International Journal of Difference Equations
ISSN 0973-6069 Volume 12, Number 1 (2017), pp. 41-53

http://www.ripublication.com

EXPONENTIAL SYNCHRONIZATION OF SWITCHED COMPLEX DYNAMICAL NETWORKS WITH TIME-VARYING DELAY VIA PERIODICALLY INTERMITTENT CONTROL

J.Yogambigai [*] **and M. Syed Ali**

Department of Mathematics, Thiruvalluvar University,
Vellore - 632 115, Tamilnadu, India.
E-mail : [*] yogambigaij@gmail.com, syedgru@gmail.com

ABSTRACT

This paper is concerned with the intermittent control problem for a class of switched complex dynamical networks with time-varying delay. Unlike the classical synchronization problem, in our synchronized scheme, the intermittents are adopted to synchronize the concerned neuron states with inter connected nodes. Introducing an appropriate Lyapunov-Krasovskii functional and using matrix inequality techniques, several novel sufficient conditions are derived to guarantee a class of switched complex dynamical networks to be exponentially synchronized, in which both the time-delay and its time variation can be fully considered. The derived criteria are expressed in terms of linear matrix inequalities (LMIS) that can be easily checked by using the standard numerical software. Finally, a numerical example is provided to illustrate the applicability of the proposed approach.

Key Words: Synchronization, Complex dynamical networks, Time-varying delay, Periodically intermittent control, Switching, Linear Matrix Inequality.

1 Introduction

A complex dynamical network (CDNs) is a set of coupled nodes interconnected by edges, in which each node is a dynamical system [1]. CDNs exist everywhere in our daily lives, including World Wide Web, power grids, food webs, ecosystems, Internet, and so on [2]. Since time delays may cause oscillation, divergence or instability, stability analysis for time delayed systems has become a topic of great theoretic and practical importance in recent years [3]. The methods for synchronization of the chaotic systems have been widely studied in recent years, and many different methods have been applied theoretically and experimentally to synchronize chaotic systems; see for instance [4].

In the past few years, the research on the synchronization of CDNs has arrested much attention [5]. Among them, exponential synchronization problem has attracted more attention [6] because of more performance information of system being given. It is hoped that the controller designed make the convergence of the synchronous error signal fast. In the case where the network cannot synchronize by itself, many control techniques including continuous feedback and discontinuous feedback have been developed to drive the network to synchronize [7]. Recently, discontinuous feedback control, such as impulsive control , switch control, and intermittent control, has received wide attention.

Moreover, compared with continuous control approaches, intermittent control is more effective because the system output is measured intermittently rather than continuously [8]. Very recently, the synchronization criterion for switched complex dynamical networks under arbitrary switching has been presented in [9]. In this paper, some periodical intermittent controllers are added to partial nodes to drive the network to synchronize, and exponential synchronization criteria are derived based on rigorous mathematical analysis.

Motivated by the aforementioned discussions, in this paper, exponential synchronization of CDNs with time-varying delays, using periodically intermittent control is studied. By constructing an appropriate Lyapunov–functional and using the refined Jensen's based inequality, a sufficient condition depending on the intermittent period is obtained under which the resulting error dynamical system is exponentially stable. Finally, the exponential synchronization criteria can be solved by LMIs. An illustrative example is given to show the effectiveness of proposed method.

Notations: Throughout the paper, R^n denotes the n dimensional Euclidean space, and $\mathsf{R}^{m\times n}$ is the set of all $m\times n$ real matrices. For symmetric matrices C and D, the notation $\mathsf{C}\geq\mathsf{D}$ means that $\mathsf{C}-\mathsf{D}$ is positive-semi definite; Q^T transpose of the matrix Q; I is the identity matrix with appropriate dimension; '*' * represents the symmetric matrix.

2 Problem statement and Preliminaries

In this section, we formulate an error system for the drive-response synchronization of delayed complex dynamical networks. Consider a class of delayed complex dynamical network models consisting of N identical nodes, which can be described by the following differential equation:

$$\dot{u}_z(t) = -\mathbf{C}_{\eta_k} u_z(t) + \mathbf{D}_{\eta_k} f(u_z(t)) + \mathbf{D}_{\sigma\eta_k} f(u_z(t-\sigma(t))) + \sum_{j=1}^{N} \mathbf{w}_{zj}^{\eta_k} \Gamma_{\eta_k} u_j(t) + J_z, \quad z = 1,2,...,N, \tag{1}$$

where $u_z(t) \in \mathsf{R}^n$ represents the state of the z^{th} node of the system (1), $f : \mathsf{R}^n \to \mathsf{R}^n$ are continuously nonlinear vector functions, $\Gamma \in R^{n\times n}$ represents the diagonal inner-coupling matrix, $\mathbf{W}^{\eta_k} = [\mathbf{w}_{zj}^{\eta_k}]_{N\times N}$ is the coupling configuration matrix of the networks representing the coupling strength and the topological structure of the system. The row sum of $\mathbf{W}^{\eta_k}$ are zero. $\Gamma_{\eta_k} = diag\{b_{a1\eta_k},\ b_{a2\eta_k},....,b_{an\eta_k}\}$ are matrices describing the inner-coupling between the subsystems at time t. $\eta_k : [0,\infty) \to \mathsf{M} = 1,2,...,m$ is the switching signal, which is a piecewise constant function continuous from the right. $J_z(r,t)$ is the input from outside of the networks. The initial conditions are given by $u_z(t) = \psi_{uz}(s) \in C([-\sigma,0],\mathsf{R})$, where $C([-\sigma,0],\mathsf{R})$ denotes the set of all continuous functions from $[-\sigma,0]$ to R. Further we assume that $\sigma(t) > 0$ differentiable and bounded and satisfy

