

Fuzzy Intuitionistic on Queuing System

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ABSTRACT

Our aim in this, we describe to deal with the intuitionistic fuzzy queueing model with uniform strength and intuitionistic fuzzy parameters. Firstly, We solve the model by effective characteristics in a crisp case. Then we apply the concept of α -cuts and Extension technique to Construct membership functions and non- membership functions of the system characteristics using NLP (Non-linear Programming models) in the intuitionistic fuzzy case.

Keywords $M/E^K/1$ queue, intuitionistic fuzzy, Membership and Non-membership functions, parametric and non-linear programming problem.

INTRODUCTION

In this paper, We consider a queue model for waiting list of the some function in which the arrival rate of the customers a fuzzy number and service rate is assumed a fuzzy decision variable. The objective is to find optimum value of service rate with minimum total cost of the system.

However, in many practical applications, the statistical information may be obtained objectively, that is the arrival pattern and service pattern are more suitably described by linguistic terms such as good, bad (or) moderate rather than by probability distributions. Thus fuzzy queues are much more realistic than the commonly used crisp queues. For queuing models with single and multiple servers under various consideration the $M/E^K/1$ vacation systems with a single-unit arrival have attracted much attention from Researchers applied the previous results to a machine serving problems and a queuing decision problem (1) J.J. Buckley, T. Feuring, Y. Hayashi queueing theory revisited.

The extensions of this model can be referred to Vinod[18], Igaki[19], Tian et al[20], Tian and Xu[21], and Zhang and Tian [22,23] studied the $M/M/C$ vacation

systems with a single-unit arrival and “Partial server vacation policy”. They proved several conditional results for the queue length and waiting time. X. Chao and Zhao [24] investigated the G1/M/C vacation models with a single-unit arrival and provided iterative algorithms for computing the stationary probability distributions. Also, multiple and single channel queuing system with finite or infinite capacity. Moreover, on the basis of Zadeh extension principle [7,8] R.R. Yager, L.A. Zadeh. The possibility concept and fuzzy Markov chains [6] R. Sharma, G.C. Sharma, Li and Lee [5] have derived analytical solutions for two fuzzy queues, namely, M/F/1 and FM/FM/1, where F denotes fuzzy time and FM denotes fuzzified time.

Moreover, Negi and Lee [14] their approach is very complicated and is generally unsuitable to computational purposes and they propose the α -cut and the variable approaches to analyse fuzzy queues. Also, their approach only provides crisp solutions. If we can derive the membership of some performance measure, we obtain a more reasonable and realistic performance measures because it maintains the fuzziness of input informations that can be used to represent the fuzzy system more accurately.

We adopt parametric programming to construct the membership functions of the performance of fuzzy queues and successfully apply to four simple fuzzy queue with one or two fuzzy variables, namely, M/F/1, F/M/1, F/F/1 and FM/FM/1.

The basic idea is to apply the α -cuts and Zadeh’s extension principle [6,7] to transform the fuzzy bulk arrival queues to a family of crisp bulk arrival queues.

The intuitionistic fuzzy set (IFS) was introduced by K. Atanassov [17] and in our regular real life. Here the degree of rejection and acceptance are considered so that the sum of both values is always less than one. Atanassov also analyzed IFS is a more efficient way. Atanassov [17] discussed an Open problems in IFS theory. Atanassov and Kreinovich [10] implemented IF interpretation of interval data. S. Banerjee and Roy. T.K. [2] considered application if the intuitionistic fuzzy optimization in the constrained multi objective stochastic inventory model. A.L. Nachammai and P. Thangaraj [4] implemented for solving intuitionistic fuzzy linear programming problem by using similarity measures.

The temporal intuitionistic fuzzy sets are discussed also by Atanassov [12]. Intuitionistic fuzzy soft sets are considered by Maji Biswas and Roy [11]. Anglov [12] implemented the optimization in an intuitionistic fuzzy optimization pramanik and Roy [13] solved a vector optimization problem using an intuitionistic fuzzy goal programming. A transportation model is solved by Jana & Roy [14] using multi-objective intuitionistic fuzzy linear programming. Banerjee and Roy [2] considered application of the intuitionistic fuzzy optimization is the constrained multi-objective stochastic inventory model with fuzzy cost components was solved by intuitionistic fuzzy optimization technique by Banerjee and Roy [15]. Banerjee and Roy [16] discussed a stochastic inventory model solving it first time by intuitionistic fuzzy geometric programming technique.

In this paper, general fuzzy Non-linear programming technique is applied and we solve a probabilistic queuing model with uniform expected waiting time in the queue and number of customer in the queue under probabilistic and imprecise constraints. Finally the optimization of the objective function a comparative study is presented

among the intuitionistic fuzzy optimization queue technique. Although, general fuzzy Non-linear programming technique optimized the average arrival cost in comparison to fuzzy and intuitionistic technique.

Multiple-objective queuing model with fuzzy component:

In this section, the Multi server dynamic queuing is modeled to optimally control with traditional single objective linear (or) Non-linear programming problem in terms combination of resources.

Definition:

An intuitionistic fuzzy set(IFS)

An intuitionistic fuzzy set(IFS), when the problem of rejection (non-membership) is defined simultaneously with problem of acceptance (membership) of the objectives and the both of these problems are not difference to each other, the intuitionistic fuzzy sets can be used as a more general tool for describing uncertainty.

