

## An Inventory control model for constant deterioration in Fuzzy environment

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### **Abstract**

This paper proposes an inventory control model for fixed deterioration and Logarithmic demand rate for the optimal stock of commodities to meet the future demand which may either arise at a constant rate or may vary with time. The analytical development is provided to obtain the fuzzy optimal solution to minimize the total cost per time unit of an inventory control system, and our proposed models under the preference of manager by using graded unit-preference integration representation method for defuzzifying the fuzzy total annual inventory cost.

**AMS subject classification:** 03E72.

**Keywords:** Fuzzy inventory, Function Principle, preference, Inventory Control, logarithmic deterioration.

### **1. Introduction and preliminaries**

An Inventory fulfills many important functions within the organization. But as the inventory levels go up to provide these functions, the cost of storing and holding inventory also increases. Therefore, companies must reach a fine balance between low and high inventory levels and cost minimization is the major factor in obtaining this delicate balance. Some of the most significant inventory costs are cost of the items, cost of ordering, cost of carrying, cost of safety stock, and cost of stock outs.

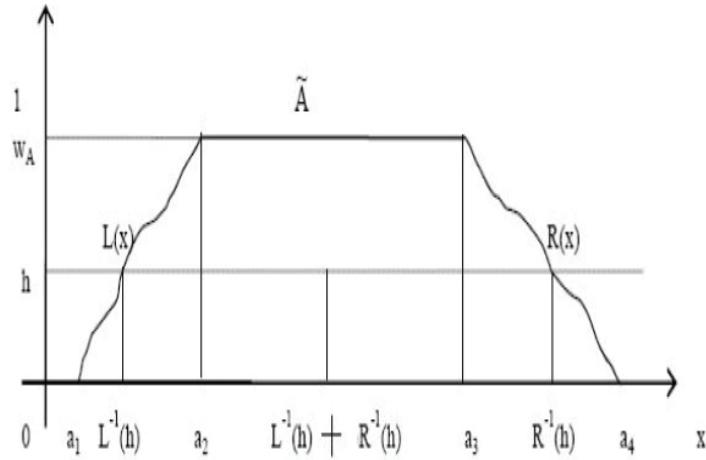


Figure 1: generalized fuzzy number

Ghare and Schrader (1963) were the first to address the inventory problem under constant demand and constant deterioration. Covert and Philip (1973) relaxed the assumption of constant deterioration rate by considering a two parameter Weibull distribution and assuming that the average carrying cost can be estimated as half of the replenishment size. Shah (1976) extended this model to allow for backlogging. Dave and Patel (1981) were the first to depart from the restrictive assumption of constant demand over an infinite planning horizon. Sharma and Kumar (2001) presented an inventory model for exponentially decaying inventory with known demand. In this model, demand is a function of selling price and rate of deterioration is a function of time. Nita H. Shah and Ankit S. Acharya (2008) presented a time dependent deteriorating order level inventory model for exponentially declining demand. In the proposed study, an inventory model has been developed for fixed deterioration and logarithmic demand rate. The objective is to obtain fuzzy optimal solution to minimize the total cost per time unit of an inventory control system based on the fuzzy arithmetical operations under Function Principle.

**Definition 1.1.** A generalized fuzzy number  $\tilde{A}$  is described as any fuzzy subset of the real line  $R$ , whose membership function  $\mu_{\tilde{A}}$  satisfies the following conditions.

- (1)  $\mu_{\tilde{A}}$  is a continuous mapping from  $R$  to the closed interval  $[0, 1]$ ;
- (2)  $\mu_{\tilde{A}} = 0$ ,  $-\infty < x \leq c$ ;
- (3)  $\mu_{\tilde{A}} = L(x)$  is strictly increasing on  $[c, a]$ ,
- (4)  $\mu_{\tilde{A}} = w$ ,  $a \leq x \leq b$ ;
- (5)  $\mu_{\tilde{A}} = R(x)$  is strictly decreasing on  $[b, d]$ ;

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$$(6) \mu_{\tilde{A}} = 0, d \leq x < \infty;$$

where  $0 < w \leq 1$ , a, b, c, and d are real numbers. We denote this type of generalized fuzzy number as  $A = (c, a, b, d; w)_{LR}$ . When  $w=1$ , we denote this type of generalized fuzzy number as  $A = (c, a, b, d)_{LR}$ .

