_1(t)\right]$
- $\left[D_T(t), M_2(t)\right] \| X(t) \sim$ Multinomial$\left[T(t); d_T(t), \mu_2(t)\right]$

To provide the system of equations in state space model, we rewrite

$$\begin{aligned} H(t+1) &= \left[1+\mu_H(t)-\gamma_2(t)-d_H(t)\right]H(t)+\alpha_1(t)B(t)+\gamma_1(t)F(t)+\varepsilon_H(t) \\ B(t+1) &= \left[1-\alpha_1(t)-\beta_2(t)-d_B(t)\right]B(t)+\mu_2(t)T(t)+\varepsilon_B(t) \\ F(t+1) &= [1-\gamma_1(t)-d_F(t)]F(t)+\gamma_2(t)H(t)+\beta_2(t)B(t)+\varepsilon_F(t) \\ T(t+1) &= [1-\mu_2(t)-d_T(t)]T(t)+\mu_1(t)H(t)+\varepsilon_T(t) \end{aligned} \tag{3}$$

Where

$$\begin{aligned} \varepsilon_H(t) &= \left[R_H(t)-\mu_H(t)H(t)\right]-\left[D_H(t)-d_H(t)H(t)\right]+\left[F_1(t)-\alpha_1(t)B(t)\right] \\ &\quad +\left[G_1(t)-\gamma_1(t)F(t)\right]-\left[G_2(t)-\gamma_2(t)H(t)\right]-\left[M_1(t)-\mu_1(t)H(t)\right] \\ \varepsilon_B(t) &= -\left[F_1(t)-\alpha_1(t)B(t)\right]-\left[D_B(t)-d_B(t)B(t)\right]-\left[B_2(t)-\beta_2(t)B(t)\right]+\left[M_2(t)-\mu_2(t)H(t)\right] \\ \varepsilon_F(t) &= \left[B_2(t)-\beta_2(t)B(t)\right]+\left[G_2(t)-\gamma_2(t)H(t)\right]-\left[D_F(t)-d_F(t)F(t)\right]-\left[G_1(t)-\gamma_1(t)F(t)\right] \\ \varepsilon_T(t) &= \left[M_1(t)-\mu_1(t)H(t)\right]-\left[M_2(t)-\mu_2(t)H(t)\right]+\left[D_T(t)-d_T(t)T(t)\right] \end{aligned} \tag{4}$$

Let $\varepsilon(t)=\left[\varepsilon_H(t),\varepsilon_B(t),\varepsilon_F(t),\varepsilon_T(t)\right]^T$. It can be shown that $\varepsilon(t)$ and $X(t)$ are uncorrelated and mean of $\varepsilon(t)$ is 0. The above stochastic difference equations will be taken as state equations in our state space model.

3. STATE SPACE MODEL:

The state space model in matrix notation is given by

$$\begin{aligned} X(t+1) &= \phi(t)X(t)+\varepsilon_t \\ Y(t) &= H(t)X(t)+\theta_t \end{aligned} \tag{5}$$

where $X(t)$ is the state vector. $Y(t)$ is the observation vector. $\phi(t)$ has been referred to as state transition matrix. $H(t)$ is the observation matrix. ε_t and θ_t are state model noise and observation noise respectively. It is observed that ε_t and θ_t are uncorrelated.

$$E\left(\varepsilon_t\right)=0, E\left(\theta_t\right)=0$$
$$E\left(\varepsilon_t\varepsilon_t^T\right)=Q(t)>0, E\left(\theta_t\theta_t^T\right)=R(t)>0$$
$$E\left(\varepsilon_t\varepsilon_s^T\right)=0, E\left(\theta_t\theta_s^T\right)=0\forall t\neq s$$

For modelling the deforestation by human population, we consider four stages:

$$X(t)=\left[H(t),B(t),F(t),T(t)\right]^T, \varepsilon(t)=\left[\varepsilon_H(t),\varepsilon_B(t),\varepsilon_F(t),\varepsilon_T(t)\right]^T$$

$$\phi(t)=\begin{bmatrix} 1+\mu_H(t)-\gamma_2(t)-d_H(t) & \alpha_1(t) & \gamma_1(t) & 0 \\ 0 & 1-\alpha_1(t)-\beta_2(t)-d_B(t) & 0 & \mu_2(t) \\ \gamma_2(t) & \beta_2(t) & 1-\gamma_1(t) & 0 \\ \mu_1(t) & 0 & 0 & [1-\mu_2(t)-d_T(t)] \end{bmatrix} \quad (6)$$

