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Prediction of DRH using Simplified Two-parameter Gamma SUH

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Abstract

The proposed procedure predicts the direct runoff hydrograph (DRH) employing the improved two-parameter gamma distribution based synthetic unit hydrograph (SUH). This method produces a hydrograph more conveniently and accurately than those due to available popular Snyder, Gray, and SCS methods by eliminating both manual and subjective fitting of a hydrograph through a few data points as well as the tiresome adjustments for unit runoff volume. Upon testing with the data of three watersheds, the model yielded average efficiency exceeding 94% and 71% in calibration and validation, respectively, indicating satisfactory model performance.

Keywords: Direct runoff hydrograph; gamma distribution; instantaneous unit hydrograph; synthetic unit hydrograph.

1. Introduction

Extension of the unit hydrograph (UH) theory to ungauged basins highlights the need for synthesizing UH from physical characteristics. Compared to a single value of design flood, SUH greatly increases the amount of information quantifying volume of flood wave and its time course. The beginning of SUH approach can be traced back to the distribution graph proposed by Bernard (1935). The prominent SUH approaches such as Snyder (1938), Gray (1961), and SCS (1972) specify a few selected points on UH through which a curve is fitted manually, which is subjective and tiresome. It may sometimes be intricate to satisfy its unit volumetric condition and therefore UH is often left unadjusted for unit runoff volume. In these methods, graphs or equations are provided to determine values of attributes such as peak flow lag time, base time, and hydrograph widths, W50 and W75. These reasons coupled with the fact that a UH can reasonably be represented by a gamma distribution as well as its shape is analogous to that of a UH comprise the basis for its fitting.

The improved two-parameter gamma distribution (2PGD)-based SUH method is easy to apply, meets the UH criterion of unity, discard the calculations for W50 and W75. Equations for calculating Nash parameters n and K from q_p and t_p of IUH were proposed by Singh (2000). n is fairly accurately expressed in terms of the non-dimensional shape factor β (= $q_p t_p$), eliminating trials (Bhunya *et.al.*, 2003). The discrete convolution derives DRH for a given depths of rainfall-excess pulses, and UH obtained from SUH. The objective of this paper is to propose and test a simple SUH method based on 2PGD and dependent on physical watershed and storm characteristics to predict DRH, and evaluate parameters for their sensitivity.

2. Two-parameter Gamma Distribution Based Suh Method

Nash (1959) and Dooge (1959) were derived 2PGD from a cascade of n-linear reservoirs of equal storage coefficient K as:

$$q = 1/K\Gamma(n)\left(t/K\right)^{n-1} e^{-t/K}, K > 0, t > 0 \quad (1)$$

where q is the instantaneous UH (IUH, h^{-1}) of unit volume and n is the shape parameter. Chow (1964) related n and K as:

$$K = t_p/(n-1) \quad (2)$$

On simplifying Eq. (1) and (2),

$$\beta = q_p t_p = (n-1)^{(n-1)} e^{-(n-1)}/\Gamma(n-1) \quad (3)$$

Bhunya *et.al.* (2003) solved Eq. (3) using optimization and a simple numerical simulation as:

$$n = 5.53\beta^{1.75} + 1.04 \ldots\ldots\ldots for \ldots\ldots\ldots 0.01 < \beta < 0.35 \quad (4a)$$

$$n = 6.29\beta^{1.998} + 1.157 \ldots\ldots\ldots for \ldots\ldots\ldots \beta \geq 0.35 \quad (4b)$$

Eqs. (4a) and (4b) can be used to estimate n for known values of q_p and t_p. It is noted that $\beta < 0.01$ is seldom experienced in field (Singh, 2000). To obtain an SUH, the IUH parameters were related to watershed characteristics (i.e. A, L and S) as:

A multitude of peak flow Q_f (m^3/s) formulae in a regression relationship with catchment area A (km^2) as for e.g. Dickens formula (1865) is available in literature and these are of the form:

$$Q_f = C_d A^m \quad (5)$$

SCS (1985) simplifies the estimation of travel time using Time of concentration t_c (h) and lag time t_l (h) as:

$$t_l = 0.6 t_c \quad (6)$$

Here t_c is estimated using Kirpich equation (1940), given the length of travel L (km) and $S = \Delta H/L$ = channel slope (m/m)

$$t_c = C_{tc}\left(L^u / S^v\right) \tag{7}$$

SCS (1985) expresses t_p in terms of t_l, and Δt (i.e. duration of rainfall-excess pulse / UH) as:

$$t_p = (\Delta t/2) + t_l = (0/2) + t_l = t_l \tag{8}$$

As $\Delta t \rightarrow 0$, the IUH is obtained. Therefore, in Eq. (8), t_p should be equal to t_l.