In 1998, Chen and Hsieh [2,3,4] propose graded mean integration representation for representing generalized fuzzy number. Now we describe graded mean integration representation (GMIR) as follows.

Suppose  $L^{-1}$  and  $R^{-1}$  are inverse functions of functions L and R, respectively, and the graded mean h-level value of generalized fuzzy number  $A = (c, a, b, d; w)_{LR}$  is  $h[L^{-1}(h) + R^{-1}(h)]/2$  as Figure 1. Then the graded mean integration representation of generalized fuzzy number based on the integral value of graded mean h-levels is

$$P(A) = \int_0^w h \left( \frac{L^{-1}(h) + R^{-1}(h)}{2} \right) dh / \int_0^w h dh \quad (1)$$

where  $h$  is between 0 and  $w$ ,  $0 < w \leq 1$ . Generalized trapezoidal fuzzy number and generalized triangular fuzzy number are denoted as  $(c, a, b, d; w)$  and  $(c, a, d; w)$  respectively. Chen and Hsieh [2,3,4] already find the general formulae of the representation of generalized trapezoidal fuzzy number, or generalized triangular fuzzy number as follows. Suppose  $A = (c, a, b, d; w)$  is a trapezoidal fuzzy number. Since,

$$L(x) = w \left( \frac{x - c}{a - c} \right), \\ c \leq x \leq a,$$

and

$$R(x) = w \left( \frac{x - d}{b - d} \right), \\ b \leq x \leq d.$$

then

$$L^{-1}(h) = c + (a - c)h/w, \quad 0 \leq h \leq w, \\ R^{-1}(h) = d - (d - b)h/w, \quad 0 \leq h \leq w,$$

and

$$\frac{L^{-1}(h) + R^{-1}(h)}{2} = \frac{c + d + (a - c - d + b)h/w}{2}.$$

By formula (1), the graded mean integration representation of  $A$  is

$$P(A) = \int_0^w h \left( \frac{c+d+(a-c-d+b)h/w}{2} \right) dh / \int_0^w h dh \\ = \frac{c+2a+2b+d}{6} \quad (2)$$

**Definition 1.2.** Suppose  $\tilde{X} = (x_1, x_2, x_3, x_4)$  and  $\tilde{Y} = (y_1, y_2, y_3, y_4)$  are two trapezoidal fuzzy numbers and  $x_1, x_2, x_3, x_4, y_1, y_2, y_3$ , and  $y_4$  are all real numbers, then the arithmetical operations under Function Principle as follows.

1.  $\tilde{X} \oplus \tilde{Y} = (x_1 + y_1, x_2 + y_2, x_3 + y_3, x_4 + y_4)$ , where
2.  $\tilde{X} \otimes \tilde{Y} = (x_1y_1, x_2y_2, x_3y_3, x_4y_4)$ .
3.  $-\tilde{Y} = (-y_4, -y_3, -y_2, -y_1)$ , then the subtraction of  $\tilde{X}$  and  $\tilde{Y}$  is  $\tilde{X} \ominus \tilde{Y} = (x_1 - y_4, x_2 - y_3, x_3 - y_2, x_4 - y_1)$ .
4.  $1/\tilde{Y} = \tilde{Y}^{-1} = (1/y_4, 1/y_3, 1/y_2, 1/y_1)$ .
5.  $\tilde{X} \oslash \tilde{Y} = (x_1/y_4, x_2/y_3, x_3/y_2, x_4/y_1)$ .
6. Let  $\alpha \in R$  then  $\alpha \geq 0, \alpha \otimes \tilde{X} = (\alpha x_1, \alpha x_2, \alpha x_3, \alpha x_4)$

$$\alpha < 0, \alpha \otimes \tilde{X} = (\alpha x_4, \alpha x_3, \alpha x_2, \alpha x_1).$$