The maximize the problem of acceptance of intuitionistic fuzzy objectives and constraints and to minimize the rejective of intuitionistic fuzzy objectives and constraints. We can write

$$\text{Max } \mu_{\tilde{A}}(x), \text{Min } \gamma_{\tilde{A}}(x), x \in X$$

Subject to

$$\gamma_{\tilde{A}}(x) \geq 0, \mu_{\tilde{A}}(x) \geq 0, \gamma_{\tilde{A}}(x) \leq 1, \mu_{\tilde{A}}(x) \leq 1, \forall x \in X, x \geq 0$$

$$\mu_{\tilde{A}}(x) + \gamma_{\tilde{A}}(x) \leq 1, \forall x \in X, x \geq 0$$

Where $\mu_{\tilde{A}}(x)$ denote the membership function of X to the intuitionistic fuzzy sets and $\gamma_{\tilde{A}}(x)$ denote the non-membership function (rejection) of X from the intuitionistic fuzzy sets.

Generally an intuitionistic fuzzy sets (IFS) denote to each value of x of the set X, the membership value

$$\mu_{\tilde{A}}: X \rightarrow [0,1] \text{ and a non-membership value}$$

$$\gamma_{\tilde{A}}: X \rightarrow [0,1] \text{ such that}$$

$$0 \leq \mu_{\tilde{A}}(x) + \gamma_{\tilde{A}}(x) \leq 1, \forall x \in X$$

and it is represented as $\{x, \mu_{\tilde{A}}(x), \gamma_{\tilde{A}}(x) / x \in X\}$

The value $\Delta_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \gamma_{\tilde{A}}(x)$ is known as the intuitionistic part of \tilde{A} to X.

Definition:

Let the confidence interval of the first intuitionistic fuzzy sets \tilde{A} and \tilde{B} . Let be $\ell_{A(\alpha)}$ and $u_{A(\alpha)}$ be a maximizing and minimizing fuzzy subset of Z defined by

$$(2) \ell_{A(\alpha)} = \min P(a, s)$$

such that $\ell_{A(\alpha)} \leq a \leq u_{A(\alpha)}$

$$(3) \quad u_{A(\alpha)} = \max P(a, s)$$

such that

$$\ell_{A(\alpha)} \leq a \leq u_{A(\alpha)}$$

$$(4) \quad \ell_{s(\alpha)} = \min P(a, s)$$

such that $\ell_{s(\alpha)} \leq a \leq u_{s(\alpha)}$

$$(5) \quad u_{s(\alpha)} = \max P(a, s)$$

such that $\ell_{s(\alpha)} \leq a \leq u_{s(\alpha)}$ respectively is defined as follows:

Both the interarrival time \tilde{A} and service times \tilde{S} are fuzzy numbers using Zadeh's extension principle (Klir and Boyun 1997, Zimmermann).

An intuitionistic fuzzy set is a subset of the real line

Queue arrival service is control, that is there is any $a \in X$ such that $\mu_{\tilde{A}}(a) = 1$ so that

$$\gamma_{\tilde{A}}(a) = 0, \text{ similarly } s \in X \text{ such that } \mu_{\tilde{S}}(s) = 1 \text{ and } \gamma_{\tilde{S}}(s) = 0$$

The membership function of the performance measure $P(\tilde{A}, \tilde{S})$ that is,

$$(6) \quad \delta_{P(\tilde{A}, \tilde{S})}(Z) = \sup_{a \in X, s \in X} \{ \min \{ \mu_{\tilde{A}}(a), \mu_{\tilde{S}}(s) \} / z = P(\tilde{A}, \tilde{S}) \} \text{ for every } a \in X, s \in X \text{ and } X \text{ in real line } a \in [0, 1]$$

The non-membership function of the performance measure $P(\tilde{A}, \tilde{S})$, that is

$$(7) \quad \delta_{P(\tilde{A}, \tilde{S})}(Z) = \inf_{a \in X, s \in X} \{ \max \{ \gamma_{\tilde{A}}(a), \gamma_{\tilde{S}}(s) \} / z = P(\tilde{A}, \tilde{S}) \} \text{ for every } a \in X, s \in X \text{ and } X \text{ in real line } a \in [0, 1]$$

The membership function of the performance measure $P(\tilde{A}, \tilde{S})$ that is, (8) $\delta_{P(\tilde{A}, \tilde{S})}(Z) = \sup_{a \in X, s \in X} \{ \min \{ \mu_{\tilde{A}}(s), \mu_{\tilde{S}}(s) \} / z = P(\tilde{A}, \tilde{S}) \}$ for every $a \in X, s \in X$ and X in real line $a \in [0, 1]$

The non-membership function of the performance measure $P(\tilde{A}, \tilde{S})$, that is

(9) $\delta_{P(\tilde{A}, \tilde{S})}(Z) = \inf_{a \in X, s \in X} \{ \max \{ \gamma_{\tilde{A}}(s), \gamma_{\tilde{S}}(s) \} / z = P(\tilde{A}, \tilde{S}) \}$ for every $a \in X, s \in X$ and X in real line $a \in [0, 1]$. Then the right shape function of a triangular fuzzy number $P(\tilde{A}, \tilde{S})$ can be determined by

(10) $R(Z) = \sup(\mu_{\tilde{A}}(z) \wedge \tilde{u}_{p(\alpha)}(z))$ and the left shape function of a triangular fuzzy number $P(\tilde{A}, \tilde{S})$ can be determined by

$$(11) \quad L(Z) = \sup(\mu_{\tilde{A}}(z) \wedge \ell_{p(\alpha)}(z))$$

If both $\ell_{p(\alpha)}$ and $\tilde{u}_{p(\alpha)}$ are invertible with respect to α , then the left shape function and right shape function

$$L(Z) = \ell_{p(\alpha)}^{-1}$$

(12) $R(Z) = \tilde{u}_{p(\alpha)}^{-1}$ (Chiang Kao et al., 1999) can be obtained from which the membership function of the fuzzy number $P(\tilde{A}, \tilde{S})$

Equivalently $R(Z)$ is the ordinate of the intersecting point of $\ell_{p(\alpha)}$ and the right

shape of $\mu_{\tilde{A}}$ and similarly $L(Z)$ is the ordinate of the intersecting point of $\tilde{\mu}_{P(\alpha)}$ and left shape of $\mu_{\tilde{A}}$.

