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Estimation of Free Edge Stresses over a Laminated Composite Plate Using Lamination Theories

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ABSTRACT:

Laminated composite materials are being increasingly used in many engineering applications because of their numerous advantages over conventional engineering materials such as metals. High stiffness and strength to weight ratio are among their most appealing advantages. Different methods have been devised by the researchers to improve the composite material performance. For laminated composites, it is well-known that at the free edges inter-laminar stresses arise from the mismatch of elastic properties between layers. In this work composites made with the help of E-glass fibre and epoxy resin and the analysis of free edges stresses of composite laminates subjected to a uniform axial load at the two ends of the laminate is performed using the 8 node rectangular element. The Free edge stresses are predicted by the use of the Lamination Theory and ANSYS. The free edge stresses values are validated of various orientations.

Keywords: **Delamination, composite, Lamination theory**

1. Introduction

The increased applications of composite materials in structural members have simulated interest in the accurate determination of the response characteristics of laminated composites. It is well known that inter-laminar stresses are developed at free edges in composite laminates where we have material discontinuity. These stresses can lead to delamination and failure of the laminate at loads that are much lower than the failure strength predicted by the classical lamination theory. Accurate determination of the stress state near the free edge is therefore crucial to correctly describe the laminate behaviour and to prevent its early failure.

New approximate theory which has been proposed to define complete stress fields within an arbitrary composite laminate. Weakness in previous laminate theories are discussed and it is demonstrated how these are overcome in the present formulation [1]. Local higher order lamination theory is proposed in this paper. The local displacement fields are expanded in terms of high order polynomial series through thickness within each ply. The displacement continuity constraints at the interface between layers are introduced into the potential energy functional by Lagrange multiplier method [2].

A solution method which was derived for determining the free edge stresses in composite laminates. A relatively simple and efficient method has been demonstrated for determining the stresses near the free edge of general composite laminates. Based on an average stress convergence criterion, a 16 ply laminate was analysed using 16 terms in the series[3].Layer wise theory is used to investigate analytically the inter laminar stresses near the free edges of general cross – ply composite laminates under uniform axial extension. Laminates with finite dimensions are considered and full three dimensional stresses in the interior and the boundary- layer regions are calculated. The results obtained from this theory are compared with those available in the literature [4].

New three dimensional hybrid stress element with a traction-free planar surface has been developed for efficient analysis of the inter-laminar stresses near traction-free straight boundaries. The use of such special element in the finite element solution has been shown to be highly accurate when only a very coarse element mesh is used near the free-edge[5].A special set of boundary conditions an elasticity solution is presented to verify the validity and accuracy of the layerwise theory. Various numerical results are then developed within the layerwise theory for the inter-laminar stresses through the thickness and across the interfaces of anti-symmetric angle-ply laminates with free edges [6].

Investigated free edge laminates subjected to uniaxial extension and uniform temperature variation by using multi-particle finite element. This two dimensional finite element approach with very coarse mesh provides very accurate three dimensional free edge stress field [7]. A higher-order triangular plate element that is based on the third-order shear deformation theory and a layer-wise plate theory of Reddy for the bending analysis of laminated composite plates. It will be shown herein that the proposed element is able to accurately predict the shear forces and twisting moments as well as the transverse shear stresses across the thickness of a laminated composite plate [8].

A composite is a structural material that consists of two or more combined constituents that are combined at a macroscopic level and are not soluble in each other. One constituent is called the reinforcing phase and the one in which it is embedded is called the matrix. The reinforcing phase material may be in the form of fibers, particles, or flakes. The matrix phase materials are generally continuous. In addition to that filler materials are added to that to increase the properties of the composite. Unidirectional – Fibers are oriented in only one direction. Bidirectional – Fibers are oriented in two direction either at right angle (as in cross ply) to one another or at some other desired angle (angle ply).

1.1 Inter Laminar Stress

Load transfer between adjacent layers in a fiber - reinforced laminate takes place by means of inter-laminar stresses, such as σ_{zz}, τ_{xz},τ_{yz}. The principal reason for the existence of inter-laminar stresses is the mismatch of poisson's ratio υ_{xy} and coefficients of mutual influence m_x and m_y between adjacent laminas.

