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An Ellipsoidal Scheme for Batsmanship in Test Cricket

Arjun Tan
Department of Physics, Alabama A & M University,
Normal, AL 35762, U. S. A.
E-mail: arjun.tan@aamu.edu; arjun.tan@mail.com

Abstract

Cricket enthusiasts often argue about the greatness of batsmen and invariably cite their batting averages in Test cricket in doing so. However, there are two other equally important statistics in batting, viz., the aggregate runs scored and the centuries made. Of these, the aggregate runs scored is the product of two quantities whose magnitude is geometrically represented by the area of the rectangle formed by the two. Its magnitude is far greater than the scoring average and is therefore incompatible with the latter for comparison. In a recent study, this quantity was compressed into a one-dimensional quantity by extracting its square root. Further, it was found that instead of centuries, a new quantity called the total centuries better correlated with the batting average than traditional count of centuries. This quantity is obtained by adding the double, triple and quadruple centuries (if any). The three quantities of the batting average, square root of aggregate runs and the total centuries were then combined into a single quantity called the batting index to serve as a measure of batsmanship in Test cricket. A vectorial-geometric scheme of composing the batting index from the three quantities were then made to rank the 10 greatest batsmen in post-war period. In this paper, a new ellipsoidal scheme of ranking the batsmen is proposed. It was found that the new batting index was more compatible in magnitude with the batting average and had smaller variances than the vectorial scheme.

Introduction

The game of ***Cricket*** is unsurpassed in terms of statistical categories recorded and easily leads all sports in terms of books and articles published on it. In the batting department, discussions and arguments frequently break out regarding selecting the greatest batsmen of say, a particular nation or the entire world; or of a particular era or of all times. For example, a recent paper in this journal selected the 93 greatest batsmen of all time with the qualification of at least 100 innings batted [1]. As is customary, in that paper, the main criterion used was the ***batting averages*** of the players in ***Test cricket***. Other important criteria such as ***aggregate runs scored*** and the ***number of centuries made*** in Test cricket are almost always overlooked because these quantities are either incompatible with the batting average and are difficult to incorporate in these discissions. An early attempt to incorporate these quantities were made by the author [2]. More recently, a ***vector-geometrical scheme*** to rank batsmen were carried out rank the 25 greatest post-war batsmen who averaged at least 50.00 in their Test careers with a minimum criterion of at least 5,000 runs made [3]. In this paper, we devise an alternative mathematical scheme to do the same.

Three Important Batting Statistics

If the ***aggregate Test runs scored by a batsman*** is ρ and the ***number of times dismissed*** (***out***) is ω, then his ***Test batting average*** is defined as:

$$\alpha - \frac{\rho}{\omega} \quad (1)$$

Thus the batting average is a ***derived quantity***. It is however, an ***intrinsic quantity***, being unique to the player's batsmanship. Equation (1) can be re-written as:

$$\rho = \alpha\omega \quad (2)$$

Equation (2) now defines the aggregate runs scored as a product of two quantities α and ω. It is thus a ***two-dimensional quantity***, much like the ***area of a rectangle*** is the product of its ***length*** and ***width***. It depends upon the length of the batsman's career. Its magnitude is far greater than the batting average and is therefore incompatible for comparison with the latter. However, by taking the square root of ρ, we can compress it into a ***one-dimensional quantity*** β whose dimension is compatible with α, which can therefore represent the ***total output*** of the batsman:

$$\beta = \sqrt{\rho} \quad (3)$$

If we assign vectorial characteristics to α and ω, then the ***cross-product*** $\vec{\rho} = \vec{\alpha} \times \vec{\omega}$ has a ***magnitude*** equal to the area of the parallelogram formed by $\vec{\alpha}$ and $\vec{\omega}$ and and a ***direction*** perpendicular to both $\vec{\alpha}$ and $\vec{\omega}$. Equation (2) is a special case when $\vec{\alpha} \perp \vec{\omega}$. A vector $\vec{\beta}$ will have the same direction as $\vec{\rho}$.