Definition:

A fuzzy number $P(\tilde{A}, \tilde{S})$ is defined to be a triangular fuzzy number if its membership function

$P(\tilde{A}, \tilde{S}):R \rightarrow [0,1]$ is equal to

$$(13) \delta_{P(\tilde{A}, \tilde{S})}(Z) = \begin{cases} L(Z) & \text{for } Z_1 \leq Z \leq Z_2, \\ 1 & \text{for } Z_2 \leq Z \leq Z_3 \\ R(Z) & \text{for } Z_3 \leq Z \leq Z_4, \\ 0 & \text{for otherwise} \end{cases}$$

Where $Z_1 \leq Z_2 \leq Z_3 \leq Z_4$ and $L(Z_1) = R(Z_4) = 0$

Where Z_1 and Z_2 stand for the lower and upper values of the support of the fuzzy number $P(\tilde{A}, \tilde{S})$ respectively and Z for the model value. This fuzzy number is denoted by $(Z_1 \ Z \ Z_2)$. Now $\frac{Z-Z_1}{Z_2-Z_1}$ and $\frac{Z-Z_3}{Z_3-Z_4}$ are known as left and right shape of the triangular fuzzy number $(Z_1 \ Z \ Z_2)$.

The complement $\delta_{P(\tilde{A}, \tilde{S})}(Z)^c$ of a fuzzy subset $\delta_{P(\tilde{A}, \tilde{S})}(Z)$ of a set $P(\tilde{A}, \tilde{S})$ is a fuzzy subset given by $\delta_{P(\tilde{A}, \tilde{S})}(Z)^c = 1 - \delta_{P(\tilde{A}, \tilde{S})}(Z)$, $z \in X$.

Definition:

Let $P(\tilde{A}, \tilde{S}) = (Z_1 \ Z \ Z_2)$ be a triangular fuzzy number. The complement $P(\tilde{A}, \tilde{S})^c$ of a triangular fuzzy number $P(\tilde{A}, \tilde{S})$ is defined by $\delta_{P(\tilde{A}, \tilde{S})}(Z)^c = 1 - \delta_{P(\tilde{A}, \tilde{S})}(Z)$. Hence the membership function $\delta_{P(\tilde{A}, \tilde{S})}(Z)^c$ is defined by

$$(14) \delta_{P(\tilde{A}, \tilde{S})}(Z)^c = \begin{cases} \frac{Z-Z_1}{Z_2-Z_1} & \text{if } x \in [Z_1, Z_2] \\ \frac{Z-Z_3}{Z_3-Z_4} & \text{if } x \in [Z, Z_2] \\ 0 & \text{Otherwise} \end{cases}$$

Definition:

An Intuitionistic fuzzy set (IFS) A in X is defined by $A = (\mu_{\tilde{A}}, \gamma_{\tilde{A}})$ of $P(\tilde{A}, \tilde{S})$ is said to be an intuitionistic fuzzy number if $\mu_{\tilde{A}}$ and $\gamma_{\tilde{A}}$ are fuzzy numbers with $\gamma_{\tilde{A}} \leq \mu_{\tilde{A}}^c$ where $\mu_{\tilde{A}}^c(x)$ denotes the complement of $\mu_{\tilde{A}}(x)$.

Definition:

Non-Membership function of fuzzy number: Let $\delta_{P(\tilde{A}, \tilde{S})}(Z)^c$ be a fuzzy number then the complement $\mu_{P(\tilde{A}, \tilde{S})}^c : R \rightarrow [0,1]$ of $\delta_{P(\tilde{A}, \tilde{S})}(Z)^c$ is given by

$$(15) \delta_{P(\tilde{A}, \tilde{S})}(Z)^c$$

$$\left\{ \begin{array}{ll} = L(Z)^c & \text{for } Z_1 \leq Z \leq Z_2 \\ 1 & \text{for } Z_2 \leq Z \leq Z_3 \\ R(Z)^c & \text{for } Z_3 \leq Z \leq Z_4 \end{array} \right\}$$

Here the queues $L(Z)^c$ in $Z_1 \leq Z \leq Z_2$ and $R(Z)^c$ in $Z_3 \leq Z \leq Z_4$ are called the right and left shape function of $\delta_{P(A,S)}(Z)^c$. Now define $R(Z)^c$ and $L(Z)^c$ are the ordinates of the intersection of $\mu_{P(A)}(a)$ and $l_{P(A)}$

With the right shape of $\delta_{P(A,S)}(Z)^c$ and left shape of $\delta_{P(A,S)}(Z)^c$ respectively.

Similarly define $R(Z)^c$ and $L(Z)^c$ are the ordinates of the intersection of $\mu_{P(S)}(s)$ with the right shape of $\delta_{P(A,S)}(Z)^c$ and left shape of $\delta_{P(A,S)}(Z)^c$ respectively.

Using the concept of α -cut the FM/FE^k/1 queueing system can be reduced as M/E^k/1 queue for which L, W, L_q, W_q .