For practical purposes, it may be sufficient to note the following,

i. Inter-laminar stresses in laminated composites develop owing mismatch in the poissson's ratios and coefficients of mutual influence between various layers. If there is no mismatch of these two engineering properties, there are no inter-laminar stresses regardless of the mismatch in elastic and shear moduli.
ii. Inter-laminar stresses can be significantly high over a region equal to the laminate thickness near the free edges of a laminate. As a result of high inter-laminar stresses, delamination may initiate at the free edges.
iii. Stacking sequence has a strong influence on the nature, magnitude and location of inter-laminar stresses.
iv. Material properties also have a strong influence on the inter-laminar shear stresses of a laminate.

1.2 Special Cases of Laminated Composites

The symmetry or anti-symmetry of a laminate is based on angle, material and thickness of plies. There are important to study because they may result in reducing or zeroing out the coupling of forces and bending moments, normal and shear forces or bending twisting moments. This is not only simplifies the mechanical analysis of composites, but also gives desired Mechanical performance.

Symmetric laminates

A laminate is called symmetric if the material, angle, and thickness of the plies are same above and below the mid plane. If it is subjected only to forces, it will have zero mid plane curvatures. Similarly, if it is subjected only to moments, it will have zero mid plane strains. It also prevents a laminate from twisting due to thermal loads, such as cooling down from processing temperatures, temperature fluctuations, during use such in the space shuttle etc.

Cross ply laminates

A laminate is called cross ply laminate, if only 0 degree and 90 degree plies are used to make a laminate. In these cases there are coupling between the normal and shear forces and also between bending and twisting moments.

Angle Ply laminates

A laminate is called angle ply laminate if it has plies of same material and thickness and only oriented at + θ and – θ directions. These angle ply laminates have higher shear stiffness and shear strength properties than cross ply laminates.

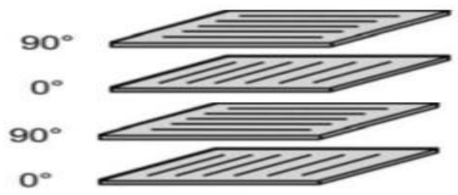

Fig 1 Laminate Layer for 0^o & 90^oOrientation

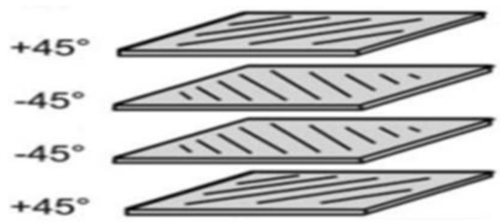

Fig 2 Laminate Layer for $+45^o$ & -45^o Orientation

1.3 Problem Description

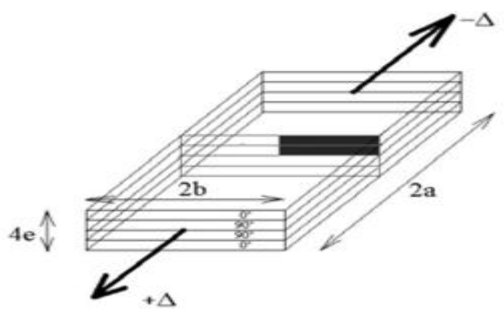

Fig 3 Laminate subjected to uniform axial load.

A four layer finite width composite laminate of length 2a, width 2b and height 4e has an uniform axial load at the two ends as shown in figure 3. The properties of the composite laminate are shown in table 1.

1.5. Objective

- Free edge stress of E-glass fibre composite laminates is found out theoretically by various theories and analytically by finite element analysis.
- This works aims to predict the inter-laminar stresses near the free edges to avoid the delamination occur in the composite lamina.

Material properties		Geometric details	Loading details
Material Name	E- Glass –Epoxy	e = 1mm h = 4e b = 16e a = 2b	$\Delta=100\times 10^3$KN/m
Youngs modulus	E_1= 44.8 GPa E_2 = E_3 = 7.27 Gpa		
Shear modulus	G_{12}=G_{13} = G_{23} = 4.86 GPa		
Poisson's ratio	υ_{12}= υ_{13}= υ_{23}= 0.28		

- The recent widespread use of laminated composites in various industrial fields necessitates analysis tools which are capable of predicting their mechanical behavior more accurately and efficiently.

2.Lamination Theory

Lamination theory is useful in calculating stresses and strains in each lamina of a thin laminated structure. Beginning with the stiffness matrix of each lamina, the step-by-step procedure in lamination theory includes,

1. Calculation of stiffness matrices for the laminate.
2. Calculation of mid-plane strains and curvature for the laminate due to a given set of applied forces and moments.
3. Calculation of in-plane strains $\varepsilon_{xx,}$ ε_{yy} and γ_{xy} for each lamina.
4. Calculation of in-plane stresses $\sigma_{xx,}$ σ_{yy} andτ_{xy} ineach lamina.