With two of the three mutually orthogonal directions of ***three-dimensional space*** already assigned, we now assign the third direction to the number of Test centuries made by the batsman. It has been pointed out that this is one of several quirky statistics of Cricket [3, 4]. ***Centuries*** (100s) are defined as all individual innings of 100 or more runs (cf. [4]). Thus, they include ***double centuries*** (200s), ***triple centuries*** (300s), ***quadruple centuries*** (400s), and so on [5]. Likewise, double centuries include triple and quadruple centuries; triple centuries include the quadruple and ***quintuple centuries*** (if any), and so on [5]. Technically, however, double centuries consist of wo centuries, triple centuries comprise three centuries, and so on. Hence counting multiple centuries as single centuries seriously deprives the batsman of his dues. In order to correct this shortcoming and account for all centuries, it was proposed to define a new category called ***total centuries*** (Σ100s) as follows:

$$\gamma = \Sigma 100s = 100s + 200s + 300s + \cdots \tag{4}$$

Along with α and β, γ now completes the triad of statistics by which batsmanship of a player is measured. In three-dimensional space, $\vec{\alpha}$, $\vec{\beta}$ and $\vec{\gamma}$, the three ***orthogonal vectors*** form a ***rectangular parallelopiped***, the magnitude of whose ***resultant*** is the ***space diagonal*** of the parallelopiped *R1*. The magnitude of this resultant is given by ***three-dimensional Pythagoras' Theorem***:

$$R1 = \sqrt{\alpha^2 + \beta^2 + \gamma^2} = \sqrt{\alpha^2 + \rho + \gamma^2} \tag{5}$$

It is $R1$ which serves as the measure of batsmanship in Test cricket in the vector-geometrical scheme. The calculated values of $R1$ from α, β and γ of the top 25 batsmen based on Test batting averages are reproduced in Table I (from [1] and [3]) for later comparison. The rankings of the top 10 batsmen due to the incorporation of β and γ: (1) Tendulkar; (2) Kallis; (3) Ponting; (4) Dravid; (5) Sangakkara; (6) Lara; (7) Chanderpaul; (8) Steve Waugh; (9) Border; and (10) Younis Khan. Only four of the original top 10 based solely on batting averages survived in this ranking of whom Tendulkar leaped from 9th to the top place; Kallis went from the 7th to the second place; Sangakkara remained in the fourth place; and Lara went from the 10th to the sixth place.

Ellipsoidal Scheme for Batsmanship in Test Cricket

In taking the vector sum of three quantities, the magnitude of the resultant is greatly enhanced, which is no longer comparable with its component magnitudes. In our new ***ellipsoidal scheme***, a mean of the components is composed, which will be compatible with the component magnitudes and free from the enhancements. It must be recalled that many types of means have been defined, among which three are the most common: ***arithmetic mean*** (*AM*), ***geometric mean*** (*GM*) and ***harmonic mean*** (*HM*) (cf. [6]). For the three positive numbers α, β and γ, we have:

$$AM = \frac{\alpha + \beta + \gamma}{3} \tag{6}$$

$$GM = \sqrt[3]{\alpha\beta\gamma} \tag{7}$$

and

$$HM = \frac{3}{\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}} \tag{8}$$

For three numbers, not all equal, the following ***inequality relation*** holds:

$$AM > GM > HM \tag{9}$$

In general, the *GM* is the preferable option, with *AM* slightly over-estimating and *HM* slightly under-estimating the value of the mean. For this reason, the *GM* is chosen as our new measure of batsmanship *R*2 in Test cricket:

$$R2 = \sqrt[3]{\alpha\beta\gamma} \tag{10}$$

Table I. New Measure of Batsmanship of the Greatest Batsmen in Test Cricket

Rank	Batsmen	Team	Ave. α	Runs ρ	Σ100s γ	Meas. $R_{\alpha\beta\gamma}$	New Rank
1	Barrington	E	58.67	6,806	21	103.39	21
2	Hammond	E	58.45	7,249	30	107.54	17
3	Sobers	WI	57.78	8,032	29	110.51	14
4	Sangakkara	SL	57.40	12,400	50	134.89	4
5	Hobbs	E	56.94	5,410	16	94.38	23
6	Hutton	E	56.67	6,971	24	103.72	20
7	Kallis	SA	55.37	13,289	47	136.25	2
8	G Chappell	A	53.86	7,110	28	104.42	19
9	Tendulkar	I	53.78	15,921	56	148.15	1
10	Lara	WI	52.88	11,953	46	129.87	6
11	Miandad	P	52.57	8,832	29	111.52	12
12	Dravid	I	52.31	13,288	41	133.06	5
13	Md Yousuf	P	52.29	7,530	28	105.11	18
14	Younis Khan	P	52.05	10,099	41	120.37	10
15	Ponting	A	51.85	13,378	47	135.19	3
16	Flower	Z	51.54	4,794	13	87.29	25
17	Hussey	A	51.52	6,235	19	96.18	22
18	Chanderpaul	WI	51.37	11,867	32	124.62	7
19	Gavaskar	I	51.12	10,122	38	119.08	11
20	S Waugh	A	51.06	10,927	33	120.93	9
21	Hayden	A	50.73	8,625	33	110.85	13
22	De Villars	SA	50.66	8,765	24	109.12	15
23	Border	A	50.56	11,174	29	120.71	8
24	Richards	WI	50.23	8,540	26	108.35	16
25	Compton	E	50.06	5,807	19	93.13	24