Convex set and concave set:

In this model the group of arrival rate \tilde{A} and service rate \tilde{S} are approximately known and it can be presented by convex sets. Note that an intuitionistic fuzzy set \tilde{A} and \tilde{S} in its universal set Z is convex set for the membership function $\mu_{\tilde{A}}(a)$, $\mu_{\tilde{S}}(s)$.

$$(16) \mu_{\tilde{A}}(\tilde{A}_1 \alpha_1 + (1 - \tilde{A}_1) \alpha_2) \geq \min \{ \mu_{\tilde{A}}(\alpha_1), \mu_{\tilde{A}}(\alpha_2) \}$$

Where $\mu_{\tilde{A}}$ is its membership function for every $\alpha_1, \alpha_2 \in \tilde{A}$ and $\tilde{A} \in [0, 1]$.

Let $\mu_{\tilde{A}}(a)$ and $\mu_{\tilde{S}}(s)$ denote the membership function of the group arrival and service time respectively. We have

$$(17) \tilde{A} = \{ (a, \mu_{\tilde{A}}(a)) / a \in X \}$$

$$\tilde{S} = \{ (s, \mu_{\tilde{S}}(s)) / s \in X \}$$

Where X is the sets of \tilde{A} and \tilde{S} which denote the universal sets of the arrival rate and service rate respectively. Let $P(\tilde{A}, \tilde{S})$ denote the characteristic of performance measure of queue. Clearly when \tilde{A} and \tilde{S} are fuzzy numbers, $P(\tilde{A}, \tilde{S})$ will be fuzzy as well. On, the basis of Zadeh's extension principle (18,20), the membership function of the performance measure $P(\tilde{A}, \tilde{S})$. Defined as

$$(18) \delta_{P(\tilde{A}, \tilde{S})}(Z) = \sup_{a \in X, s \in X} \{ \min \{ \mu_{\tilde{A}}(a), \mu_{\tilde{S}}(s) \} / Z = P(\tilde{A}, \tilde{S}) \}$$

The non-membership function of the performance measure $P(\tilde{A}, \tilde{S})$ Queuing system with bulk arrival

$$(19) \delta_{L_q}(Z) = \sup_{a \in X, s \in X} \{ \min \{ \mu_{\tilde{A}}(a), \mu_{\tilde{S}}(s) \} / Z = \frac{K+1}{2K} + \frac{\lambda^k}{\mu(\mu-\lambda)} \}$$

$$(20) \delta_{L_q}(Z) = \inf_{a \in X, s \in X} \{ \max \{ \mu_{\tilde{A}}(a), \mu_{\tilde{S}}(s) \} / Z = \frac{K+1}{2K} + \frac{\lambda^k}{\mu(\mu-\lambda)} \}$$

Thus, the membership functions in (17)-(19) are true, then they are not used in the practical. Hence, it is very tedious to determine the type of the shape. Parametric NLPs are developed to find α -cut of $P(\tilde{A}, \tilde{S})$ based in the extension principle.

Parametric Non-linear programming problem:

To define the membership function of PNLP. $\tilde{b}_{L_s}(Z)$ of \tilde{L}_s is understandable and usable form, we will apply the Zadeh's principle, to derive the interarrival times \tilde{A} and \tilde{S} are approximately known and are represented by the following fuzzy sets.

$$(21) A(\alpha) = \{a \in X / \mu_{\tilde{A}}(a) \geq \alpha, \gamma_{\tilde{A}}(a) \geq \alpha\}$$

$$(22) S(\alpha) = \{s \in X / \mu_{\tilde{S}}(s) \geq \alpha, \gamma_{\tilde{S}}(s) \geq \alpha\}$$

Then the fuzzy intervals \tilde{A} and \tilde{S} of a single server queue system are fuzzy numbers.

The fuzzy interval times are represents the membership function $\tilde{b}_{L_s}(Z)$ of \tilde{L}_s is clear level and usable form. We adopt Zadeh's approach, which relies on α -cut of \tilde{L}_s . By the definitions for the α -cut of \tilde{A} and \tilde{S} in equation(29)-(30) are crisp sets which can be written in the following form:

$$(23) A(\alpha) = \{\min_{a \in X} \{a / \mu_{\tilde{A}}(a) \geq \alpha\}, \max_{a \in X} \{a / \gamma_{\tilde{A}}(a) \geq \alpha\}\}$$

$$(24) S(\alpha) = \{\min_{s \in X} \{s / \mu_{\tilde{S}}(s) \geq \alpha\}, \max_{s \in X} \{s / \gamma_{\tilde{S}}(s) \geq \alpha\}\}$$

From the equation(31) and (32) indicates that \tilde{A} and \tilde{S} are lying the ranges $(\ell_{A(\alpha)}, u_{A(\alpha)})$ and $(\ell_{S(\alpha)}, u_{S(\alpha)})$ respectively, when the membership function are not less than a possible level α . The queue reduces to a family of crisp queue with different level α where $0 < \alpha \leq 1$. Both $A(\alpha)$ and $S(\alpha)$ crisp sets. Using α -cut the interarrival times and service times can be represented by different levels of confidence intervals (Zimmermann, 1991).

Hence a fuzzy queue can be reduced with different α levels cuts $\{A(\alpha) / 0 < \alpha \leq 1\}$ and $\{S(\alpha) / 0 < \alpha \leq 1\}$.

