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Motion and Stability of Two Interconnected Satellites System in Circular Orbit of the Centre of Mass

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Abstract

Effects of the Solar Radiation Pressure, Magnetic force, Air resistance force parameter and Earth shadow on the motion and stability of two satellites connected by Light flexible and inextensible string in the central gravitational field of the earth. The equations of motion of the system have been deduced with respect to the centre of mass of the system, which is assumed to move along Keplerian elliptic orbit (in particular circular)

Keyword: Satellites, Circular, BKM, equilibrium, stable.

Introduction

This paper is devoted on the study of the stability of the equilibrium position of two satellites connected by light, flexible and inextensible cable under the influence of magnetic force, solar radiation pressure and shadow of the earth in case of circular orbit of the centre of mass. First of all equations of motion have been derived by using Lagrange's equations of motion of first kind in elliptical orbit in Nechvill's coordinate system. Then Jacobi integral for the problem in case of circular orbit of the centre of mass of the system is obtained and, it has been shown that the equilibrium point is stable. This work is direct generalization of the works done by Das, S.K. and Bhattacharya, Sinha, S.K. and Singh A.K.P.

Equations of motion in case of circular orbit of the centre of mass of the system

The equations of motion of one of the two satellites when the centre of mass moves along Keplerian elliptical orbit in Nechvill's coordinate system have been derived in the form:

$$x'' - 2y' - 3\rho x - A\rho^3\psi_1 \cos\in \cos(v-\alpha) = \lambda_\alpha \rho^4 x - \frac{B\cos i}{\rho} - f\rho\rho'$$

$$y'' - 2x' - A\rho^3\psi_1 \cos\in \sin(v-\alpha) = \lambda_\alpha \rho^4 y - \frac{B\rho^1\cos i}{\rho^2} - f\rho 2$$

$$z'' + z + A\rho^3\psi_1 \sin\in = \lambda_\alpha \rho^4 z - \frac{B}{\rho}\left[\frac{\rho'}{\rho}\cos(v+w) + \frac{1}{\mu_E}(3p^3\rho^2 - \mu_E)\sin(v+w)\right]\sin i \qquad (1)$$

Where

$$A = \frac{p^3}{\mu\ell}\left[\frac{B_1}{m_1} - \frac{B_2}{m_2}\right] = \textit{Solar radiation pressure parameter}$$

$$B = \frac{m_1}{m_1+m_2}\left[\frac{\vartheta_1}{m_1} - \frac{\vartheta_2}{m_2}\right]\frac{\mu_E}{\sqrt{\mu p}} = \textit{Magnetic force parameter}$$

f = Air resistance force parameter

ψ_1 = *Shadow of the earth parameter*

$$\lambda_\alpha = \frac{p^3\lambda}{\mu} = \frac{-p^3\lambda}{\mu}\frac{(m_1+m_2)}{m_1 m_2}\ ; \lambda \textit{ being Leagrange's multiphor}$$

l is the length of cable connecting the two satellites of masses m_1 and m_2 respectively, μ is the product of gravitational constant with mass of the attracting centre (the earth), p is the semi latus rectum or focal parameter and μ_E is the volume of the magnetic moment of the earth dipole.

The condition of constraint is given by

$$x^2 + y^2 + z^2 \le \frac{1}{\rho^2} \qquad (2)$$

Since general solution of (1) is beyond our reach, so we restrict myself to the case of circular orbit of the centre of mass of the system in equatorial orbit (i = 0).

For circular orbit, we have

e = 0,

So

$$\rho = \frac{1}{1+e\cos v} = 1 \qquad \textit{and} \qquad \rho^1 = 0 \qquad (3)$$

It is also to be noted that the averaged values of periodic terms in equations of motion (1) can be put in the form

$$\frac{1}{2\pi}\left[\int_{\substack{-\vartheta \\ \psi_1=0}}^{\vartheta} A\cos\in\cos(v-\alpha)dv + \int_{\substack{\vartheta \\ \psi_1=1}}^{2\pi-\vartheta} A\cos\in\cos(v-\alpha)dv\right] = \frac{-A\cos\in\cos\alpha\sin\vartheta}{\pi}$$

and

$$\frac{1}{2\pi}\left[\int_{\substack{-\vartheta \\ \psi_1=0}}^{\vartheta} A\cos\in\sin(v-\alpha)dv+\int_{\substack{\vartheta \\ \psi_1=1}}^{2\pi-\vartheta} A\cos\in\sin(v-\alpha)dv\right]=\frac{A\cos\in\sin\alpha\sin\vartheta}{\pi} \quad (4)$$

on putting $\in = 0$, $\rho = 1$, $\rho^1 = 0$ and $i = 0$ for equatorial orbit (circular), Equations of motion (1), with averaged values given by (4), take the form:

$$x'' - 2y' - 3x + \frac{A\cos\alpha\sin\vartheta}{\pi} = \lambda_\alpha\ x - B$$

$$y'' + 2x' + \frac{A\sin\alpha\sin\vartheta}{\pi} = \lambda_\alpha\ y - f \quad (5)$$

$$z'' + z = \lambda_\alpha\ z$$

The condition of constraint given by (2) takes the form

$$x^2 + y^2 + z^2 \le 1 \quad (6)$$

If the inequality sign holds in (6) then the motion takes place with loose string and the motion is called free motion. If equality sign holds in (6) then the motion takes place with tight string and the motion in this case is called constrained motion.

