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Cluster Analysis of Planets, Satellites and Dwarf Planets in the Solar System

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Abstract

Objects of the solar system revolving around the Sun fall into distinct categories such as Jovian planets, Terrestrial Planets, Dwarf planets and Satellites based upon size, mass and other physical properties. In this paper, Cluster analyses of the planets, dwarf planets and large satellites are performed with their radii and a mass parameter as variables and dendrograms of the two analyses are constructed. Broad agreements between the two dendrograms are observed, but also important differences. The dendrogram based on the mass parameter is deemed more appropriate in the classification of the solar system objects.

1. OBJECTS IN THE SOLAR SYSTEM

The solar system was formed by the gravitational contraction and coalescence of the primordial dust cloud. At the center of the solar system is the dominant object **Sun**. The next tier of objects are the planets, asteroids and comets which all revolve around the Sun. Finally, there are satellites, large and small, which revolve around planets. All these objects which ultimately revolve around the Sun are members of the solar system. They can be classified as follows.

(1) **Jovian Planets**. The four large gas planets – **Jupiter**, **Saturn**, **Uranus** and **Neptune** – are known as the Jovian planets. They are large in size but have low densities. They also possess many natural satellites.

(2) **Terrestrial Planets**. The four smaller rocky planets – **Earth**, **Venus**, **Mars** and **Mercury** – are called Terrestrial planets. They have higher densities than the Jovian planets.

(3) The **Asteroids**. The asteroids are thousands of small rocky objects revolving around the Sun between and beyond the orbits of Mars and Jupiter. They have also been referred to as minor planets or planetoids. The largest asteroid named **Ceres** was discovered through the telescope in 1801 (before the discoveries of Uranus, Neptune and Pluto) when it was referred to as the seventh planet. However, that reference was short-lived when other large asteroids **Juno**, **Pallas** and **Vesta** were subsequently discovered.

(4) **Pluto**. Since its discovery in 1930, Pluto enjoyed the status as the ninth planet for over 75 year. That status was revoked in 2006, when other Pluto-like objects belonging to the Kuiper belt (to which Pluto is now believed to belong) were discovered through charge-coupled device cameras, many of which rivalled the size of Pluto. The largest of these and asteroid belt objects (including Pluto and Ceres) are now accorded the designation of a **Dwarf Planet**.

(5) **Satellites**. Satellites are continually being discovered around the Jovian planets. There are seven satellites in the solar system (four belonging to Jupiter and one each belonging to Saturn, Neptune and the Earth) which are larger than Pluto. Two satellites (**Ganymede** belonging to Jupiter and **Titan** belonging to Saturn) are larger than Mercury, even though Mercury is heavier than the two.

In this study, the largest objects in the solar system orbiting the Sun are grouped according to **Cluster Analysis**. Twenty eight of the largest objects were considered, which included 8 planets, 2 minor planets and 18 large satellites orbiting several planets. These objects are listed in Table I in descending order of size. Two sets of cluster analyses are performed based upon sizes and masses of the objects. **Sizes** of celestial objects are generally expressed in terms of the **volume** or **radius**. For spherical objects, the volume is proportional to the cube of the radius. Hence choosing the volume or the radius does not affect the size ranks of the objects. Here we choose the radius R as the representative parameter of the size. Analogously, we define a **mass parameter** m which is the cube root of the mass M of the object: $m = \sqrt[3]{M}$. In Table I, the radii of the objects are taken from [1, 2] and the mass parameters are taken from [3]. Note that the size rank and mass rank of an object are not always the same. For example, Uranus is larger than Neptune, but Neptune is more massive than Uranus. The satellites Ganymede and Titan are larger than the planet Mercury, but Mercury is heavier. Likewise, the size and mass ranks of the following objects are also interchanged: Rhea and Oberon; Umbriel and Ariel; Tethys and Ceres; and Palls and Vesta.