Let $Y_1(t)$ be the deforestation prevalance at time *t* and $Y_2(t)$ be the total population size of system at time *t*.

$$\begin{aligned} Y_1(t) &= B(t)+\eta_1(t) \\ Y_2(t) &= H(t)+B(t)+F(t)+\eta_2(t) \end{aligned} \quad (7)$$

$\eta_1(t)$ is associated with the error in reports of deforestation, whereas $\eta_2(t)$ is associated with the error in population survey errors. Here ε_t and $\eta(t)$ are uncorrelated.

We have $Y(t)=\left[Y_1(t),Y_2(t)\right]^T$, $\eta(t)=\left[\eta_1(t),\eta_2(t)\right]^T$. Then we have the observation matrix

$$H(t)=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad (8)$$

4. RECURSIVE ESTIMATION:

The kalman filter is one of the most popular estimation techniques that has been largely investigated for state estimation of non linear systems[7].

We use the following notations:

- $\hat{X}_{t+1|t+1}$ - The best linear estimator of the state vector at time $t+1$ based on the observations $Y_1, Y_2 .....Y_{t+1}$.
- $\hat{X}_{t+1|t}$ - The one step ahead prediction of the state vector at time $t+1$ based on the dynamic model.
- $P_{t+1|t+1}=E\left(X_{t+1}-\hat{X}_{t+1|t+1}\right)\left(X_{t+1}-\hat{X}_{t+1|t+1}\right)^T$;The covariance matrix of best state estimator.
- $R_{t+1|t}=E\left(X_{t+1}-\hat{X}_{t+1|t}\right)\left(X_{t+1}-\hat{X}_{t+1|t}\right)^T$;The covariance matrix of predictor.

- K_t - Gain matrix.

The kalman filter equations are given by:

$$\hat{X}_{t+1|t+1} = \hat{X}_{t+1|t} + K_{t+1}\left[Y(t+1) - H(t+1)\,\hat{X}_{t+1|t}\right]$$

$$\hat{X}_{t+1|t} = \phi(t)\,\hat{X}_{t|t}$$

$$K_{t+1} = P_{t+1|t}H^T(t+1)\left[H(t+1)P_{t+1|t}H^T(t+1) + R(t+1)\right]^{-1} \qquad (9)$$

$$P_{t+1|t} = \phi(t)P_{t|t}\phi^T(t) + Q(t)$$

$$P_{t+1|t+1} = \left[I - K_{t+1}H(t+1)\right]P_{t+1|t}$$

Where I is the Identity matrix.

5 . APPLICATION TO THE DEFORESTATION DUE TO HUMAN POPULATION

AND ITS EFFECT ON FARM FIELDS (RICE CROPS):

India is one of the world's largest producers of white rice and brown rice, accounting for 20% of all world rice production. Rice is India's pre-eminent crop, and is the staple food of the people of the eastern and southern parts of the country. Since the production of rice is essential in India, this section deals with how the production of rice in India is affected due to deforestation in India. We have considered the Total forest cover in India 1990-2010 and the deforestation percent in 2005-2010. And we consider the average yield of rice in India 2009 and total rice consumption in India 2015.

Parameters	Sources
$\alpha_1(t) = 0.21$	[4]
$\gamma_1(t) = 0.6$	[5]
$\gamma_2(t) = 0.25$	[6]
$\beta_2(t) = 0.001, \mu_2(t) = 0.001$ $d_H(t) = 0.0007, d_B(t) = 0.008$ $d_F(t) = 0.0003, \mu_1(t) = 0.01$ $d_T(t) = 0.0001, \mu_H(t) = 0.001$	Assumption
Initial Values	**Sources**
X_0 =[50 30 10 3]^T	Dynamic models