The Δt-h UH can be derived by averaging the known SUH ordinates at Δt-h intervals. The discrete convolution Eq. (9) estimates direct runoff Q_n for given depth of m^{th} rainfall-excess pulse P_m and Δt-h UH ordinates U_{n-m+1} as:

$$Q_n = \sum_{m=1}^{n \le M} P_m U_{n-m+1} \tag{9}$$

Statistical indices, Nash-Sutcliffe (1972) Efficiency (η_{NS}) and Relative Error (RE) were used for performance evaluation. Proposed method is tested on the data of Goodwin Creek (GC) sub-watersheds (Table 1).

Table 1: Hydroclimatic and Physiographic Characteristics of Goodwin Creek Sub-Watersheds, Mississippi River, U.S.A.

S. No.	Watershed	Climate & Soil (%)	Avg. Annual Rainfall (mm)	A (km2)	L (km)	S (m/m)	No. of Events Used For
1	W6 GC	Humid & Silty = 100	1440	1.25	2.216	0.012	C: 11 & V: 4
2	W7 GC			1.66	2.456	0.009	C: 6 & V: 4
3	W14 GC			1.66	2.253	0.016	C: 4 & V: 3
Note: C = Calibration & V = Validation							

Table 2: Calibration Results of W6 Goodwin Creek Watershed (Avg. η_{NS} = 94.24%).

Event	**Δt (h)**	**Calibration Parameters**						**Computed variables**			**Performance in DRH Computation**			
		ERPmax (mm)	**Cd**	**m**	**Ctc**	**u**	**v**	**qp (h-1)**	**tc (h)**	**tp (h)**	**ηNS (%)**	**RE (%) in QP**	**RE (%) in TP**	**RE (%) in Qv**

1	0.50	5.12	1.15	0.90	0.19	0.78	0.40	0.79	2.12	1.27	97.47	-0.32	0.00	0.12
2	0.50	1.87	0.19	0.65	0.08	0.77	0.36	0.33	0.73	0.44	96.05	15.84	-20.0	6.78
3	1.00	2.02	0.20	0.76	0.16	0.77	0.38	0.33	1.63	0.98	93.33	16.97	-3.85	8.45
4	0.25	1.36	0.14	0.75	0.08	0.77	0.39	0.35	0.82	0.49	94.36	1.60	12.50	2.77
5	0.50	2.24	0.14	0.75	0.04	0.77	0.39	0.21	0.37	0.22	96.14	0.16	0.00	5.89
6	0.50	4.42	0.72	0.66	0.08	0.79	0.48	0.54	1.21	0.72	97.60	-0.44	-4.00	3.58
7	0.50	8.15	1.40	0.91	0.29	0.76	0.33	0.60	2.23	1.34	89.04	-2.49	-14.29	-0.10
8	0.25	7.09	7.19	4.26	0.29	0.58	0.11	7.56	0.73	0.44	96.98	14.62	-20.00	2.21
9	0.17	0.58	0.40	0.88	0.14	0.77	0.35	2.42	1.26	0.76	98.36	4.43	0.00	0.00
10	1.00	2.39	0.59	1.25	0.62	0.74	0.20	0.94	2.63	1.58	80.62	25.40	0.00	9.18
11	0.50	18.93	4.31	1.03	0.21	0.77	0.39	0.82	2.13	1.28	96.68	3.95	0.00	0.13

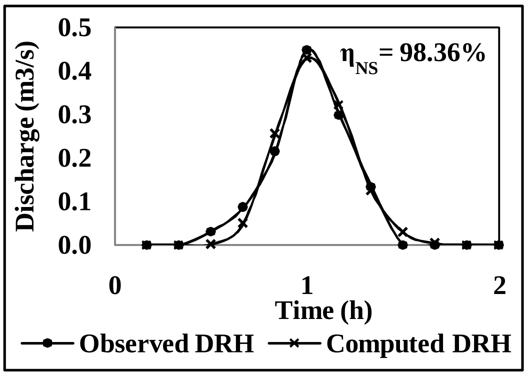

Figure 1: Calibration on a storm event of W6 GC.