As a result that

$$(25) a_{\alpha}^L = \min \mu_{\tilde{A}}^{-1}(\alpha)$$

$$(26) a_{\alpha}^U = \max \mu_{\tilde{A}}^{-1}(\alpha)$$

$$(27) s_{\alpha}^L = \min \mu_{\tilde{S}}^{-1}(\alpha)$$

$$(28) s_{\alpha}^U = \max \mu_{\tilde{S}}^{-1}(\alpha)$$

$\{A(\alpha) / 0 < \alpha \leq 1\}$ and $\{S(\alpha) / 0 < \alpha \leq 1\}$ are the two sets represents set of movable boundaries and they form nested structure for expressing the relationship between the crisp sets and fuzzy sets (Klir and Boyun 1997). We can use the α levels cuts of \tilde{L}_s to construct the membership function defined in (24) is parametrized by α . Using Zadeh's extension principle.

$$\tilde{b}_{L_s}(Z) = \sup_{z \in X} \{\min \{s, \mu_{\tilde{A}}(s), \mu_{\tilde{S}}(s)\} / z = L = \frac{N+1}{2N} \frac{L^2}{\mu(\mu-\lambda)} + \frac{1}{\mu} \} \}$$

To derive the membership function and non-membership function $\tilde{b}_{L_s}(Z)$, we need at least one of the following cases to hold, such that Z satisfies α

$$\left. \begin{array}{l} \text{case(1)} \mu_{\tilde{A}}(a) = \alpha, \mu_{\tilde{B}}(s) \geq \alpha \\ \text{case(2)} \mu_{\tilde{A}}(a) \geq \alpha, \mu_{\tilde{B}}(s) = \alpha \\ \text{case(1)} \gamma_{\tilde{A}}(a) = 1 - \alpha, \gamma_{\tilde{B}}(s) \geq 1 - \alpha \\ \text{case(2)} \gamma_{\tilde{A}}(a) \geq 1 - \alpha, \gamma_{\tilde{B}}(s) = 1 - \alpha \end{array} \right\} \quad (29)$$

that is, true, it satisfies the $0 < \mu_{\tilde{A}}^{-1}(\alpha) + \mu_{\tilde{B}}^{-1}(\alpha) \leq 1$ and $0 < \gamma_{\tilde{A}}^{-1}(\alpha) + \gamma_{\tilde{B}}^{-1}(\alpha) \leq 1$. This can be achieved using parametric non linear programming technique. The non-linear programming technique to find the lower and upper bound of

α -cut of $\tilde{\delta}_{\tilde{Z}}(Z)$ for case(1) are

$$\begin{aligned} (30) \alpha \tilde{L}_{\tilde{Z}_l} &= \min \left\{ \frac{K+1}{2k} + \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} \right\} \\ \alpha \tilde{L}_{\tilde{Z}_u} &= \max \left\{ \frac{K+1}{2k} + \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} \right\} \end{aligned}$$

From the definition of $A(\alpha)$ and $S(\alpha)$ such that at a α $A(\alpha)$, $s \in S(\alpha)$ can be replaced by a $\alpha(\tilde{L}_{\tilde{A}}, \tilde{U}_{\tilde{A}})$, $s \in \alpha(\tilde{L}_{\tilde{S}}, \tilde{U}_{\tilde{S}})$. Using the operation define on the intervals (Klin and Boyoun, 1997). Let the confidence interval of the first fuzzy set \tilde{A} and \tilde{S} be $(\tilde{L}_{\tilde{A}(\alpha)}, \tilde{U}_{\tilde{A}(\alpha)})$ and $(\tilde{L}_{\tilde{S}(\alpha)}, \tilde{U}_{\tilde{S}(\alpha)})$ respectively. To find the membership function of $\tilde{\delta}_{\tilde{Z}}(Z)$ it suffices to find the lower and upper shape functions of $\tilde{\delta}_{\tilde{Z}}(Z)$, which is equal to finding the lower bound $\tilde{L}_{\tilde{Z}_l}$ and $\tilde{L}_{\tilde{Z}_u}$ of the α -cut of $\tilde{L}_{\tilde{Z}}$ which can be written as (30)

And as a result, the non-membership functions of $\tilde{L}_{\tilde{Z}}$ become

$$\tilde{\delta}_{\tilde{Z}}(Z) = \inf_{a \in X, s \in Y} \{ \max \{ a, s, \mu_{\tilde{A}}(a), \mu_{\tilde{B}}(s) \} / Z = \frac{K+1}{2k} + \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} \}$$

Upper bound and lower bound of α -cut of $\tilde{L}_{\tilde{Z}}$, $\tilde{L}_{\tilde{Z}}$, $\tilde{W}_{\tilde{Z}}$, $\tilde{W}_{\tilde{Z}}$. Can be obtained as Where $\theta = \frac{K+1}{2k}$ and

$$\left. \begin{array}{l} \alpha \tilde{L}_{\tilde{Z}_l} = \min \left\{ \theta + \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} \right\} \\ \alpha \tilde{L}_{\tilde{Z}_u} = \max \left\{ \theta + \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} \right\} \\ \alpha \tilde{L}_{\tilde{Z}_l} = \min \left\{ \theta + \frac{\lambda^2}{\mu(\mu-\lambda)} \right\} \\ \alpha \tilde{L}_{\tilde{Z}_u} = \max \left\{ \theta + \frac{\lambda^2}{\mu(\mu-\lambda)} \right\} \\ \alpha \tilde{L}_{\tilde{W}_l} = \min \left\{ \theta + \frac{\lambda}{\mu(\mu-\lambda)} + \frac{1}{\mu} \right\} \\ \alpha \tilde{L}_{\tilde{W}_u} = \max \left\{ \theta + \frac{\lambda}{\mu(\mu-\lambda)} + \frac{1}{\mu} \right\} \\ \alpha \tilde{L}_{\tilde{W}_l} = \min \left\{ \theta + \frac{\lambda}{\mu(\mu-\lambda)} \right\} \\ \alpha \tilde{L}_{\tilde{W}_u} = \max \left\{ \theta + \frac{\lambda}{\mu(\mu-\lambda)} \right\} \end{array} \right\} \quad (31)$$