Table I. Size and Mass Ranks of Objects circling the Sun					
Size Rank	Object	Nature of Object	Radius R, km	$m = \sqrt[3]{M}$, 10^6 kg$^{1/3}$	Mass Rank
1	Jupiter	Jovian Planet	71,492	1,238.3	1
2	Saturn	Jovian Planet	60,268	828.4	2
3	Uranus	Jovian Planet	25,559	442.8	3
4	Neptune	Jovian Planet	24,766	467.9	4
5	Earth	Terrestrial Planet	6,378	181.4	5
6	Venus	Terrestrial Planet	6,052	169.5	6
7	Mars	Terrestrial Planet	3,397	86.3	7
8	Ganymede	Satellite of Jupiter	2,634	52.9	9
9	Titan	Satellite of Saturn	2,575	51.2	10
10	Mercury	Terrestrial Planet	2,440	69.1	8
11	Callisto	Satellite of Jupiter	2,403	47.6	11
12	Io	Satellite of Jupiter	1,821	44.7	12
13	Moon	Satellite of Earth	1,738	41.9	13
14	Europa	Satellite of Jupiter	1,565	36.3	14
15	Triton	Satellite of Neptune	1,353	27.8	15
16	Pluto	Dwarf Planet; Ex-Planet	1,137	23.6	16
17	Titania	Satellite of Uranus	789	15.2	17
18	Rhea	Satellite of Saturn	764	13.2	19
19	Oberon	Satellite of Uranus	761	14.4	18
20	Iapetus	Satellite of Saturn	718	12.5	20
21	Charon	Satellite of Pluto	586	11.5	21
22	Umbriel	Satellite of Uranus	585	10.6	23
23	Ariel	Satellite of Uranus	581	11.1	22
24	Dione	Satellite of Saturn	560	10.3	24
25	Tethys	Satellite of Saturn	530	8.5	26
26	Ceres	Dwarf Planet; Asteroid	512	9.8	25
27	Pallas	Dwarf Planet; Asteroid	291	6.0	28
28	Vesta	Dwarf Planet; Asteroid	277	6.4	27

2. CLUSTER ANALYSIS

Cluster analysis consists of the ***classification of a group of objects*** according to certain similarities and criteria [4 − 6]. Cluster analysis has been applied to all fields of empirical science and has led to important discoveries. For example, classification of animals by Darwin resulted in his theory of evolution. Similarly, classification of elements by Mendeleyev gave rise to the periodic table discovery of missing elements. In astrophysics, the classification of stars by the Russell-Hertzsprung diagram led to the theory of stellar evolution. More recently, cluster analysis has been applied to determine the planetary status of Pluto [7].

The basic scheme of cluster analysis consists of the following steps.

Step 1. Suppose that there are *n* ***objects*** which are ***to be grouped*** according to certain ***properties***, which are generally quantitative in nature.

Step 2. The ***similarity*** two objects is determined by one of many ***resemblance coefficient*** (*r*) found in the literature [8]. The value of *r* ranges between 0 and 1 (100%). For variables having non-negative values, a particularly simple resemblance coefficient has been used [7]:

$$r = \frac{1}{m}\sum_{i=1}^{m}\frac{\min(x_i, y_i)}{\max(x_i, y_i)} \tag{1}$$

Step 3. The resemblance coefficients are calculated between every two objects and their ***average values*** determined. Amongst the averaging schemes, the ***arithmetic mean*** (*AM*) and the ***geometric mean*** (*GM*) are the most common:

$$AM = \frac{x_i + y_i}{2} \tag{2}$$

and

$$GM = \sqrt{x_i y_i} \tag{3}$$

For properties having a wide range of values, the geometric mean is preferable [7]. However, for properties having comparable values, the arithmetic mean suffices.

Step 4. The two objects having the greatest value of *r* are deemed the ***most similar***. They are then ***joined to form a single new object*** or ***cluster*** in the group, whereby themselves disappearing in the process. The number of objects in the group is then reduced by one. The new object is assigned the average values of the disappearing objects.

Step 5. Steps 2 − 4 are repeated and each time the number of objects is reduced by one. The process is continued until all the objects have been clustered into one.

Step 6. A tree diagram called a ***dendrogram*** is constructed to illustrate the entire clustering process.

3. RESULTS

In this study, two clustering schemes were undertaken with the radius *R* and mass parameter *m* in Table I as the physical parameters. The resemblance coefficients in the two schemes are denoted by ***r*** and ***R*** (not to be confused with the radius *R*), respectively. The entire clustering schemes in the two cases with their dendrograms are shown in Figures 1 and 2.