$$0 \le \sigma(t) \le \sigma, \quad \dot{\sigma}(t) \le \sigma_d,$$

where σ and σ_d are known constants. The corresponding response system is established as follows:

$$\dot{v}_z(t) = -\mathbf{C}_{\eta_k} v_z(t) + \mathbf{D}_{\eta_k} f(v_z(t)) + \mathbf{D}_{\sigma\eta_k} f(v_z(t-\sigma(t))) + \sum_{j=1}^{N} \mathbf{w}_{zj}^{\eta_k} \Gamma_{\eta_k} v_j(t) + J_z + w_z(t),$$

$$z = 1,2,...,N, \tag{2}$$

where $v_z(t) \in \mathsf{R}^n$ represents the state of the z^{th} node of the response system. $w_z(t)$ is the control signals and the initial conditions are given by $v_z(t) = \varphi_{vz}(s) \in C([-\sigma,0],\mathsf{R})$. For simplicity, the drive system is supposed to be without disturbances and disturbance dynamics are exhibited in the response system.

In order to give a better explanation, subtract the drive system (1) from the response system (2) and define the synchronization errors as $\mu_z(t) = u_z(t) - v_z(t)$. The error dynamical system is expressed as

$$\left.\begin{aligned} &\dot{\mu}_z(t) = -\mathbf{C}_{\eta_k}\mu_z(t) + \mathbf{D}_{\eta_k} g(\mu_z(t)) + \mathbf{D}_{\sigma\eta_k} g(\mu_z(t-\sigma(t))) + \sum_{j=1}^{N} \mathbf{w}_{zj}^{\eta_k}\Gamma_{\eta_k}\mu_j(t) - w_z(t), \\ &\mu_z(t) = \psi_z(s), s \in [-\sigma, 0], \quad z = 1,2,...,N, \end{aligned}\right\} \tag{3}$$

where $\psi_z(s) = \phi_z(s) - \varphi_z(s)$,
$g(\mu_z(t)) = f(u_z(t)) - f(v_z(t), g(\mu_z(t-\sigma(t))) = f(u_z(t-\sigma(t))) - f(v_z(t-\sigma(t))$

Assumption (H): The activation function $g_i(\cdot)$ is bounded and there exist constants l_i^-, l_i^+ such that $l_i^- < l_i^+$ and

$$l_i^- \le \frac{g(\eta)}{\eta} \le l_i^+, g(0) = 0, \textit{ for all } \eta \ne 0, \tag{4}$$

For the sake of simplicity, we denote

$$L_1^- = diag\{l_1^- l_1^+, l_2^- l_2^+, ..., l_n^- l_n^+\}, L_1^+ = diag\{\frac{l_1^- + l_1^+}{2}, \frac{l_2^- + l_2^+}{2}, ..., \frac{l_n^- + l_n^+}{2}\},$$

Now, we consider an intermittent state feedback controller as follows:

$$w_z(t) = \begin{cases} w_z(t) = \mathbf{K}_{\eta_k}(u_z(t) - v_z(t)) = \mathbf{K}_{\eta_k}\mu_z(t), & t_k \le t < t_k + d_k, \\ 0, \quad t_k + d_k \le t < t_k + 1, \end{cases} \tag{5}$$

where $k \in N$, $\mathbf{K}_{\eta_k}$ is the control gain matrix, $t_{k+1} - t_k$ is control period , d_k is the control width. The control instant is defined by $0 = t_1 < t_2 < ...t_k..., \lim_{k\to\infty} t_k = \infty$, t_k is the updating instant time of the Zero-Order-Hold (ZOH).

Under control law (5), system (3) can be written as

$$\begin{cases} \dot{\mu}_z(t) & = -(\mathbf{C}_{\eta_k} + \mathbf{K}_{\eta_k})\mu_z(t) + \mathbf{D}_{\eta_k} g(\mu_z(t)) + \mathbf{D}_{\sigma\eta_k} g(\mu_z(t-\sigma(t))) \\ & \quad + \sum_{j=1}^{N} \mathbf{w}_{zj}^{\eta_k}\Gamma_{\eta_k}\mu_j(t), \quad t_k \le t < t_k + d_k, \\ \dot{\mu}_z(t) & = -\mathbf{C}_{\eta_k}\mu_z(t) + \mathbf{D}_{\eta_k} g(\mu_z(t)) + \mathbf{D}_{\sigma\eta_k} g(\mu_z(t-\sigma(t))) \\ & \quad + \sum_{j=1}^{N} \mathbf{w}_{zj}^{\eta_k}\Gamma_{\eta_k}\mu_j(t), \quad t_k + d_k \le t < t_k + 1, \end{cases} \tag{6}$$

Before giving our main result, we present the following definition and lemmas which will be used in the proof of the main theorem.

Definition 2.1 *[10] Drive-response systems (1) and (2) are said to be exponentially synchronized under switching signal* η_k *if the error system (6) is exponentially stable, i.e., there exist scalars* $\alpha > 0$ *and* $\delta \geq 1$ *such that*

$$\|\mu_z(t,\psi)\| \leq \delta\psi e^{-\alpha t}, \psi = \sup_{-\sigma\leq\theta\leq 0} \|\mu_z(\theta)\|, \quad 0.2cm \forall\, t \geq 0, \tag{7}$$

where the constant α, is defined as the exponential synchronization rate.