Consider an Fm/ FE_k / 1 queue, where both arrival rate and service rate are fuzzy number represented by

$$\left\{ \begin{array}{l} \tilde{A} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4) \quad \tilde{S} = (\mu_1, \mu_2, \mu_3, \mu_4) \\ \tilde{D}_\lambda(\lambda) = \begin{cases} 0 & \text{if } \lambda < \lambda_1 \\ \frac{\lambda - \lambda_1}{\lambda_2 - \lambda_1} & \text{if } \lambda_1 \leq \lambda \leq \lambda_2 \\ \frac{\lambda_3 - \lambda}{\lambda_3 - \lambda_2} & \text{if } \lambda_2 \leq \lambda \leq \lambda_3 \\ \frac{\lambda_4 - \lambda}{\lambda_4 - \lambda_3} & \text{if } \lambda_3 \leq \lambda \leq \lambda_4 \\ 0 & \text{if } \lambda \geq \lambda_4 \end{cases} \end{array} \right\} \quad (32)$$

$$\left\{ \begin{array}{l} \tilde{D}_\mu(\mu) = \begin{cases} 0 & \text{if } \mu < \mu_1 \\ \frac{\mu - \mu_1}{\mu_2 - \mu_1} & \text{if } \mu_1 \leq \mu \leq \mu_2 \\ \frac{\mu_3 - \mu}{\mu_3 - \mu_2} & \text{if } \mu_2 \leq \mu \leq \mu_3 \\ \frac{\mu_4 - \mu}{\mu_4 - \mu_3} & \text{if } \mu_3 \leq \mu \leq \mu_4 \\ 0 & \text{if } \mu \geq \mu_4 \end{cases} \end{array} \right\} \quad (33)$$

using the concept of α -cut method

$$\begin{aligned} \alpha_\lambda &= [\alpha(\lambda_1 - \lambda_1) + \alpha(\lambda_2 - \lambda_1) + \lambda_1, \lambda_4 - \alpha(\lambda_4 - \lambda_3) - \alpha(\lambda_3 - \lambda_2)] \\ \alpha_\mu &= [\alpha(\mu_1 - \mu_1) + \alpha(\mu_2 - \mu_1) + \mu_1, \mu_4 - \alpha(\mu_4 - \mu_3) - \alpha(\mu_3 - \mu_2)] \end{aligned} \quad (34)$$

Let the system performance \tilde{L}_s, \tilde{W}_s are given by

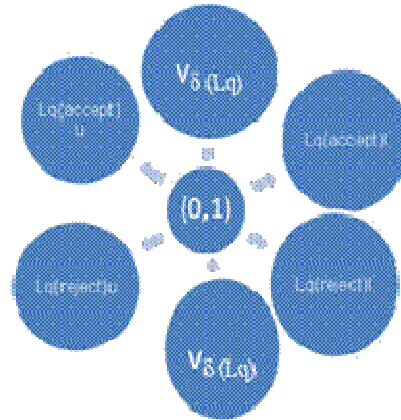
$$\left\{ \begin{array}{l} \tilde{L}_s = \frac{K+1}{2k} + \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} \\ \tilde{L}_q = \frac{K+1}{2k} + \frac{\lambda^2}{\mu(\mu-\lambda)} \\ \tilde{W}_s = \frac{K+1}{2k} \frac{\lambda}{\mu(\mu-\lambda)} + \frac{1}{\mu} \\ \tilde{W}_q = \frac{K+1}{2k} + \frac{\lambda}{\mu(\mu-\lambda)} \end{array} \right\} \quad (35)$$

Using α -cut of λ and μ

$$(36) \alpha_{\tilde{L}_q} = \left[\frac{K+1}{2k} + \frac{(\alpha(\lambda_2 - \lambda_1) + \alpha(\lambda_3 - \lambda_2) + \lambda_1)^2}{(\mu_4 - \alpha(\mu_4 - \mu_3) - \alpha(\mu_3 - \mu_2))(\mu_4 - \alpha(\mu_4 - \mu_3) - \alpha(\mu_3 - \mu_2) - (\lambda_4 - \alpha(\lambda_4 - \lambda_3) - \alpha(\lambda_3 - \lambda_2)))}, \right. \\ \left. \frac{K+1}{2k} + \frac{(\lambda_4 - \alpha(\lambda_4 - \lambda_3) - \alpha(\lambda_3 - \lambda_2))^2}{(\alpha(\mu_4 - \mu_3) + \alpha(\mu_3 - \mu_2) + \mu_4)(\alpha(\mu_4 - \mu_3) + \alpha(\mu_3 - \mu_2) + \mu_4 - (\alpha(\lambda_4 - \lambda_3) + \alpha(\lambda_3 - \lambda_2) + \lambda_4))} \right]$$

Similarly $\tilde{L}_q, \tilde{L}_s, W_q, \tilde{W}_s$ and $\alpha \in [0,1]$ in [1] we get an triangular fuzzy numbers

$$(37) \tilde{L}_q = \left[\frac{K+1}{2k} + \frac{\lambda_1^2}{\mu_4(\mu_4 - \lambda_4)}, \frac{K+1}{2k} + \frac{\lambda_2^2}{\mu_3(\mu_3 - \lambda_3)}, \frac{K+1}{2k} + \frac{\lambda_3^2}{\mu_2(\mu_2 - \lambda_2)}, \frac{K+1}{2k} + \frac{\lambda_4^2}{\mu_1(\mu_1 - \lambda_1)} \right]$$



Procedure to solve IFQ

Step-1: $\tilde{a} = (\mu_{\tilde{a}}, \nu_{\tilde{a}})$ be an intuitionistic fuzzy number. We use $z = \mu_{\tilde{a}} - \nu_{\tilde{a}}$ where $z \in (0-1)$ variables.