Displacement Equation for basis of Lamination Theory are,

$$u = u^0(x,y) + z\, F_1(x,y)$$

$$v = v^0(x,y) + z\, F_2(x,y)$$

$$w = w^0(x,y) = w(x,y)$$

4.2Formulae used in Lamination Theory

Stiffness matrix:

$$[\bar{Q}] = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}$$

Here,

$$\bar{Q}_{11} = U_1 + U_2 \cos 2\theta + U_3 \cos 4\theta$$

$$\bar{Q}_{12} = \bar{Q}_{12} = U_4 - U_3 \cos 4\theta$$

$$\bar{Q}_{22} = U_1 - U_2 \cos 2\theta + U_3 \cos 4\theta$$

$$\bar{Q}_{16} = \frac{1}{2} U_2 \sin 2\theta + U_3 \sin 4\theta$$

$$\bar{Q}_{26} = \frac{1}{2} U_2 \sin 2\theta - U_3 \sin 4\theta$$

$$\bar{Q}_{66} = U_5 - U_3 \cos 4\theta$$

$$U_1 = \frac{1}{8}(3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66})$$

$$U_2 = \frac{1}{2}(Q_{11} - Q_{22})$$

$$U_3 = \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66})$$

$$U_4 = \frac{1}{8}(Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66})$$

$$U_5 = \frac{1}{2}(U_1 - U_4)$$

$$Q_{11} = \frac{E_{11}}{1-\upsilon_{12}\upsilon_{21}}$$

$$Q_{22} = \frac{E_{22}}{1-\upsilon_{12}\upsilon_{21}}$$

$$Q_{12} = Q_{21} = \frac{\upsilon_{12}E_{22}}{1-\upsilon_{12}\upsilon_{21}}$$

$$Q_{66} = G_{12}$$

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} [A][B][D]$$

$$A_{mn} = \sum_{j=1}^{N} (\bar{Q}_{mn})_j (h_j - h_{j-1})$$

$$B_{mn} = \frac{1}{2} \sum_{j=1}^{N} (\bar{Q}_{mn})_j (h_j^2 - h_{j-1}^2)$$

$$D_{mn} = \frac{1}{3} \sum_{j=1}^{N} (\bar{Q}_{mn})_j (h_j^3 - h_{j-1}^3)$$

If $A_{16} = A_{26} = 0$ and $[B]=0$

The mid-plane strains are,

$$\varepsilon_{xx}^{\cdot} = \frac{A_{22}}{A_{11}A_{22} - A_{12}^2} N_{XX}, \varepsilon_{YY}^{\cdot} = \frac{-A_{12}}{A_{11}A_{22} - A_{12}^2} N_{XX},$$
$$\gamma_{XY}^{\cdot} = 0$$

Here, N_{XX} is the tensile load in X direction.

3.Finite Element Analysis

In this chapter, Fig 4 shows the geometry of the specimen with four layers with orientation $0°\&90°$ of thickness 0.002 m andresults of stress in X-direction, Y-direction and shear stress of E-glass fibers compared with ANSYS & Lamination theory.

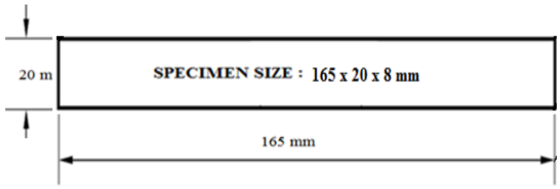

Fig 4 Geometry of the Specimen

3.1 Estimation of Free Edge Stresses Using ANSYS

Shell section of the laminated composite are to be ploted for the layers of the E-glass Fiber with orientation $0°\&90°$ of thickness 0.002 m.Laminated composite with various layers of the E-glass Fiber with orientation as shown in Fig 8. For the First and Third layer with an orientation of $0°$. Second and fourth layer has angle of $90°$.