lemma 2.2 *(Jensen's inequality [11]) For any constant matrix* $N > 0$ *and scalars* $\beta > \alpha > 0$ *such that the following integrations are well defined, then*

$$-(\gamma-\beta)\int_\beta^\gamma \mu^T(s) N \mu(s)ds \leq -\int_\beta^\gamma \mu^T(s)ds\, N \int_\beta^\alpha \mu(s)ds,$$

lemma 2.3 *(Refined Jensen-based inequality [12]) For a given matrix* $R \in S_n^+$ *and a function* $\varphi:[a,b] \to \mathbb{R}^n$ *whose derivative* $\dot{\varphi} \in C([a,b],\mathbb{R}^n)$ *,the following inequalities hold:* $\int_a^b \dot{\varphi}^T(s)R\dot{\varphi}(s)ds \geq \frac{1}{b-a}\hat{\chi}\bar{R}\hat{\chi},$ *where* $\bar{R} = diag\{R, 3R, 5R\}$,

$\hat{\chi} = [\chi_1^T\ \chi_2^T\ \chi_3^T]^T$, $\chi_1 = \varphi(b)-\varphi(a)$, $\chi_2 = \varphi(b)+\varphi(a)-\frac{2}{b-a}\int_a^b \varphi(s)ds,$

$\chi_3 = \varphi(b)-\varphi(a)+\frac{6}{b-a}\int_a^b \varphi(s)ds - \frac{12}{(b-a)^2}\int_a^b\int_s^b \varphi(u)duds.$

3 Main results

By utilizing the Kronecker product, the error dynamical networks (6) can be written in a compact form as

$$\begin{cases} \dot{\mu}(t) & = -(\mathbf{C}_{\eta_k}+\mathbf{K}_{\eta_k})\mu(t)+\mathbf{D}_{\eta_k}g(\mu(t))+\mathbf{D}_{\sigma\eta_k}g(\mu(t-\sigma(t))) \\ & \quad +(\mathbf{W}^{\eta_k}\otimes\Gamma_{\eta_k})\mu(t), \quad t_k \leq t < t_k+d_k, \\ \dot{\mu}(t) & = -\mathbf{C}_{\eta_k}\mu(t)+\mathbf{D}_{\eta_k}g(\mu(t))+\mathbf{D}_{\sigma\eta_k}g(\mu(t-\sigma(t))) \\ & \quad +(\mathbf{W}^{\eta_k}\otimes\Gamma_{\eta_k})\mu(t), \quad t_k+d_k \leq t < t_k+1, \end{cases} \tag{8}$$

where $\mu(t) = [\mu_{1z}^T(t)\ \mu_{2z}^T(t)\ .....\mu_{Nz}^T(t)]^T, \mathbf{K}_{\eta_k} = diag\{\mathbf{K}_{1\eta_k}\ \mathbf{K}_{2\eta_k}\ .....\mathbf{K}_{N\eta_k}\},$

$$g(\mu_z(t)) = [g^T(\mu_{1z}(t))\ g^T(\mu_{2z}(t))\ .....g^T(\mu_{Nz}(t))]^T,$$

$$g(\mu_z(t-\sigma(t))) = [g^T(\mu_{1z}(t-\sigma(t)))\ g^T(\mu_{2z}(t-\sigma(t)))\ .....g^T(\mu_{Nz}(t-\sigma(t)))]^T.$$

Theorem 3.1 *Consider the drive system (1) and the response system (2) with time-varying delays,* $\sigma(t)$*, satisfying (4) and (5). For given constants* α*,* β*, if there exist matrices* $\mathbf{P}_{\eta_k} > 0, \mathbf{R}_{\eta_k} > 0, \mathbf{S}_{\eta_k} > 0$*,* $\mathbf{S}_{1\eta_k}, \mathbf{S}_{2\eta_k}$ $n \times n$*-matrices.*

$$\Omega_{\eta_k} = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & 0 & \Xi_{15} & \mathbf{Q}_{\eta_k}\mathbf{D}_{\sigma\eta_k} & \Xi_{17} & \Xi_{18} \\ * & \sigma\mathbf{S}_{2\eta_k}\mathbf{Q}_{\eta_k} & 0 & 0 & \mathbf{Q}_{\eta_k}\mathbf{D}_{\eta_k} & \mathbf{Q}_{\eta_k}\mathbf{D}_{\sigma\eta_k} & 0 & 0 \\ * & * & \Xi_{33} & 0 & 0 & 0 & \Xi_{37} & \Xi_{38} \\ * & * & * & \Xi_{44} & 0 & \mathbf{S}_{2\eta_k}\Sigma_{2\eta_k} & 0 & 0 \\ * & * & * & * & T-\mathbf{S}_{1\eta_k} & 0 & 0 & 0 \\ * & * & * & * & * & -\mathbf{S}_{2\eta_k} & 0 & 0 \\ * & * & * & * & * & * & \Xi_{77} & \Xi_{78} \\ * & * & * & * & * & * & * & \Xi_{88} \end{bmatrix} < 0, \quad (9)$$

$$\tilde{\Omega}_{\eta_k} = \begin{bmatrix} \tilde{\Xi}_{11} & \tilde{\Xi}_{12} & \tilde{\Xi}_{13} & 0 & \tilde{\Xi}_{15} & \mathbf{Q}_{\eta_k}\mathbf{D}_{\sigma\eta_k} & \tilde{\Xi}_{17} & \tilde{\Xi}_{18} \\ * & \sigma\mathbf{S}_{2\eta_k}\mathbf{Q}_{\eta_k} & 0 & 0 & \mathbf{Q}_{\eta_k}\mathbf{D}_{\eta_k} & \mathbf{Q}_{\eta_k}\mathbf{D}_{\sigma\eta_k} & 0 & 0 \\ * & * & \tilde{\Xi}_{33} & 0 & 0 & 0 & \tilde{\Xi}_{37} & \tilde{\Xi}_{38} \\ * & * & * & \tilde{\Xi}_{44} & 0 & \mathbf{S}_{2\eta_k}\Sigma_{2\eta_k} & 0 & 0 \\ * & * & * & * & -\mathbf{S}_{1\eta_k} & 0 & 0 & 0 \\ * & * & * & * & * & -\mathbf{S}_{2\eta_k} & 0 & 0 \\ * & * & * & * & * & * & \tilde{\Xi}_{77} & \tilde{\Xi}_{78} \\ * & * & * & * & * & * & * & \tilde{\Xi}_{88} \end{bmatrix} < 0, \quad (10)$$