## 2. Mathematical Model

**Notations:** The inventory model is developed on the basis of the following assumption and notations:

1. Replenishment size is constant and the replenishment rate is infinite.
2. The lead time is zero.
3. T is the fixed length of each production cycle.
4.  $\log(1+t)$  is the demand rate at time t.
5.  $\theta$  is the constant rate of deterioration.
6.  $H$  is the inventory holding cost per unit time.
7.  $A$  is the shortage cost per unit time.
8.  $C$  is the cost of each deteriorated unit.
9.  $D$  is the total amount of deteriorated unit.
10.  $q(t)$  is the on hand inventory at any time t.
11.  $Q$  is the total amount of inventory produced at the beginning of each period.
12.  $S (> 0)$  is the initial inventory after fulfilling back order.
13.  $T_{avg}$  is the total average cost.

### Assumptions:

1.  $Q$  as total amount of inventory at the beginning of each period.

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2.  $S$  as the initial inventory after fulfilling backorders.
3. Inventory level gradually decreases during time  $(0, t_1)$ ,  $(t_1 < T)$  due to the reasons of market demand and deterioration.
4. Inventory level falls to zero at time  $t = t_1$ .
5. Shortages occur during time period  $(t_1, T)$  which are fully backlogged..

This model has been shown graphically in Figure 2. The differential equation to which the on hand inventory  $q(t)$  satisfying in two different parts of the cycle time  $T$  are given by:

$$\frac{dq(t)}{dt} + \theta q(t) = -\log(1+t) \quad 0 \leq t \leq t_1. \quad (3)$$

$$\frac{dq(t)}{dt} = -\log(1+t) \quad t_1 \leq t \leq T. \quad (4)$$

The equation (3) is linear differential equation

$$q(t)e^{\theta t} = - \int \log(1+t)e^{\theta t} dt + c$$

$$q(t) = \left( \frac{(1-2\theta)}{6}t^3 + \frac{(3\theta-2)}{24}t^4 - \frac{t^2}{2} \right) e^{-\theta t} + ce^{-\theta t}$$

By the boundary condition at  $t = 0, q(t) = S$  it gives  $S = c$

$$\therefore q(t) = \left( \frac{(1-2\theta)}{6}t^3 + \frac{(3\theta-2)}{24}t^4 - \frac{t^2}{2} \right) e^{-\theta t} + Se^{-\theta t}$$

this gives the solution as

$$q(t) = (1+t)\log(1+t) - t + c$$

By boundary condition at  $t = t_1, q(t) = 0$ , we get

$$(1+t_1)\log(1+t_1) - t_1 + c = 0$$

$$c = t_1 - (1+t_1)\log(1+t_1)$$

$$q(t) = (1+t)\log(1+t) - t + t_1 - (1+t_1)\log(1+t_1) \quad (5)$$

Now, equation (3) becomes

$$\left( \frac{(1-2\theta)}{6}t_1^3 + \frac{(3\theta-2)}{24}t_1^4 - \frac{t_1^2}{2} \right) e^{-\theta t_1} + Se^{-\theta t_1} = 0$$

This gives

$$S = \left( \frac{(2\theta - 1)}{6} t_1^3 + \frac{(2 - 3\theta)}{24} t_1^4 + \frac{t_1^2}{2} \right). \quad (6)$$

Hence, the unit deteriorated is given by

$$D = S - \int_0^{t_1} \log(1+t) dt = S - [(1+t_1) \log(1+t_1) - t_1]$$

$$D = \left( \frac{(2\theta - 1)}{6} t_1^3 + \frac{(2 - 3\theta)}{24} t_1^4 + \frac{t_1^2}{2} \right) - [(1+t_1) \log(1+t_1) - t_1]$$

Now for the interval  $0 < t < t_1$ .

Total average inventory is given by

$$I_1(t_1) = \frac{1}{T} \int_0^{t_1} q(t) dt$$

$$I_1(t_1) = \frac{1}{T} \int_0^{t_1} \left[ \left( \frac{(1-2\theta)}{6} t^3 + \frac{(3\theta-2)}{24} t^4 - \frac{t^2}{2} \right) e^{-\theta t} + S e^{-\theta t} \right] dt$$

After integrating and use the value of S, we get

$$I_1(t_1) = \frac{1}{T} \left[ \left( \frac{(2\theta-1)}{8} t_1^4 + \frac{(2-3\theta)}{30} t_1^5 + \frac{t_1^3}{3} \right) + \theta \left( -\frac{3t_1^4}{8} + \frac{t_1^5}{20} - \frac{t_1^6}{32} \right) \right].$$

Now the the interval  $t_1 < t < T$ .