We are interested in the motion and stability of constrained motion. Thus, the motion takes place on the unit sphere given by

$$x^2 + y^2 + z^2 = 1 \quad (7)$$

From (7), we get

$$xx' + yy' + zz' = 0 \quad (8)$$

Multiplying 1st , 2nd and 3rd equation of (5) by $2x'$, $2y'$ and $2z'$ respectively and adding them together and integrating w.r. to v, we get on using

$$x'^2 + y'^2 + z'^2 - (3x^2 - z^2) = h - 2Bx - \frac{2A\sin\vartheta}{\pi}(x\cos\alpha + y\sin\alpha) - 2fy \quad (9)$$

where h is the constant of integration which is known as Jacobi integral and can be interpreted as energy equation with modified potential v given by

$$v = -\frac{1}{2}(3x^2 - z^2) + \frac{A}{\pi}\sin\upsilon(x\cos\alpha + y\sin\alpha) + Bx + fy \quad (10)$$

Differentiating (8) w.t to v, we get

$$x'^2 + y'^2 + z'^2 = -(xx'' + yy'' + zz'') \quad (11)$$

Multiplying 1st , 2nd and 3rd equations of (5) by x, y and z respectively and adding, we get on using (7) and (11)

$$-\lambda_\alpha = (x'^2 + y'^2 + z'^2) + 2(xy' - x'y) - Bx - \frac{A\sin\vartheta}{\pi}(x\cos\alpha + y\sin\alpha) - fy \quad (12)$$

Let us introduce the polar spherical coordinates given by

$$x = \cos\phi \cos\psi$$
$$y = \cos\varphi \sin\psi \tag{13}$$

and
$z = \sin\varphi$
on the unit sphere (7) using (13) in (9) , we get after some simplification.

$$\phi'^2 + \psi'^2 + \cos^2\phi = \cos^2\phi\,[\cos^2\psi + 1] - 2B\cos\varphi\cos\psi - 2f\cos\phi\sin\psi - \frac{2A\sin\vartheta}{\pi}(\cos\psi\cos\alpha + \sin\psi\sin\alpha)\cos\varphi + h_1 \tag{14}$$

where $h_1 = h - 1$
(14) is known as Jacobian integral of the equation of motion.

Particular solution and its stability

For the equilibrium positions, we can take the modified potential energy as

$$V(\varphi, \psi) = -\cos^2\varphi\,(3\cos^2\psi + 1) - 1 + 2B\cos\varphi\cos\psi + 2f\cos\phi\sin\psi + \frac{2A\sin\vartheta}{\pi}\cos\varphi\,[\cos\psi\cos\alpha + \sin\psi\sin\alpha] \tag{15}$$

The equilibrium positions are given by the stationary values of v(φ, ψ) and hence we must have

$$\frac{\partial v}{\partial \varphi} = 0 \tag{16}$$

$$\frac{\partial v}{\partial \Psi} = 0 \tag{17}$$

From (16) it follows that, $\sin\varphi = 0 \Rightarrow \varphi = 0$ (18)

$$\text{or } \cos\varphi\,(3\cos^2\psi + 1) - B\cos\psi - \frac{A\sin\vartheta}{\pi}(\cos\psi\cos\alpha + \sin\psi\sin\alpha) - f\sin\Psi = 0 \tag{19}$$

From (17), we get

$$\left[\frac{\partial v}{\partial \Psi}\right]_{\Psi=\Psi_0}^{\phi=0} = 3\cos\Psi_0\sin\Psi_0 - B\sin\Psi_0 - \frac{A\sin\vartheta}{\pi}\sin\Psi_0\cos\alpha + f\cos\Psi_0 + \frac{A\sin\vartheta}{\pi}\sin\alpha\cos\Psi_0$$

Thus

$$\left[\frac{\partial v}{\partial \Psi}\right]_{\Psi=\Psi_0}^{\phi=0} = O$$

$$\Rightarrow 3\cos\Psi_0 \sin\Psi_0 - (B + \frac{A\sin\vartheta}{\pi}\cos\alpha)\sin\Psi_0 + (f + \frac{A\sin\vartheta}{\pi}\sin\alpha)\cos\Psi_0 = 0. \tag{20}$$