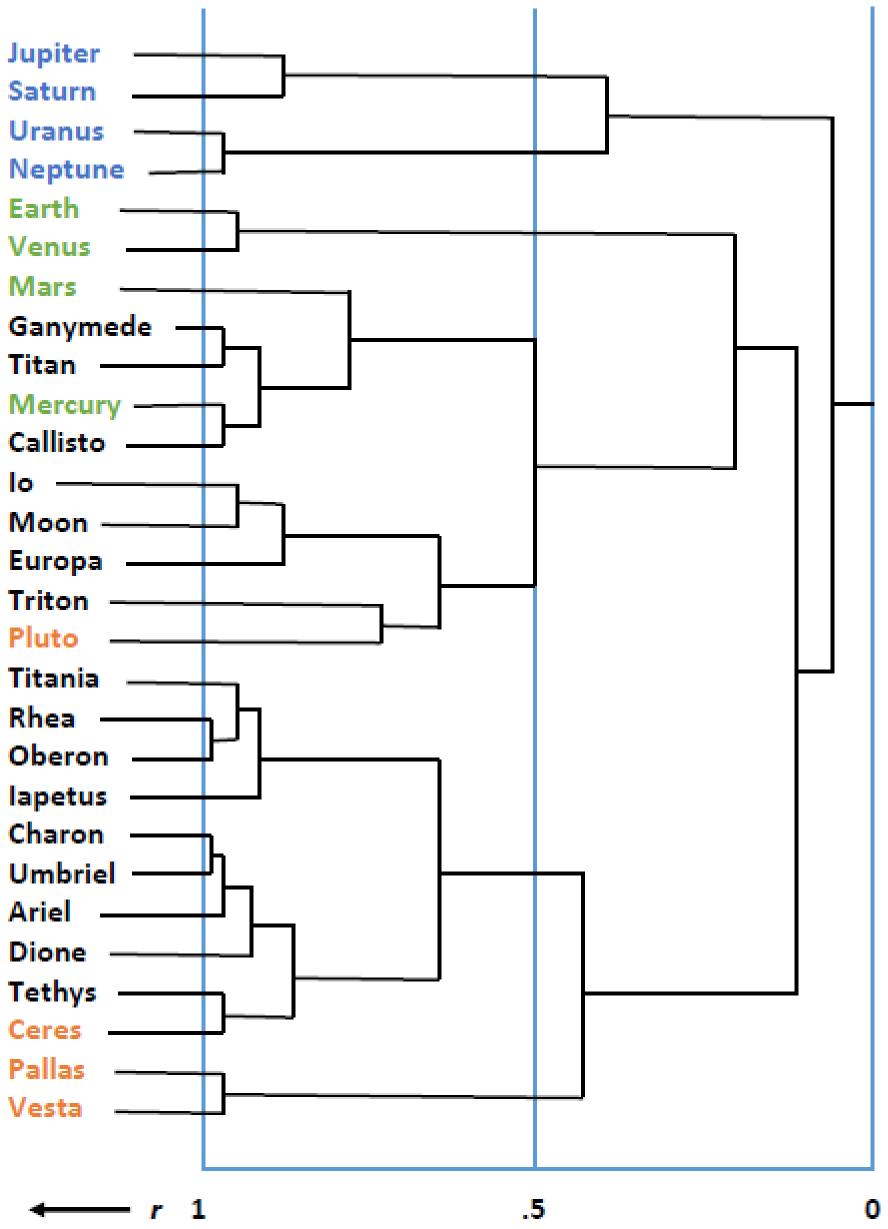

Fig. 1

Figure 1 is the size dendrogram of the solar system objects with *R* as the physical parameter. Since the objects are listed in declining order in size, an object necessarily clusters with its neighbor. In the first round of clustering, 22 objects form 11 clusters including: the Jovian planets Jupiter and Saturn; and Uranus and Neptune; the Terrestrial planets Earth and Venus; the largest satellites Ganymede and Titan; etc. The remaining 6 objects remain unclustered: Mars, Europa, Titania, Iapetus, Ariel and Umbriel. The number of objects is reduced to 17. In the second round 5 more clusters are formed and the number of objects is reduced to 12. In the third round, the 4 more objects including the remaining original objects (Mars, Iapetus and Dione) are clustered and the number of objects is reduced to 8. In the fourth round, 2 clusters are formed, reducing the number of objects to 6. In the fifth round, 2 more clusters are formed, reducing the number of objects to 4. A sole cluster is formed in the next three rounds, at the end of which all the initial 28 objects in the solar system are clustered into one.