Total average inventory is given by

$$I_2(t_1) = \frac{1}{T} \int_{t_1}^T q(t) dt$$

$$I_2(t_1) = \frac{1}{T} \int_{t_1}^T [(1+t) \log(1+t) - t + t_1 - (1+t_1) \log(1+t_1)] dt$$

$$I_2(t_1) = \frac{1}{T} \left[ \left( \frac{(1+t)^2}{2} \log(1+t) - \frac{(1+t)^2}{4} \right) - t(1+t_1) \log(1+t_1) - \frac{t^2}{2} + tt_1 \right]_{t_1}^T$$

$$I_2(t_1) = \frac{1}{T} [T_1 + T_2 + T_3]$$

Where

$$T_1 = \frac{(1+T)^2}{2} \left( \log(1+T) - \frac{1}{2} \right),$$

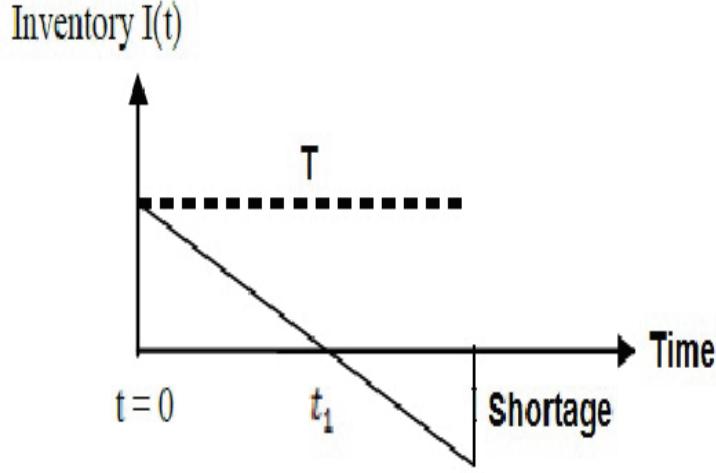


Figure 2: Graphical Representation of Proposed Inventory System.

$$T_2 = (1 + t_1) \log(1 + t_1) \left( \frac{t_1 - 1}{2} - T \right)$$

and

$$T_3 = \frac{(1 + t_1)^2}{4} + \frac{t_1^2}{2} - \frac{T^2}{2} + Tt_1$$

$\therefore$  The total average cost per unit time is given by

$$T_{avg}(t_1) = HI_1(t_1) + AI_2(t_1) + CD$$

### 3. Fuzzy Model and Solution Procedure

We consider the model in fuzzy environment. Due to fuzziness, it is not easy to define all the parameters precisely. we use the following variables:  $\tilde{\theta}$  : fuzzy rate of deterioration  $\tilde{C}$  : fuzzy cost of each deterioration,  $\tilde{H}$  : fuzzy carrying cost,  $\tilde{A}$  : fuzzy shortage cost,

Suppose  $\tilde{A} = (a_1, a_2, a_3, a_4)$ ,  $\tilde{\theta} = (\theta_1, \theta_2, \theta_3, \theta_4)$ ,  $\tilde{H} = (h_1, h_2, h_3, h_4)$ ,  $\tilde{C} = (c_1, c_2, c_3, c_4)$ , are nonnegative trapezoidal fuzzy numbers.