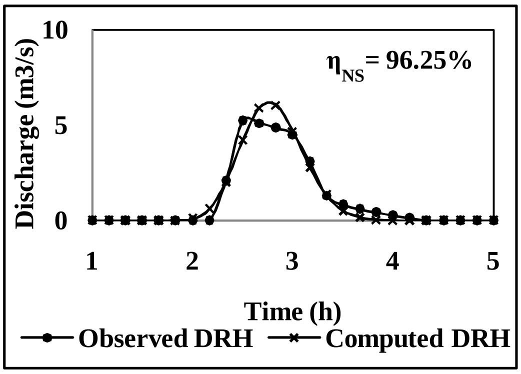

Figure 2: Validation on a storm event of W14 GC.

Table 3: Average Values Calibration Parameters used in Validation.

Watershed	Average Calibration Parameters					ERPmax (mm)
	Cd	m	Ctc	u	v	
W6 GC	1.49	1.16	0.20	0.75	0.34	4.92
W7 GC	1.00	0.76	0.20	0.77	0.40	4.44
W14 GC	3.14	0.69	0.20	0.77	0.37	5.95

Table 4. Validation Results of W14 Goodwin Creek Watershed (Avg. η_{NS} = 79.12%).

Event	Δt (h)	Avg. Calibration Parameters						Computed variables			Performance in DRH Computation			
		ERPmax (mm)	Cd	m	Ctc	u	v	qp (h-1)	tc (h)	tp (h)	ηNS (%)	RE (%) in QP	RE (%) in TP	RE (%) in Qv
1	0.17	5.95	3.14	0.69	0.20	0.77	0.37	1.63	1.76	1.06	54.34	31.52	-12.50	0.00
2	0.25	5.95	3.14	0.69	0.20	0.77	0.37	1.63	1.76	1.06	86.76	-17.77	-16.67	0.00
3	0.17	5.95	3.14	0.69	0.20	0.77	0.37	1.63	1.76	1.06	96.25	-15.72	-13.33	0.00

3. Analysis and Discussion of Results

3.1. Determination of Rainfall-Excess and Direct Runoff

In order to separate the observed DRH from the base-flow and rainfall-excess hyetograph from the infiltration, a constant discharge base-flow separation and phi-index ϕ (mm/h or cm/h, Eq. 10) methods were used.

$$\phi = P_t - Q_t / t_e \tag{10}$$

where P_t and Q_t = total rainfall and DRH depths (mm or cm), and t_e = duration of total rainfall-excess (h).

3.2. Derivation of SUH to Predict DRH

The above procedure was calibrated using 21 of 32 randomly selected storm events, and validated on the others. The calibration results of W6 GC are shown in Table 2. Model parameters were calibrated using SOLVER routine of EXCEL based on the generalized reduced gradient nonlinear programming algorithm (Lasdon *et.al.*, 1978), with objective function of maximizing η_{NS}.

The calibration results are explained, as an example, for DRH (η_{NS} = 98.36%, Figure 1) derived using the data of event (9) of the W6 GC. For deriving Gamma SUH from known A (= 1.25 km^2), L (= 2.216 km), and S (=$\Delta H/L$ = 0.012 m/m) as given in Table 1, the regional Q_f -A relationship (Eq. 5) and SCS (1985) approach (Eqs. 6-8) were used.