Step-2: $\mu_{\tilde{a}} + \nu_{\tilde{a}} < 1$, $\mu_{\tilde{a}} \geq \nu_{\tilde{a}}$, $\nu_{\tilde{a}} \geq 0$ the value of the intuitionistic fuzzy

$A_{\alpha}(a) = 1 - \mu_{\alpha}(a) - \nu_{\alpha}(a)$ and $S_{\alpha}(s) = 1 - \mu_{\alpha}(s) - \nu_{\alpha}(s)$

To find intuitionistic fuzzy arrival and service rate of acceptance and rejection based on intuitionistic fuzzy number are defined as follows:

$$(38) A_{\alpha}^{accept} = \{\mu_{\alpha}(a)^{accept}, \nu_{\alpha}(a)^{accept}\}; A_{\alpha}^{reject} = \{\mu_{\alpha}(a)^{reject}, \nu_{\alpha}(a)^{reject}\}$$

Where $\mu_{\alpha}(a)^{accept} = \max\{\mu_{\alpha}(a)\}; \nu_{\alpha}(a)^{accept} = \min\{\nu_{\alpha}(a)\}$

$$\mu_{\alpha}(s)^{accept} = \max\{\mu_{\alpha}(s)\}; \nu_{\alpha}(s)^{accept} = \min\{\nu_{\alpha}(s)\}; \mu_{\alpha}(a)^{reject} = \max\{\mu_{\alpha}(a)\};$$

$$\nu_{\alpha}(a)^{reject} = \min\{\nu_{\alpha}(a)\}$$

$$\mu_{\alpha}(s)^{reject} = \max\{\mu_{\alpha}(s)\}; \nu_{\alpha}(s)^{reject} = \min\{\nu_{\alpha}(s)\}$$

Step-3: Now calculate intuitionistic fuzzy queue, the value of rejection (non-membership function) and the value of acceptance (membership) are considered. So that the sum of the both values is less than unity.

Multi-variate theorem:

$$\lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \vdots \\ \lambda_n \end{pmatrix} \quad \text{and} \quad \mu = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \vdots \\ \mu_n \end{pmatrix}$$

Let X_i , ($i=1,2,3,\dots,n$) be independent $\alpha \in (0-1)$ variate, then the conditional distribution $\chi^2 = \sum_{i=1}^n X_i^2$ subject to $m(<n)$, independent homogeneous linear constraints $\alpha_{11} X_1 + \alpha_{12} X_2 + \alpha_{13} X_3 + \dots + \alpha_{1n} X_n = 0$

$$\alpha_{21} X_1 + \alpha_{22} X_2 + \alpha_{23} X_3 + \dots + \alpha_{2n} X_n = 0$$

$$\dots$$

$$\alpha_{m1} X_1 + \alpha_{m2} X_2 + \alpha_{m3} X_3 + \dots + \alpha_{mn} X_n = 0$$

is also χ^2 distribution with (n-m)degrees of freedom.

Proof: Equivalently the constraints can be expressed as

$$\alpha_{11} X_1 + \alpha_{12} X_2 + \alpha_{13} X_3 + \dots + \alpha_{1n} X_n = 0$$

$$\alpha_{21} X_1 + \alpha_{22} X_2 + \alpha_{23} X_3 + \dots + \alpha_{2n} X_n = 0$$

$$\dots$$

$$\alpha_{m1} X_1 + \alpha_{m2} X_2 + \alpha_{m3} X_3 + \dots + \alpha_{mn} X_n = 0$$

Where $\alpha_i = (\alpha_{i1} \alpha_{i2} \alpha_{i3} \dots \alpha_{in})$, $i=1,2,\dots,m$ are m unitary mutually orthogonal of arrivals. Let us now transform the variables.

$$\lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \vdots \\ \lambda_n \end{pmatrix} \quad \text{and} \quad \mu = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \vdots \\ \mu_n \end{pmatrix}$$

By means of a linear orthogonal transformation $y=AX$ where

$$A = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \dots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \dots & \alpha_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ \alpha_{m1} & \alpha_{m2} & \alpha_{m3} & \dots & \alpha_{mn} \end{pmatrix}$$

This implies that the constraints in Y are equivalently to $Y_i = 0$, ($i=1,2,\dots,m$)

By Fisher's lemma $Y_i = 0$, ($i=1,2,\dots,m$) are also independent $\alpha \in (0,1)$ variables and

$$(39) \chi^2 = \sum_1^n X_i^2 = \sum_1^n Y_i^2 = \sum_{m+1}^n Y_i^2 \quad (\text{Transformation } Y \text{ is orthogonal})$$

Thus the conditional distribution of $\sum_1^n X_i^2$ subject to the constraints Y is same as the unconditional distribution of $\sum_1^n Y_i^2$, where Y_i ($i= m+1, \dots, n$) are independent standard normal variates without any constraints on them. Hence

$$\chi^2 = \sum_1^n X_i^2 = \sum_1^n Y_i^2$$

Being the sum of squares of (n-m) independent standard normal variates follows χ^2 distribution with n-m degrees of freedom. The expected number of busy servers

$$(40) \delta_Q(Z) = \chi^2 = \sum_{i=1}^n X_i^2 = \sum_{i=1}^n Y_i^2 = \sum_{i=1}^n (\alpha_{m1} X_1 + \alpha_{m2} X_2 + \alpha_{m3} X_3 + \dots + \alpha_{mn} X_n)^2$$