The total average cost per unit time is given by

$$\tilde{T}_{avg}(t_1) = (L \otimes (\tilde{\theta} \otimes \tilde{H})) \oplus (\alpha \otimes \tilde{H}) \oplus (M \otimes \tilde{A}) \oplus (\beta \otimes (\tilde{C} \otimes \tilde{\theta})) \oplus (N \otimes \tilde{C})$$

Where

$$L = \frac{1}{T} \left( -\frac{t_1^4}{8} - \frac{t_1^5}{20} - \frac{t_1^6}{32} \right), \quad \alpha = \frac{1}{T} \left( -\frac{t_1^4}{8} + \frac{t_1^5}{15} \right) \text{ and } \beta = \left( \frac{t_1^3}{3} - \frac{t_1^4}{8} \right)$$

$$M = \frac{1}{T} (T_1 + T_2 + T_3) , N = \left( \frac{t_1^4}{12} - \frac{t_1^3}{6} + \frac{t_1^2}{2} - 2t_1 - 2(1+t_1)\log(1+t_1) \right)$$

$$\tilde{T}_{avg}(t_1) = (\tilde{T}_{avg_1}(t_1), \tilde{T}_{avg_2}(t_1), \tilde{T}_{avg_3}(t_1), \tilde{T}_{avg_4}(t_1))$$

$$\tilde{T}_{avg_1}(t_1) = (L\theta_1 h_1 + \alpha h_1 + Ma_1 + \beta c_1 \theta_1 + Nc_1)$$

$$\tilde{T}_{avg_2}(t_1) = (L\theta_2 h_2 + \alpha h_2 + Ma_2 + \beta c_2 \theta_2 + Nc_2)$$

$$\tilde{T}_{avg_3}(t_1) = (L\theta_3 h_3 + \alpha h_3 + Ma_3 + \beta c_3 \theta_3 + Nc_3)$$

$$\tilde{T}_{avg_4}(t_1) = (L\theta_4 h_4 + \alpha h_4 + Ma_4 + \beta c_4 \theta_4 + Nc_4)$$

Defuzzify the fuzzy total average cost  $\tilde{T}_{avg}(t_1)$ .

$$\begin{aligned} P(\tilde{T}_{avg}(t_1)) &= \frac{1}{6} (L\theta_1 h_1 + 2L\theta_2 h_2 + 2L\theta_3 h_3 + L\theta_4 h_4) + \frac{1}{6} (\alpha h_1 + 2\alpha h_2 + 2\alpha h_3 + \alpha h_4) \\ &\quad + \frac{1}{6} (Ma_1 + 2Ma_2 + 2Ma_3 + Ma_4) + \frac{1}{6} (\beta c_1 \theta_1 + 2\beta c_2 \theta_2 + 2\beta c_3 \theta_3 + \beta c_4 \theta_4) \\ &\quad + \frac{1}{6} (Nc_1 + 2Nc_2 + 2Nc_3 + Nc_4). \end{aligned}$$

$$\begin{aligned} P(T_{avg}(t_1)) &= \frac{L}{6} (\theta_1 h_1 + 2\theta_2 h_2 + 2\theta_3 h_3 + \theta_4 h_4) + \frac{\alpha}{6} (h_1 + 2h_2 + 2h_3 + h_4) \\ &\quad + \frac{M}{6} (a_1 + 2a_2 + 2a_3 + a_4) + \frac{\beta}{6} (c_1 \theta_1 + 2c_2 \theta_2 + 2c_3 \theta_3 + c_4 \theta_4) \\ &\quad + \frac{N}{6} (c_1 + 2c_2 + 2c_3 + c_4). \end{aligned}$$

To minimize the average total cost per unit time, the optimal value of  $t_1$  can be obtained by solving the following equation:

$$\begin{aligned} \frac{dT_{avg}(t_1)}{dt_1} &= \frac{L'}{6} (\theta_1 h_1 + 2\theta_2 h_2 + 2\theta_3 h_3 + \theta_4 h_4) + \frac{\alpha'}{6} (h_1 + 2h_2 + 2h_3 + h_4) \\ &\quad + \frac{M'}{6} (a_1 + 2a_2 + 2a_3 + a_4) + \frac{\beta'}{6} (c_1 \theta_1 + 2c_2 \theta_2 + 2c_3 \theta_3 + c_4 \theta_4) \\ &\quad + \frac{N'}{6} (c_1 + 2c_2 + 2c_3 + c_4) = 0 \end{aligned}$$

Where

$$L' = \frac{1}{T} \left( -\frac{t_1^3}{2} - \frac{t_1^4}{4} - \frac{3t_1^5}{16} \right), \alpha' = \frac{1}{T} \left( -\frac{t_1^3}{2} + \frac{t_1^4}{3} \right) \text{ and } \beta' = \left( t_1^2 - \frac{t_1^3}{2} \right)$$