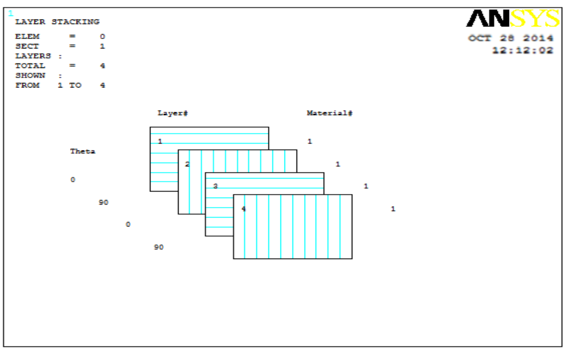

Fig 5 Layer Orientation

2D view& 3D view of theLaminated composite with various layers combined together.as shown in the Fig 6.

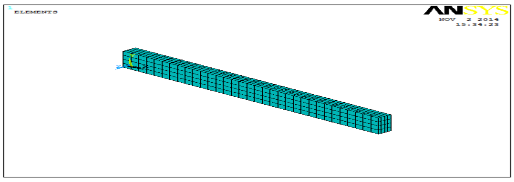

Fig 6 Meshed View

By Nodal solution result, shows the Stress in X-direction with an max value of 40.9N/mm^2 show in Fig 7.

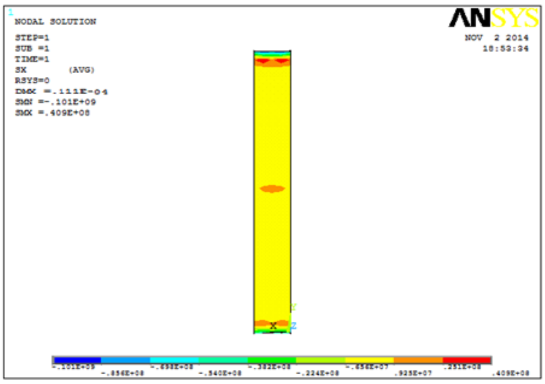

Fig 7 Stress in X-direction

By Nodal solution result, shows the Stress in Y-direction with an max value of 1.418N/mm^2 show in Fig 8.

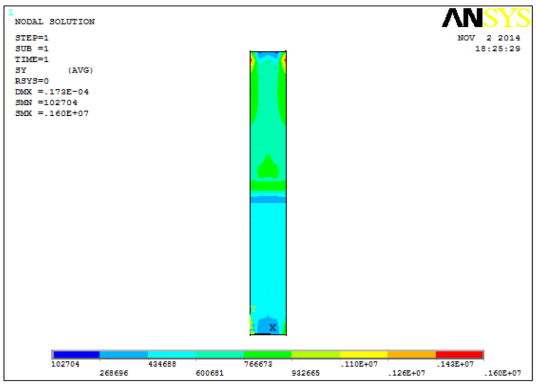

Fig 8 Stress in Y-Direction

By Nodal solution result, shows the Shear Stress with an max value asshown in Fig 9 which has negative values

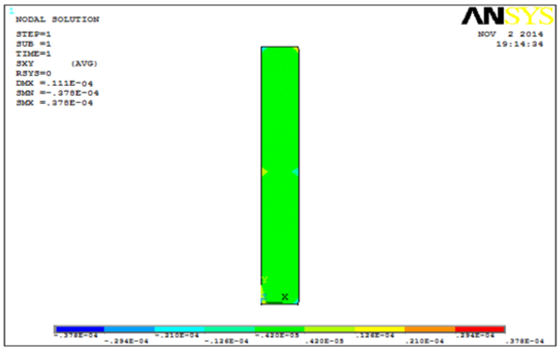

Fig 9 Shear stress

4. Conclusions

Composites are preferred because of their low specific weight, better stiffness and compression property than some of the metals that commonly used.Lamination theory of E-glass fiber of stress in X & Y direction and shear stress are calculated by the standard material properties for orientation 0^0, 90^0& 45^0,-45^0.Numerical analysis is done for E-GLASS/EPOXY composites and the results were validated &compared for suitable applications.

	Orientation of 0^0& 90^0		**Orientation of +45^0& -45^0**	
Units N/mm^2	Ansys	Lamination Theory	Ansys	Lamination Theory
Stress in X directi-on (σxx)	40.9	43.24	26.42	25.144
Stress in Y direction (σxx)	1.418	1.38	0.428	0.242
Shear Stress in XY Plane (τxy)	0.00004	0	0.0002	0

The free edge stress value of the composite laminates is predicted by the use of ANSYS and Lamination Theory. By comparing both ANSYS and lamination Theory with Orientation of 0^0& $90^0$and Orientation of +45^0& -45^0,the stress values of 0^0& 90^0 is higher than +45^0& -45^0.

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