where

$$\Xi_{11} = \alpha\mathbf{P}_{\eta_k} + \mathbf{R}_{\eta_k} - e^{-\alpha\sigma}9\mathbf{S}_{2\eta_k}\mathbf{Q}_{\eta_k}\mathbf{C}_{\eta_k} - 2\mathbf{Q}_{\eta_k}\mathbf{K}_{\eta_k} - \mathbf{S}_{1\eta_k}\Sigma_{1\eta_k}, \Xi_{12} = \mathbf{P}_{\eta_k} - \mathbf{C}^T_{\eta_k}\mathbf{Q}^T_{\eta_k} - \mathbf{K}^T_{\eta_k}\mathbf{Q}^T_{\eta_k},$$

$$\Xi_{13} = e^{-\alpha\sigma}3\mathbf{S}_{\eta_k}, \Xi_{15} = \mathbf{Q}_{\eta_k}\mathbf{W}_{1\eta_k} + \mathbf{S}_{1\eta_k}\Sigma_{2\eta_k}, \Xi_{17} = -e^{-\alpha\sigma}\frac{24}{\sigma}\mathbf{S}_{\eta_k}, \Xi_{18} = e^{-\alpha\sigma}\frac{60}{\sigma^2}\mathbf{S}_{\eta_k}, \Xi_{33} = -e^{-\alpha\sigma}9\mathbf{S}_{\eta_k},$$

$$\Xi_{37} = e^{-\alpha\sigma}\frac{36}{\sigma}\mathbf{S}_{\eta_k}, \Xi_{38} = e^{-\alpha\sigma}\frac{-60}{\sigma^2}\mathbf{S}_{\eta_k}, \Xi_{44} = -\mathbf{R}_{\eta_k}(1-\sigma_d)e^{-\alpha\sigma} - \mathbf{S}_{\eta_k}\Sigma_{2\eta_k}, \Xi_{77} = -e^{-\alpha\sigma}\frac{192}{\sigma^2}\mathbf{S}_{\eta_k},$$

$$\Xi_{78} = -e^{-\alpha\sigma}\frac{360}{\sigma^3}\mathbf{S}_{\eta_k}, \Xi_{88} = -e^{-\alpha\sigma}\frac{-720}{\sigma^4}\mathbf{S}_{\eta_k}, \tilde{\Xi}_{11} = \alpha\mathbf{P}_{\eta_k} + \mathbf{R}_{\eta_k} - 9\mathbf{S}_{\eta_k} - 2\mathbf{Q}_{\eta_k}\mathbf{C}_{\eta_k} - \mathbf{S}_{1\eta_k}\Sigma_{1\eta_k},$$

$$\tilde{\Xi}_{12} = \mathbf{P}_{\eta_k} - \mathbf{C}^T_{\eta_k}\mathbf{Q}^T_{\eta_k}, \tilde{\Xi}_{13} = e^{-\alpha\sigma}3\mathbf{S}_{\eta_k}, \tilde{\Xi}_{15} = 2\mathbf{Q}_{\eta_k}\mathbf{D}_{\eta_k} + \mathbf{S}_{1\eta_k}\Sigma_{2\eta_k}, \tilde{\Xi}_{17} = -e^{-\alpha\sigma}\frac{24}{\sigma}\mathbf{S}_{\eta_k},$$

$$\tilde{\Xi}_{18} = e^{-\alpha\sigma}\frac{60}{\sigma^2}\mathbf{S}_{\eta_k}, \tilde{\Xi}_{33} = -e^{-\alpha\sigma}9\mathbf{S}_{\eta_k}, \tilde{\Xi}_{37} = \frac{36\mathbf{S}_{\eta_k}}{\sigma}e^{-\alpha\sigma}, \tilde{\Xi}_{38} = \frac{-60\mathbf{S}_{\eta_k}}{\sigma^2}e^{-\alpha\sigma},$$

$\tilde{\tilde{\Xi}}_{44} = -\mathbf{R}_{\eta_k}(1-\sigma_d)e^{-\alpha\sigma} - \mathbf{S}_{2\eta_k}\Sigma_{2\eta_k}, \tilde{\tilde{\Xi}}_{77} = -e^{-\alpha\sigma}\frac{192}{\sigma^2}\mathbf{S}_{\eta_k}, \tilde{\tilde{\Xi}}_{78} = -e^{-\alpha\sigma}\frac{360}{\sigma^3}\mathbf{S}_{\eta_k}, \tilde{\tilde{\Xi}}_{88} = -e^{-\alpha\sigma}\frac{-720}{\sigma^4}\mathbf{S}_{\eta_k},$
then the intermittent state-feedback controller (5) can make the response system (2) synchronize with the drive system (1) and the exponential synchronization rate is α. Moreover, desired controller gain matrix is given as $\mathbf{K}_{\eta_k} = \mathbf{Q}_{\eta_k}^{-1}\mathbf{X}_{\eta_k}$.

