

Second Order Harshad Number

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Abstract

Harshad numbers are well known in Mathematics. Here a new definition is given for second order Harshad numbers.

Introduction

In mathematics, a Harshad number in a given number base is an integer that is divisible by the sum of its digits when written in that base [1]. Harshad numbers were defined by D. R. Kaprekar. A mathematician from India [2]. The term harshad comes from a Sanskrit word harsha, meaning joy, and da, meaning 'give'.

If one looks at the number for the current year, 2023, one can easily find that this is a Harshad number. The sum of its digits is 7, and 2023 is divisible by 7. In fact, the numbers 2022, 2023, 2024 and 2025 are four consecutive Harshad numbers. One more interesting fact to note about the number 2023 is that it is also divisible by the sum of squares of its digits. The sum of squares of its digits is 17, and 2023 is divisible by 17. One may notice that 2023 is also divisible by 17^2 . This fact gives the idea, that there are many such numbers, which are divisible by the square of the sum of squares of its digits. Let us call those numbers as second order Harshad numbers. Here we shall discuss about the numbers only in base 10.

Definition

Second order Harshad number is an integer which is divisible by the square of the sum of squares of its digits.

Discussion

All single digit numbers except 1 will not be second order Harshad number. The number 10 and 100 will be the obvious second order Harshad number. Some Harshad

numbers may also fall in the category of second order Harshad numbers, but not all Harshad numbers will satisfy the criterion for second order Harshad number. Also not all second order Harshad numbers will satisfy the criterion for Harshad number.

Let us look at some examples. One can easily verify that the numbers 2023 and 4332 are Harshad numbers, as they are divisible by 7 and 12 respectively which is the sum of their digits. At the same time they both are second order Harshad numbers, as 2023 is divisible by 17^2 and 4332 is divisible by 38^2 . Similarly, as mentioned earlier, 2022, 2024 and 2025 are also Harshad numbers, but they are not second order Harshad numbers. Just as another interesting fact, here it may be mentioned that 2025 is the sum of cubes of all digits from 1 to 9, or otherwise 2025 can be expressed as the square of the sum of all digits from 1 to 9.

On the other hand, the number 52022 and 71824 are two second order Harshad numbers, as they are divisible by 37^2 and 134^2 respectively, but they are not Harshad numbers, as they are not divisible by 11 and 22 respectively. There may be many such numbers. It may be noticed that if any Harshad number or second order Harshad number is multiplied by 10 or by multiple powers of 10, it will remain in the same category, as adding multiple 0 at the right end will not change the criteria. It is not known whether there are any consecutive second order Harshad numbers. Here is a list of second order Harshad numbers up to 5000.

10,100,1000,1100,1200,1300,2000,2023,2100,2400,3100,4332,5000

One may notice that many of these numbers are ending with zeroes. Exception is only two numbers 2023 and 4332 up to 5000. All the above listed numbers are also Harshad numbers. Up to 100000 only two numbers 52022 and 71824 are second order Harshad numbers, which are not Harshad numbers.

Conclusion

From the above discussion, it may be concluded that there are infinite numbers of second order Harshad numbers. The set of second order Harshad numbers is not a subset of the set of Harshad numbers and the intersection of the sets of Harshad numbers and second order Harshad numbers is not empty, but there are numbers which are second order Harshad numbers, but not Harshad numbers.

It may be mentioned here that similarly one can define n-th order Harshad numbers and those numbers will be rare except many obvious numbers ending with zeroes.

Reference

- [1] https://en.wikipedia.org/wiki/Harshad_number
- [2] D. R. Kaprekar, Multidigital Numbers, Scripta Mathematica, 21, 1955, 27