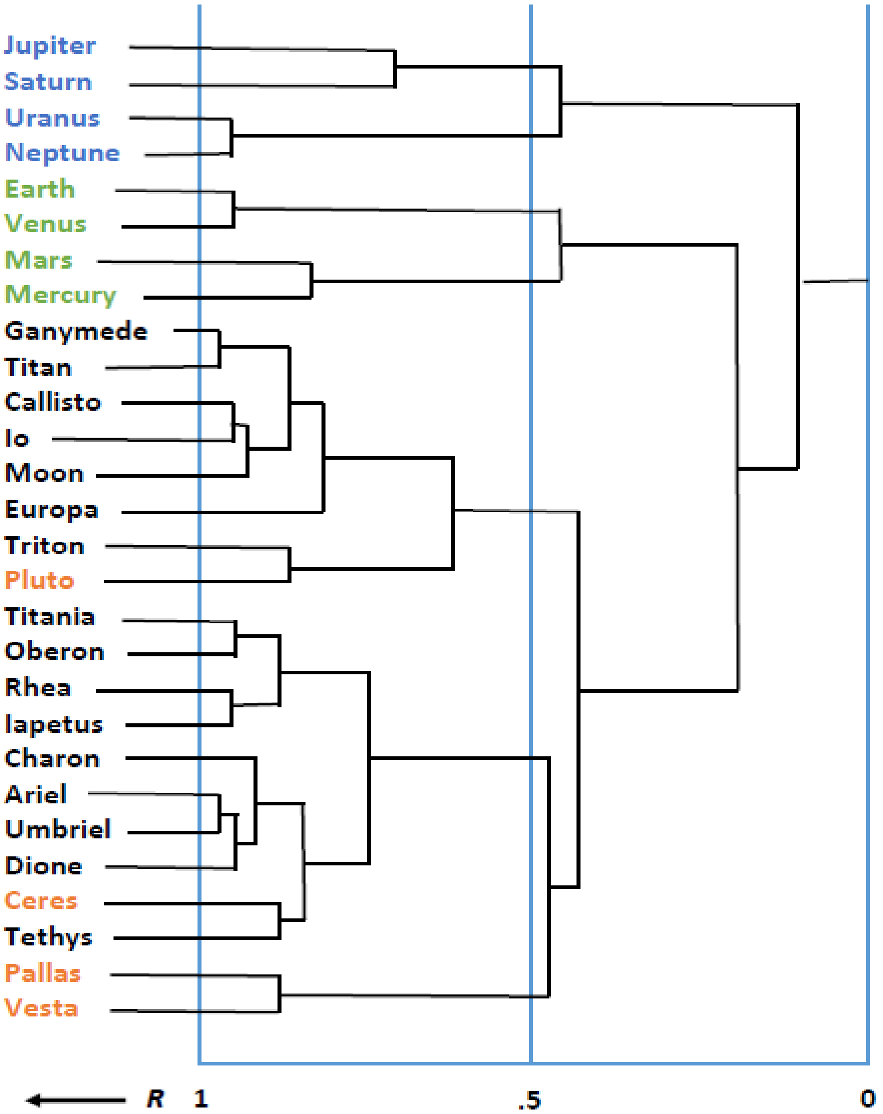

Fig. 2

Figure 2 is the mass dendrogram of the 28 solar system objects with *m* as parameter. The resulting clusters are similar to those of Fig. 1 with important differences. First, in Fig.1, Mercury, the smallest planet clustered with the largest satellites in the first two rounds, whereupon this cluster joined with Mars. Thus Mercury and Mars both clustered with satellites before they clustered with the Terrestrial planets based upon size. However, in Fig. 2, all the four Terrestrial planets clustered together based on the mass parameter. Even though Ganymede, the largest satellite is 8% larger in diameter than Mercury, Mercury is 31% more massive than Ganymede. It is more fitting for Mercury (and Mars) to cluster with the other Terrestrial planets than with satellites. Hence, mass is deemed to be the more appropriate parameter in cluster analysis of solar system objects than size.

Next, Pluto is ranked in the middle of the satellites in Table I based on both size and mass parameters. Thus, Pluto is more satellite-like in composition, even though it orbits the Sun and has its own satellite system. It clusters with Triton, the largest satellite of Neptune in both of the dendrograms. In fact the latter is widely believed to have been a Pluto-like Kuiper belt object which was captured by Neptune [9]. In other words, Triton would have been a dwarf planet today, had it not become a satellite of Neptune. We should add that additional Pluto-like objects are continually being discovered which can alter the tail-end of Table I in the future.

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