Proof: Choose a Lyapunov–Krasovskii functional as follows

$$V(t) = \sum_{i=1}^{3} V_i(t), \tag{11}$$

where $V_1(t) = \mu^T(t)\mathbf{P}_{\eta_k}\mu(t),$

$$V_2(t) = \int_{t-\sigma(t)}^{t} e^{\alpha(s-t)}\mu^T(s)\mathbf{R}_{\eta_k}\mu(s)ds,$$

$$V_3(t) = \int_{-\sigma}^{0}\int_{t+\theta}^{t} e^{\alpha(s-t)}\dot{\mu}^T(s)\mathbf{S}_{\eta_k}\dot{\mu}(s)dsd\theta.$$

The time derivative of $V(t)$ along the trajectories of (8) satisfy

$$\dot{V}_1(t) = -\alpha V_1(t) + \alpha\mu^T(t)\mathbf{P}_{\eta_k}\mu(t) + 2\mu^T(t)\mathbf{P}_{\eta_k}\dot{\mu}(t), \tag{12}$$

$$\dot{V}_2(t) \le -\alpha V_2(t) + [\mu^T(t)\mathbf{R}_{\eta_k}\mu(t) - (1-\mu)e^{-\alpha\sigma}\mu^T(t-\sigma(t))\mathbf{R}_{\eta_k}\mu(t-\sigma(t))], \tag{13}$$

$$\dot{V}_3(t) = -\alpha V_3(t) + \sigma\dot{\mu}^T(t)\mathbf{S}_{\eta_k}\dot{\mu}(t) - e^{-\alpha\sigma}\int_{t-\sigma}^{t}\dot{\mu}^T(s)\mathbf{S}_{\eta_k}\dot{\mu}(s)ds. \tag{14}$$

Using Lemma 2.3, we obtain

$$-\int_{t-\sigma}^{t}\dot{\mu}^T(s)\mathbf{S}_{\eta_k}\dot{\mu}(s)ds \le \begin{bmatrix} \mu(t) \\ \mu(t-\sigma(t)) \\ \frac{1}{\sigma}\int_{t-\sigma}^{t}\mu(s)ds \\ \frac{2}{\sigma^2}\int_{t-\sigma}^{t}\int_{\theta}^{t}\mu(s)dsd\theta \end{bmatrix}^T \begin{bmatrix} -9\mathbf{S}_{\eta_k} & 3\mathbf{S}_{\eta_k} & -28\mathbf{S}_{\eta_k} & 30\mathbf{S}_{\eta_k} \\ 3\mathbf{S}_{\eta_k} & -9\mathbf{S}_{\eta_k} & 36\mathbf{S}_{\eta_k} & -30\mathbf{S}_{\eta_k} \\ -24\mathbf{S}_{\eta_k} & 36\mathbf{S}_{\eta_k} & -192\mathbf{S}_{\eta_k} & 180\mathbf{S}_{\eta_k} \\ 30\mathbf{S}_{\eta_k} & -30\mathbf{S}_{\eta_k} & 180\mathbf{S}_{\eta_k} & -180\mathbf{S}_{\eta_k} \end{bmatrix}$$

$$\times \begin{bmatrix} \mu(t) \\ \mu(t-\sigma(t)) \\ \frac{1}{\sigma}\int_{t-\sigma}^{t}\mu(s)ds \\ \frac{2}{\sigma^2}\int_{t-\sigma}^{t}\int_{\theta}^{t}\mu(s)dsd\theta \end{bmatrix} \tag{15}$$

From Assumption (H), we can get

$$0 \le \begin{bmatrix} \mu(t) \\ g(\mu(t)) \end{bmatrix}^T \begin{bmatrix} -\mathbf{S}_{1\eta_k}\Sigma_{1\eta_k} & \mathbf{S}_{1\eta_k}\Sigma_{2\eta_k} \\ * & -\mathbf{S}_{1\eta_k} \end{bmatrix} \begin{bmatrix} \mu(t) \\ g(\mu(t)) \end{bmatrix}, \tag{16}$$

$$0 \le \begin{bmatrix} \mu(t-\tau(t)) \\ g(\mu(t-\tau(t))) \end{bmatrix}^T \begin{bmatrix} -\mathbf{S}_{2\eta_k}\Sigma_{1\eta_k} & \mathbf{S}_{2\eta_k}\Sigma_{2\eta_k} \\ * & -\mathbf{S}_{2\eta_k} \end{bmatrix} \begin{bmatrix} \mu(t-\tau(t)) \\ g(\mu(t-\tau(t))) \end{bmatrix}. \tag{17}$$

When $t_k \le t < t_k + d_k$ from the first of model (8), we have

$$0 = 2(\mu^T(t) + \dot{\mu}^T(t))\mathbf{Q}_{\eta_k}(-\dot{\mu}(t) - (\mathbf{C}_{\eta_k} + \mathbf{K}_{\eta_k})\mu(t) + \mathbf{D}_{\eta_k} g(\mu(t))$$

$$+\mathbf{D}_{\sigma\eta_k} g(\mu(t-\sigma(t))) + (\mathbf{W}^{\eta_k} \otimes \Gamma_{\eta_k})\mu(t)). \tag{18}$$

Therefore, we obtain from (11)-(18) that

$$dV(t) \le \mu^T(t)(-2\mathbf{Q}_{\eta_k}\mathbf{K}_{\eta_k})\mu(t) + \mu^T(t)(-2\mathbf{K}^T_{\eta_k}\mathbf{Q}^T_{\eta_k})\dot{\mu}(t)$$

$$= -\alpha V(t) + \xi^T(t)\Omega_{\eta_k}\xi(t)$$

$$\le -\alpha V(t), \tag{19}$$

where

$$\xi(t) = [(\mu^T(t)\ \dot{\mu}^T(t)\ \mu^T(t-\tau)\ g^T(\mu(t))\ g^T(\mu(t-\tau(t)))\int_{t-\tau}^{t}\mu(s)ds\int_{t-\tau}^{t}\int_{\theta}^{t}\mu^T(s)ds)]^T,$$