For every set of variable, $(X_1 \ X_2 \ X_3 \dots\dots\dots X_n)$

if we write

$$\delta_Q(Z) = \sum_{k=1}^n \alpha_{ik} \alpha_{kj}, \quad (i, j = 1, 2, \dots, n),$$

then this implies that $\delta_Q(Z)$ is a kronecker delta so that

$$(41) \delta_Q(Z) = \begin{cases} 1, & i=j \\ 0, & i \neq j \end{cases}$$

Further the membership function

$$(42) \mu_{\delta_Q}(Z) = \begin{cases} L(Z), & Z_1 \leq Z \leq Z_2 \\ 1, & Z_2 \leq Z \leq Z_3 \\ R(Z), & Z_3 \leq Z \leq Z_4 \end{cases}$$

non-membership function

$$(43) \gamma_{\delta_Q}(Z) = \begin{cases} 1-L(Z), & Z_1 \leq Z \leq Z_2 \\ 1, & Z_2 \leq Z \leq Z_3 \\ 1-R(Z), & Z_3 \leq Z \leq Z_4 \end{cases}$$

It is noted that the membership functions for other system characteristics such as expected waiting time of the customers in the queuing can be derived in a similar manner.

Numerical Example

Suppose that we have a queuing system M/M/1 with arrival rate, service rate are $\tilde{\lambda} = (1, 2, 3, 4)$ and $\tilde{\mu} = (8, 9, 11, 12)$ per hour respectively. $\tilde{\lambda}, \tilde{\mu}$ are triangular fuzzy intuitionistic numbers are described for the threshold value, we take $K=1$ and it is simple to find

$$A_\alpha = [\min \mu_A^{-1}(\alpha), \max \mu_A^{-1}(\alpha)] = [1+\alpha, 4-\alpha] \quad (44)$$

$$S_\alpha = [\min \mu_S^{-1}(\alpha), \max \mu_S^{-1}(\alpha)] = [11+\alpha, 14-\alpha]$$

$$\tilde{L}_q(\alpha) = 1 + \frac{(2\alpha+1)^2}{(12-\alpha-\alpha)(12-2\alpha-4+2\alpha)}, \quad 1 + \frac{(4-\alpha-\alpha)^2}{(\alpha+8)(\alpha+\alpha+8-\alpha-\alpha-1)}$$

$$\tilde{L}_q(\alpha) = \left\{ 1 + \frac{(2\alpha+1)^2}{(8)(12-2\alpha)}, 1 + \frac{(4-2\alpha)^2}{7(\alpha+8)} \right\} \quad (45)$$

Substitute $\alpha=0$ and $\alpha=1$ in $\tilde{L}_q(\alpha)$ we get

$$\begin{aligned} \tilde{L}_q(\alpha) &= 1 + \frac{(2(0)+1)^2}{(8)(12-0)}, 1 + \frac{(4-0)^2}{7(0+8)}, 1 + \frac{(2(1)+1)^2}{(8)(12-1)}, 1 + \frac{(4-2(1))^2}{7(1+8)} \\ &= [1.01041667, 1.2857, 1.1125, 1.06349] \end{aligned} \quad (46)$$

Similarly substitute in $\tilde{L}_s(\alpha), \tilde{W}_s(\alpha), \tilde{W}_q(\alpha)$ we get the fuzzy solution. Next to write IFQ

$$\begin{aligned}\tilde{L}_q(\infty) &= \frac{1.01041667}{1.01041667 + 1.1125} + \frac{1.2857}{2.12291667} \\ &= [0.475956821, 0.605629047]\end{aligned}$$

Using IFQ

$$\begin{aligned}l_k^{rsj} &= l_k^{acc} + t(u_k^{acc} - l_k^{acc}) \text{ where } 0 \leq t \leq 1 \\ u_k^{rsj} &= u_k^{acc} + t(u_k^{acc} - l_k^{acc}) \text{ for } t=0,\end{aligned}$$

$$\begin{aligned}\text{Where } u_k^{acc} &= \max_{1 \leq r \leq k} z_k(\overline{X}_r) \\ l_k^{acc} &= \min_{1 \leq r \leq k} z_k(\overline{X}_r) \\ z_k(\overline{X}_r) &= u_k^{acc} - t(u_k^{acc} - l_k^{acc}) \\ &= 1.1125 - 0.7(1.1125 - 1.06349) \\ &= 1.078193 \\ z_k(\overline{X}_r) &= l_k^{rsj} - t(u_k^{rsj} - l_k^{rsj}) \\ &= 1.01041667 + 0.2(1.2857 - 1.01041667) \\ &= 1.065473336\end{aligned} \quad (47)$$

Membership function and non- membership function

$$\begin{aligned}\frac{u_k^{acc} - z^A}{u_k^{acc} - l_k^{acc}} &= \frac{1.1125 - 1.078193}{1.1125 - 1.06349} = 0.7 \\ \frac{z - l_k^{rsj}}{u_k^{rsj} - l_k^{rsj}} &= \frac{1.065473336 - 1.01041667}{1.2857 - 1.01041667} = 0.2\end{aligned} \quad (48)$$

Finally $\lambda = 2, \mu = 9$

$$L_q = \frac{\mu + 1}{2\mu} + \frac{\lambda^2}{\mu(\mu - \lambda)} = 1.063492063 \quad (49)$$

Conclusions and Future of Research:

This paper is to analyze intuitionistic queueing model by NLP technique rather than usual IFNLP technique. Comparison of the membership and non-membership function which of the fuzzy and intuitionistic cost shows that minimizes cost function. This concept of intuitionistic fuzzy set can be viewed as an alternative methods to define a convex fuzzy set. In this paper we introduced a new method to solve IFNLP using characteristics of the performance functions. We proposed IF algorithm steps and solved with an illustrative example. Thus the method is very useful in the real world problems, this provides more information to help design effect of intuitionistic fuzzy queueing system.

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