The peak flow Q_f is computed by fitting known A and optimized regression constants C_d = 0.40 and m = 0.88 in Eq. (5). It yields q_p (= 2.42 h^{-1}) for given A and rainfall-excess pulse of maximum depth (ERP_{max} = 0.58 mm). IUH is response due to an instantaneous (zero duration) rainfall-excess hence, Δt = 0 h. Therefore, t_p (Eq. 8) should be equal to t_l = 0.6 t_c = 0.76 h (Eq. 6). Here, t_c = 1.26 h is estimated using Eq. (7) with known L and S, where optimized C_{tc} = 0.14, u =0.77, v =0.35. The resulting parameters are: K = 0.04 (Eq. 2), $\beta = q_p t_p$ = 1.84 (Eq. 3), and n = 22.33 (Eq. 4b). With estimated q_p, t_p, n, and K a Gamma SUH is derived from Eq. (1). REs in DRHs runoff volumes (Q_v), peaks (Q_p), and time to peaks (T_p) are 0, 4.43%, and 0, respectively, indicating almost perfect mass conservation. However, for the same reason, the overestimation in rising phase caused due to low discharge values at the head end leads to underestimation of computed peaks. Similarly, overestimation in receding phase leads to high discharge values at the tail end of DRH.

The calibration results reveal that the model performance to be generally excellent with average η_{NS}: 94.24%, 95.08% and 94.20% for W6, W7 and W14 GC, respectively. As seen, η_{NS} in calibration varies from 80.62% (W6 GC) to 98.36% (W6 GC), exhibiting satisfactory to excellent performance. REs in DRHs Q_v, Q_p, and T_p varies from 0 (W7 GC) to 9.18% (W6 GC), 0.16% (W6 GC) to 25.40% (W6 GC), and 0 (W6/W7/W14 GC) to -20% (W6/W7 GC), respectively. Sensitivity analysis evaluates the impact of q_p and t_p on η_{NS} when varied from -50% to +50% of their original value. As seen, q_p is more sensitive than t_p on the events of W6 & W14 GC, and it is otherwise on W7 GC.

3.3 Validation using Average Parameters Values

The proposed procedure was validated using the average values calibration parameters (Table 3). Here, average C_d and m values were used to estimate Q_f (Eq.5) from which q_p is derived using A (km^2) and average ERP_{max} value (mm). Similarly, average C_{tc}, u and v values are fitted with L (km) and S (m/m) in SCS approach (Eqs. 6-8), to estimate t_p. The results are explained for DRH (η_{NS} = 96.25%, Figure 2) derived using the data of event (3) of W14 GC (Table 4). The average values of calibrated parameters used for estimating q_p (= 1.63 h^{-1}) and t_p (= 1.06 h) are: C_d = 3.14, m = 0.69, ERP_{max} = 5.95 mm, and C_{tc} = 0.20, u = 0.77, v =0.37 respectively. REs in Q_v, Q_p, and T_p are 0, -15.72%, and -13.33%, respectively.

The validation results reveal that model appears to have performed well on all 3 watersheds with average η_{NS}: 71.68% (W6 GC), 83.10% (W7 GC), and 79.12% (W14 GC). As seen, η_{NS} in validation varies from 54.34% (W14 GC) to 96.25% (W14 GC), exhibiting average to excellent performance. REs in DRHs Q_v, Q_p, and T_p varies from 0 (W14 GC) to -21.52% (W6 GC), -3.99% (W6 GC) to 31.52% (W14 GC), and 0 (W6/W7 GC) to -57.14% (W7 GC), respectively.

3.4 Validation for Proximate Watersheds

To verify predictive ability of the proposed model on nearby watersheds, average parameter values of W6 & W7 GC (Table 3) were used to predict DRH using data of storm events recorded at W14 (4 events) & W6 (8 events) GC watersheds. W14 GC performs reliably using average parameter values of W6 GC with average η_{NS} attained is 84.56%. The estimated q_p and t_p values are 1.19 $h^{-1}$and 0.90 h, respectively. W6 GC also performs reliably considering average parameter values of W7 GC with average η_{NS} achieved is 79.32%. The q_p and t_p used for prediction are 0.77 $h^{-1}$and 1.28 h, respectively. Hence, we can say that the proposed approach can be applicable to nearby ungauged watershed nevertheless; the characteristics of both the watersheds must be identical.

4. Conclusion

The proposed procedure is easy to grasp and understand, simple to use, and gives accurate results. It enables determination of SUH for ungauged watersheds with little and easily available information on *A*, *L*, and *S*. In calibration, 90% of events resulted in η_{NS} greater than 91%, exhibiting outstanding performance, whereas in validation, the model performed satisfactorily with 81% of events resulting in η_{NS} greater than 72%.

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