$$\Pi_{\eta_k}(t) = -\alpha V(t) + \mu^T(t)(\alpha\mathbf{P}_{\eta_k} + \mathbf{R}_{\eta_k} - e^{-\alpha\sigma}9\mathbf{S}_{\eta_k} - 2\mathbf{Q}_{\eta_k}\mathbf{C}_{\eta_k} + 2\mathbf{Q}_{\eta_k}(\mathbf{W}^{\eta_k} \otimes \Gamma_{\eta_k})$$

$$-\mathbf{S}_{1\eta_k}\Sigma_{1\eta_k})\mu(t) + \mu^T(t)(\mathbf{P}_{\eta_k} - 2\mathbf{C}^T_{\eta_k}\mathbf{Q}^T_{\eta_k} - 2\mathbf{K}^T_{\eta_k}\mathbf{Q}^T_{\eta_k} + 2(\mathbf{W}^{\eta_k} \otimes \Gamma_{\eta_k})^T\mathbf{Q}^T_{\eta_k})\dot{\mu}(t)$$

$$+\mu^T(t)(e^{-\alpha\sigma}3\mathbf{S}_{\eta_k})\mu(t-\sigma) + \mu^T(t)(2\mathbf{Q}_{\eta_k}\mathbf{D}_{\eta_k} + \mathbf{S}_{1\eta_k}\Sigma_{1\eta_k})g(\mu(t)) + \mu^T(t)(2\mathbf{Q}_{\eta_k}\mathbf{D}_{\sigma\eta_k})$$

$$\times g(\mu(t-\sigma(t))) + \mu^T(t)(e^{-\alpha\sigma}\frac{-28\mathbf{S}_{\eta_k}}{\sigma})\int_{t-\sigma}^{t}\mu(s)ds + \mu^T(t)((e^{-\alpha\sigma}\frac{60\mathbf{S}_{\eta_k}}{\sigma^2})\int_{t-\sigma}^{t}\int_{\theta}^{t}\mu(s)dsd\theta$$

$$+\dot{\mu}^T(t)(\sigma\mathbf{S}_{\eta_k} - \mathbf{Q}_{\eta_k})\dot{\mu}(t) + \dot{\mu}^T(t)(2\mathbf{Q}_{\eta_k}\mathbf{D}_{\eta_k})g(\mu(t)) + \dot{\mu}^T(t)(2\mathbf{Q}_{\eta_k}\mathbf{D}_{\sigma\eta_k})g(\mu(t-\sigma(t)))$$

$$+\mu^T(t-\sigma)(-e^{-\alpha\sigma}9\mathbf{S}_{\eta_k})\mu(t-\sigma) + \mu^T(t-\sigma)(-e^{-\alpha\sigma}\frac{36\mathbf{S}_{\eta_k}}{\sigma})\int_{t-\sigma}^{t}\mu(s)ds + \mu^T(t-\sigma)$$

$$\times(-e^{-\alpha\sigma}\frac{60\mathbf{S}_{\eta_k}}{\sigma^2})\int_{t-\sigma}^{t}\int_{\theta}^{t}\mu(s)dsd\theta+\mu^T(t-\sigma(t))(-\mathbf{R}_{\eta_k}(1-\sigma_d)e^{-\alpha\sigma}-\mathbf{S}_{2\eta_k}\Sigma_{2\eta_k})$$

$$\times\mu(t-\sigma(t))+\mu^T(t-\sigma(t))(\mathbf{S}_{2\eta_k}\Sigma_{2\eta_k})g(\mu(t-\sigma(t))+g^T(e(t))(-\mathbf{S}_{1\eta_k})g(\mu(t))$$

$$+g^T(\mu(t-\sigma(t))(-\mathbf{S}_{2\eta_k})g(\mu(t-\sigma(t))+\int_{t-\sigma}^{t}\mu^T(s)ds((-e^{-\alpha\sigma}\frac{-192\mathbf{S}_{\eta_k}}{\sigma^2}))\int_{t-\sigma}^{t}\mu(s)ds$$

$$+\int_{t-\sigma}^{t}\mu^T(s)ds((-e^{-\alpha\sigma}\frac{360\mathbf{S}_{\eta_k}}{\sigma^3}))\int_{t-\sigma}^{t}\int_{\theta}^{t}\mu(s)dsd\theta$$

$$+\int_{t-\sigma}^{t}\int_{\theta}^{t}\mu^T(s)ds((-e^{-\alpha\sigma}\frac{-720\mathbf{S}_{\eta_k}}{\sigma^4}))\int_{t-\sigma}^{t}\int_{\theta}^{t}\mu(s)dsd\theta$$

Namely, we have $V(t)\le V(t_k)e^{-\alpha(t-t_k)},\quad t_k\le t<t_k+d_k.$ (20)

When $t_k+d_k\le t<t_k+1$, from second equation of model (8) we have

$$0=2(\mu^T(t)+\dot{\mu}^T(t))\mathbf{Q}_{\eta_k}(-\dot{\mu}(t)-\mathbf{C}_{\eta_k}\mu(t)+\mathbf{D}_{\eta_k}g(\mu(t))+\mathbf{D}_{\sigma\eta_k}g(\mu(t-\sigma(t)))+(\mathbf{W}^{\eta_k}\otimes\Gamma_{\eta_k})\mu(t)). \quad (21)$$

It follows from (11)-(17), (19) and (21)

$$dV(t)\le\Pi_{\eta_k}(t)$$

$$=-\alpha V(t)+\xi^T(t)\tilde{\Omega}_{\eta_k}\xi(t)+(\alpha+\beta)\mu^T(t)\mathbf{P}\mu(t)$$

$$\le\beta V(t). \quad (22)$$

Namely,wehave $V(t)\le V(t_k+d_k)e^{\beta}(t-t_k-d_k),\quad t_k+d_k\le t<t_k+1,k\in N$ (23)