Transition Matrix: $\phi(t)=\begin{pmatrix}0.74 & 0.21 & 0.6 & 0\\ 0 & 0.78 & 0 & 0.001\\ 0.25 & 0.001 & 0.39 & 0\\ 0.01 & 0 & 0 & 0.99\end{pmatrix}$	
State Variables $X_t=[H(t),B(t),F(t),T(t)]^T \quad t=1,2,..N$	**Sources** To be estimated by KF Recursion

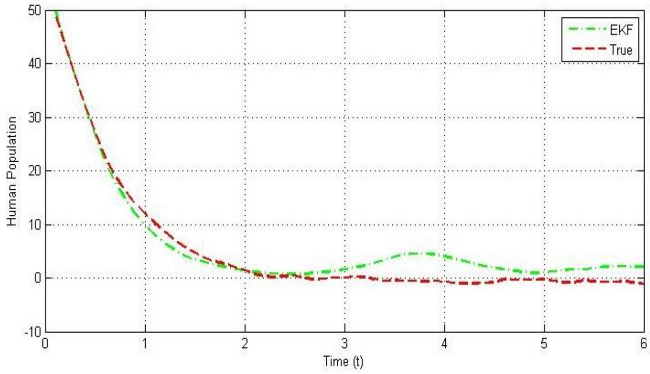

Fig 1: Plots of Observed(True) and Estimated(KF) of Human Population.

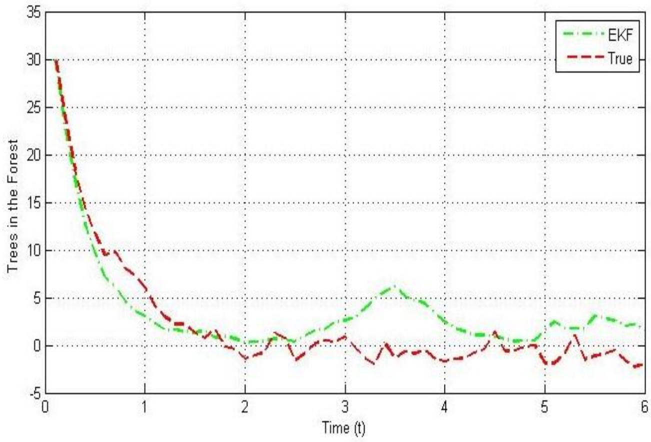

Fig 2: Plots of Observed(True) and Estimated(KF) of Trees in forests.

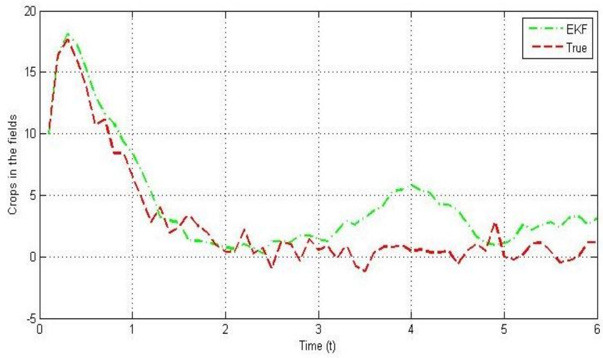

Fig 3: Plots of Observed(True) and Estimated(KF) of Crops in fields.

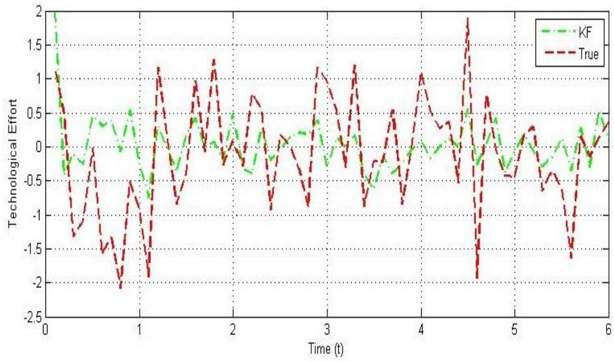

Fig 4: Plots of Observed(True) and Estimated(KF) of Technological effort.

6. CONCLUSION:

In this paper, we have proposed a state space approach for a model on deforestation due human population. We have applied Kalman filter to our model to estimate and predict the extinction of forests and its effects on farm fields. Finally, we provide an Numerical Illustration of Deforestation in India in the year 2009 and its effect on Rice crops and have provided the observed and estimated values of the human population, forest and rice crops in India.

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