For the smallest value of Ψ_0, we have
Sin Ψ_0 = Ψ_0 and cos Ψ_0 =1

Thus, the equation (20), gives us

$$\Psi_0 = \frac{-f - \frac{A}{\pi}\sin\vartheta\sin\alpha}{3 - B - \frac{A}{\pi}\sin\vartheta\cos\alpha} \tag{21}$$

Thus, the equilibrium point is given by

$$\phi = \phi_0 = 0 \quad ; \quad \Psi = \Psi_0 = \frac{-f - \frac{A}{\pi}\sin\vartheta\sin\alpha}{3 - B - \frac{A}{\pi}\sin\vartheta\cos\alpha} \tag{22}$$

For equilibrium point (ϕ_0, Ψ_0) given by equation (22)

$$\phi = \phi_0 = 0 \quad and \quad \Psi = \Psi_0 = \frac{-f - \frac{A}{\pi}\sin\vartheta\sin\alpha}{3 - B - \frac{A}{\pi}\sin\vartheta\cos\alpha}$$

To be stable, we must have to show that 'V' is positive definite

$$V = \begin{vmatrix} \left[\frac{\partial^2 v}{\partial\Psi^2}\right]^{\phi=0}_{\Psi=\Psi_0} & \left[\frac{\partial^2 v}{\partial\varphi\partial\Psi}\right]^{\phi=0}_{\Psi=\Psi_0} \\ \left[\frac{\partial^2 v}{\partial\Psi\partial\varphi}\right]^{\phi=0}_{\Psi=\Psi_0} & \left[\frac{\partial^2 v}{\partial\phi^2}\right]^{\phi=0}_{\Psi=\Psi_0} \end{vmatrix} > 0 \tag{23}$$

But we have

$$\left[\frac{\partial^2 v}{\partial\Psi^2}\right]^{\phi=0}_{\Psi=\Psi_0} = [6\cos 2\Psi_0 - 2B\cos\Psi_0 - 2f\sin\Psi_0 + \frac{2A}{\pi}\sin\theta(\cos\Psi_0\cos\alpha + \sin\Psi_0\sin\alpha)]$$

$$\left[\frac{\partial^2 v}{\partial\Psi^2}\right]^{\phi=0}_{\Psi=\Psi_0} = 2[(3\cos^2\Psi_0 + 1) - B\cos\Psi_0 - f\sin\Psi_0 - \frac{A}{\pi}\sin\vartheta(\cos\Psi_o\cos\alpha + \sin\Psi_o\sin\alpha)] \tag{24}$$

$$\text{and}\left[\frac{\partial^2 v}{\partial\phi\partial\Psi}\right]^{\phi=0}_{\Psi=\Psi_0} = \left[\frac{\partial^2 v}{\partial\Psi\partial\varphi}\right]^{\phi=0}_{\Psi=\Psi_0} = 0 \tag{25}$$

Using equation (24) and equation (25) in equation (23), we get

$$[6\cos 2\Psi_0 - 2B\cos\Psi_0 - 2f\sin\Psi_0 + \frac{2A}{\pi}\sin\theta(\cos\Psi_0\cos\alpha + \sin\Psi_0\sin\alpha)]$$

$$X\ 2[(3\cos^2\Psi_0 + 1) - B\cos\Psi_0 - f\sin\Psi_0 - \frac{A}{\pi}\sin\vartheta(\cos\Psi_0\cos\alpha + \sin\Psi_0\sin\alpha)] > 0 \qquad (26)$$

$$\Rightarrow [3\cos 2\Psi_0 - B\cos\Psi_0 - f\sin\Psi_0 + \frac{A}{\pi}\sin\vartheta(\cos\Psi_0\cos\alpha + \sin\Psi_0\sin\alpha)]$$

$$X\,[3\cos^2\Psi_0 + 1 - B\cos\Psi_0 - f\sin\Psi_0 - \frac{A}{\pi}\sin\vartheta(\cos\Psi_0\cos\alpha + \sin\Psi_0\sin\alpha)] > 0 \qquad (27)$$

Thus on putting

$$\Psi_0 = \frac{-f - \frac{A}{\pi}\sin\vartheta\sin\alpha}{3 - B - \frac{A}{\pi}\sin\vartheta\cos\alpha}$$

In (27), it can be easily seen that (27) is satisfied
Hence V is positive definite and accordingly

$$\phi = \phi_0 = 0 \quad ; \quad \Psi = \Psi_0 = \frac{-f - \frac{A}{\pi}\sin\vartheta\sin\alpha}{3 - B - \frac{A}{\pi}\sin\vartheta\cos\alpha}$$

Is a stable equilibrium point.

Conclusion

We can easily see that the equilibrium point is stable in the sense of Lyapunov.

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