From (18) and (21), it follows that

1. When $0=t_1\le t<d_1,$ we have from (11) that

$$V(t)\le V(0)e^{-\alpha(t)} \quad (24)$$

2. When $d_1\le t<t_2$ we have from (21) and (22) that

$$V(t)\le V(d_1)e^{\beta(t-d_1)}\le V(0)e^{-\alpha d_1+\beta(t-d_1)} \quad (25)$$

3. When $t_2\le t<d_2$ we have from (18) and (23) that

$$V(t)\le V(t_2)e^{\alpha(t-t_2)}$$

$$\leq V(0)e^{-\alpha d_1+\beta(t-d_1)-\alpha(t-t_2)} \tag{26}$$

4. When $t_2+d_2 \leq t < t_3$ we have from (21) and (24) that

$$V(t) \leq V(t_2+d_2)e^{\beta(t-t_2-d_2)}$$

$$\leq V(0)e^{-\alpha(d_1+d_2)+\beta(t_2-d_1)+\beta(t-t_2-d_2)} \tag{27}$$

5. When $t_3 \leq t < t_3+d_3$ we have from (18) and (25) that

$$V(t) \leq V(t_3)e^{\beta(t-t_3)}$$

$$\leq V(0)e^{-\alpha(d_1+d_2)+\beta(t_3-d_1-d_2)-\alpha(t-t_3)} \tag{28}$$

6. When $t_3+d_3 \leq t < t_4$ we have from (21) and (26) that

$$V(t) \leq V(t_3+d_3)e^{\beta(t-t_3-d_3)}$$

$$\leq V(0)e^{-\alpha(d_1+d_2+d_3)+\beta(t_3-d_1-d_2)+\beta(t-t_3-d_3)} \tag{29}$$

By induction, we have the following estimate of $V(t)$

7. When $t_k \leq t < t_k+d_k$ we have

$$V(t) \leq V(0)e^{-\alpha(d_1+d_2+...d_k-1)} + \beta(t_k-d_1-d_2-...d_k-1)-\alpha(t-t_k)$$

$$\leq V(0)e^{\beta t_k-(\alpha+\beta)}\Sigma_{i=1}^{k-1}d_i \tag{30}$$

8. When $t_k+d_k \leq t < t_k+1$ we have from that

$$V(t) \leq V(0)e^{-\alpha(d_1+d_2+...d_k)+\beta(t_k-d_1-d_2-...d_k-1)+\beta(t-t_k-d_k)}$$

$$\leq V(0)e^{-\alpha d_k+\beta t_k-(\alpha+\beta)\Sigma_{i=1}^{k-1}d_i+|\beta|(t_k+1-t_k-d_k)}$$

$$\leq e^{|\beta|}cV(0)e^{\beta t_k-(\alpha+\beta)\Sigma_{i=1}^{k-1}d_i} \tag{31}$$

It follows from(28) and(29) that

$$V(t) \leq e^{|\beta|c}V(0)e^{\beta t_k} - (\alpha+\beta)\Sigma_{i=1}^{k-1}d_i. \tag{32}$$

Additionally, it follows from (11) that

$$V(0) = \mu^T(0)\mathbf{P}\mu(0) + \int_{0-\sigma(0)}^{0} e^{\alpha(s-0)}\mu^T(s)\mathbf{R}\mu(s)ds + \int_{-\sigma}^{0}\int_{0+\theta}^{0} e^{\alpha(s-0)}\dot{\mu}^T(s)\mathbf{S}\dot{\mu}(s)dsd\theta$$

$$\leq (\lambda_{max}(\mathbf{P}) + \sigma e^{-\alpha\sigma}\lambda_{max}(\mathbf{R}) + \frac{\sigma^2}{2} e^{-\alpha\sigma}\lambda_{max}(\mathbf{S})\|\psi(t)\|^2, \tag{33}$$

and $\quad V(t) \geq \lambda_{min}(P)\|\mu(t)\|^2. \qquad (34)$

Now, from (32)-(34), we get

$$V(t) \leq \delta e^{-\alpha t}, \tag{35}$$

which means $\|\mu(t)\| \leq \delta e^{-\alpha t}$. This proof is complete.

4 Numerical Examples

Example 4.1 *Consider a delayed complex dynamical network as follows:*

$$\dot{\mu}(t) = -\mathbf{C}_{\eta_k}\mu(t) + \mathbf{D}_{\eta_k} g(\mu(t)) + \mathbf{D}_{\sigma\eta_k} g(\mu(t-\sigma(t))) + (\mathbf{W}^{\eta_k} \otimes \Gamma_{\eta_k})\mu(t) - w(t) \tag{36}$$

$$\mathbf{C}_1 = \begin{bmatrix} 1.4 & 0 \\ 0 & 1.5 \end{bmatrix}, \quad \mathbf{D}_1 = \begin{bmatrix} 0.4 & 0.7 \\ 0.1 & 0 \end{bmatrix}, \quad \mathbf{D}_{\sigma 1} = \begin{bmatrix} 0.2 & 0.6 \\ 0.5 & 0.1 \end{bmatrix}, \mathbf{C}_2 = \begin{bmatrix} 1.2 & 0 \\ 0 & 1.3 \end{bmatrix},$$

$$\mathbf{D}_2 = \begin{bmatrix} 1.2 & 0.1 \\ 0.4 & -1.3 \end{bmatrix}, \mathbf{D}_{\sigma 2} = \begin{bmatrix} 1.18 & 0 \\ 0 & 1.9 \end{bmatrix}, \quad \Gamma_1 = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 1 & -0 \\ 0 & 1 \end{bmatrix},$$

$$\Sigma_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \Sigma_{21} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \Sigma_{12} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad \Sigma_{22} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\mathbf{W}^1 = \mathbf{W}^2 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}.$$