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$$M' = \frac{1}{T} (2t_1 + (t_1 - T) \log(1 + t_1)) , N' = \left( \frac{t_1^3}{3} - \frac{t_1^2}{2} + t_1 - 4 - 2 \log(1 + t_1) \right)$$

Optimal amount of the initial inventory after fulfilling backorder  $\tilde{S}$  denoted by  $\tilde{S}^*$  is

$$S^* = \left[ \left( \frac{t_1^3}{3} - \frac{t_1^4}{8} \right) \frac{(\theta_1 + 2\theta_2 + 2\theta_3 + \theta_4)}{6} + \left( \frac{t_1^4}{12} - \frac{t_1^3}{6} + \frac{t_1^2}{2} \right) \right].$$

Optimal amount of the unit deteriorated  $\tilde{D}$  denoted by  $\tilde{D}^*$  is

$$D^* = S^* - [(1 + t_1) \log(1 + t_1) + t_1]$$

Thus minimum value of the total cost  $T_{avg}(t_1)$  denoted by  $T_{avg}^*(t_1)$  is

$$\begin{aligned} T_{avg}^*(t_1) &= \frac{1}{6} [L\theta_1 h_1 + \alpha h_1 + Ma_1 + \beta c_1 \theta_1 + Nc_1] \\ &\quad + \frac{1}{3} [L\theta_2 h_2 + \alpha h_2 + Ma_2 + \beta c_2 \theta_2 + Nc_2] \\ &\quad + \frac{1}{3} [L\theta_3 h_3 + \alpha h_3 + Ma_3 + \beta c_3 \theta_3 + Nc_3] \\ &\quad + \frac{1}{6} [L\theta_4 h_4 + \alpha h_4 + Ma_4 + \beta c_4 \theta_4 + Nc_4]. \end{aligned}$$

## 4. Numerical Examples

To illustrate the proposed method, let us consider the following input data:

**Crisp Model:** Consider  $C = Rs200$  per order,  $H = Rs5$  per unit per year,

$$A = Rs15 \text{ per unit per year}, \quad \theta = 0.01 \text{ per year}, \quad t_1 = 0.5 \text{ and } T = 1.$$

The solution of crisp model is:

$$T_{avg}^* = 116.8603, \quad S^* = 0.1097, \quad \text{and} \quad D^* = 0.5814$$

**Fuzzy Model:** We can apply the fuzzy inventory model with fuzzy order quantity to find the optimal fuzzy total average cost and the fuzzy backorder. First, we represent the case of vague value as the type of trapezoidal fuzzy number.

Suppose  $\tilde{C} = (c_1, c_2, c_3, c_4) = (100, 200, 200, 300)$ ,  $\tilde{A} = (a_1, a_2, a_3, a_4) = (7.5, 15, 15, 22.5)$ ,  $\tilde{H} = (h_1, h_2, h_3, h_4) = (2.5, 5.0, 5.0, 7.5)$ ,  $\tilde{\theta} = (\theta_1, \theta_2, \theta_3, \theta_4) = (.005, .01, .01, .015)$ ,  $t_1 = 0.5$  and  $T = 1$ . Then the fuzzy total average cost and the fuzzy optimal backorder quantity:

$$\tilde{T}_{avg} = (58, 116, 116, 174) \quad \tilde{S}^* = (0.055, 0.110, 0.110, 0.165)$$

$$\tilde{D}^* = (0.29, 0.58, 0.58, 0.87)$$

## 5. Conclusion

This paper presented a fuzzy inventory control model for logarithmic demand rate, fixed deteriorating items with shortages under fully backlogged condition. The proposed model is developed in both the crisp and fuzzy environments. In fuzzy environment, all related inventory parameters were assumed to be trapezoidal fuzzy numbers. The optimum results of fuzzy model are defuzzified under the preference of manager by using graded mean  $h = 1$  level integration representation method. So, the decision maker, after analyzing the result, can plan for the optimal value for total cost, and for other related parameters.

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