Let $\sigma = 0.9, \alpha = 0.1,$ and $\sigma_d = 0.5$ applying Theorem 3.1 the feasible solutions of LMIs (10)-(11) are,

$$\mathbf{P}_1 = \begin{bmatrix} 27.0591 & 5.6001 \\ 5.6001 & 25.8565 \end{bmatrix}, \quad \mathbf{P}_2 = \begin{bmatrix} 20.9461 & 0.5722 \\ 0.5722 & 18.8554 \end{bmatrix}, \quad \mathbf{R}_1 = \begin{bmatrix} 24.6731 & -5.6337 \\ -5.6337 & 22.0174 \end{bmatrix},$$

$$\mathbf{R}_2 = \begin{bmatrix} 31.5279 & -21.8946 \\ -21.8946 & 36.7379 \end{bmatrix}, \quad \mathbf{S}_1 = \begin{bmatrix} 30.1205 & -15.6843 \\ -15.6843 & 28.7725 \end{bmatrix}, \quad \mathbf{S}_2 = \begin{bmatrix} 12.8312 & -5.1212 \\ -5.1212 & 10.0670 \end{bmatrix},$$

$$\mathbf{S}_{11} = \begin{bmatrix} 30.6455 & -0.7511 \\ -0.7511 & 31.0991 \end{bmatrix}, \quad \mathbf{S}_{12} = \begin{bmatrix} 32.7119 & -0.1027 \\ -0.1027 & 32.6227 \end{bmatrix}, \quad \mathbf{S}_{21} = \begin{bmatrix} 31.7055 & -3.0134 \\ -3.0134 & 30.9819 \end{bmatrix},$$

$$\mathbf{K}_1 = \mathbf{Q}_1^{-1}\mathbf{X}_1 = \begin{bmatrix} -0.5417 & 0.1272 \\ 0.1252 & -0.4029 \end{bmatrix}, \quad \mathbf{K}_2 = \mathbf{Q}_2^{-1}\mathbf{X}_2 = \begin{bmatrix} -0.6556 & 0.0623 \\ 0.0680 & -0.6821 \end{bmatrix}.$$

Therefore, we know from Theorem 3.1 that the origin of system (1) and (2) is globally exponentially synchronized.

5 conclusion

In this paper, we have considered the problem of exponential synchronization for a class of complex dynamical networks with time-varying delays via periodically intermittent control. By utilizing a appropriate Lyapunov-Krasovskii functional, we have shown that the exponential synchronization problem of complex dynamical networks is solvable if a set of linear matrix inequalities (LMIs) are feasible. A unified LMI approach has been developed to establish sufficient conditions for the delayed complex dynamical networks to be exponentially synchronized, and a numerical example has been provided to show the usefulness of the proposed synchronization condition.

REFERENCES

[1] Bullmore, E., Sporns, O., 2009, "Complex brain networks: graph theoretical analysis of structural and functional systems," Nature Reviews Neuroscience **10** pp. 186–198.

[2] Yi, J. W., Wang, Y. W., Xiao, J. W., Huang, and Y., 2013, "Exponential synchronization of complex dynamical networks with Markovian jumping parameters and stochastic delays and its application to multi-agent systems," Commun. Nonlinear Sci. Numer. Simul., **18** pp. 1175–1192.

[3] Syed Ali, M., Balasubramaniam, P., 2009, "Stability analysis of uncertain fuzzy Hopfield neural networks with time delays," Commun. Nonlinear Sci. Numer. Simul., **14** pp. 2776-2783.

[4] Wang, Y., Hao, J.,and Zuo, Z., 2010, "A new method for exponential synchronization of chaotic delayed systems via intermittent control, *Phys. Lett. A*., **374** pp. 2024–-2029.

[5] Syed Ali, M., Yogambigai, J., 2016, "Synchronization of complex dynamical networks with hybrid coupling delays on time scales by handling multitude Kronecker product terms," Appl. Math. Comput., **291** pp. 244–258.

[6] Yang, Y., Cao, J., 2010, "Exponential synchronization of the complex dynamical networks with a coupling delay and impulsive effects," Nonlinear Anal. RWA., **11** pp. 1650–1659.

[7] Syed Ali, M., Yogambigai, and J., Cao, J., 2017, "Synchronization of master-slave Markovian switching complex dynamical networks with time-varying delays in nonlinear function via sliding mode control," Acta Math. Sci. Ser. B Engl. Ed., **37** pp. 368–384.

[8] Hu, C., Yu, J., Jiang, H., and Teng, Z., 2010, "Exponential stabilization and synchronization of neural networks with time-varying delays via periodically intermittent control," Nonlinearity ., **23** pp. 2369–-2391.

[9] Wang, Y. W., Yang, M., Wang, H. O., and Guan, Z. H., 2009, "Robust stabilization of complex switched networks with parametric uncertainties and delays via impulsive control," IEEE Trans. Circuits Syst. I, Reg. Papers., **56** pp. 2100–2108.

[10] Li, N., Zhang, Y., Hu, J., and Nie, Z., 2011, "Synchronization for general complex delayed networks with sampled-data," Neurocomputing., **74**, pp. 805–811.

[11] Gu, K., 2000, "An integral inequality in the stability problem of time-delay systems,in:Proceeding 39th IEEE Conference on Decision and control, sydney," Australia, December pp. 2805–2810.

[12] Park, P. Y., Lee, W. I., and Lee, and S. Y., 2015, "Auxiliary function-based integral inequalities for quadratic functions and their applications to time-delay systems," J. Frankl. Ins., **35**, pp. 1378–1396.