A = Australia; E = England; I = India; P = Pakistan; SA = South Africa; SL = Sri Lanka; WI = West Indies; Z = Zimbabwe

Table II. Ellipsoidal Measure of Batsmanship in Test Cricket

Rank	Batsmen	Team	Ave. α	Runs ρ	Σ100s γ	Meas. $R2$	New Rank
1	Barrington	E	58.67	6,806	21	46.67	21
2	Hammond	E	58.45	7,249	30	53.05	14
3	Sobers	WI	57.78	8,032	29	53.15	13
4	Sangakkara	SL	57.40	12,400	50	68.37	2
5	Hobbs	E	56.94	5,410	16	40.62	24
6	Hutton	E	56.67	6,971	24	48.43	20
7	Kallis	SA	55.37	13,289	47	66.94	3
8	G Chappell	A	53.86	7,110	28	50.60	16
9	Tendulkar	I	53.78	15,921	56	72.43	1
10	Lara	WI	52.88	11,953	46	64.31	5
11	Miandad	P	52.57	8,832	29	52.33	15
12	Dravid	I	52.31	13,288	41	62.76	6
13	Md Yousuf	P	52.29	7,530	28	50.27	17
14	Younis Khan	P	52.05	10,099	41	59.86	7
15	Ponting	A	51.85	13,378	47	65.57	4
16	Flower	Z	51.54	4,794	13	35.93	25
17	Hussey	A	51.52	6,235	19	42.60	22
18	Chanderpaul	WI	51.37	11,867	32	56.37	9
19	Gavaskar	I	51.12	10,122	38	58.03	8
20	S Waugh	A	51.06	10,927	33	56.06	10
21	Hayden	A	50.73	8,625	33	53.77	11
22	De Villars	SA	50.66	8,765	24	48.46	19
23	Border	A	50.56	11,174	29	53.72	12
24	Richards	WI	50.23	8,540	26	49.42	18
25	Compton	E	50.06	5,807	19	41.69	23

A = Australia; E = England; I = India; P = Pakistan; SA = South Africa; SL = Sri Lanka; WI = West Indies; Z = Zimbabwe

Geometrical Significance of Batting Index

If α, β and γ are identified with the ***semi-principal-axes*** of an ellipsoid (Fig. 1), the volume of that ellipsoid V is:

$$V = \frac{4}{3}\pi\alpha\beta\gamma \tag{11}$$

Now the volume of a sphere of radius r having the same volume of the ellipsoid is:

$$V = \frac{4}{3}\pi r^3 \tag{12}$$

From Eqs. (11) and (12), one gets:

$$r = \sqrt[3]{\alpha\beta\gamma} \ , \tag{13}$$

which, by Eq. (10) is the new batting index $R2$. In other words, the ellipsoidal batting index is numerically equal to the radius of a sphere having a volume equal to that of the ellipsoid having semi-principal-axes of α, β and γ.

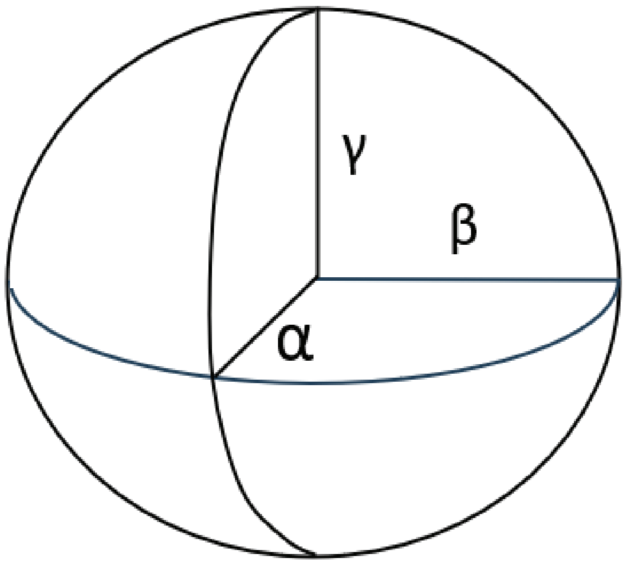

Figure 1

Results and Discussion

The $R2$ indices of the 25 batting greats are calculated and entered in Table II. Based on their values, the new rankings of the top 10 batsmen are as floows: (1) Tendulkar; (2) Sangakkara; (3) Kallis; (4) Ponting; (5) Lara; (6) Dravid; (7) Younis Khan; (8) Gavaskar; (9) Chanderpaul; and (10) Steve Waugh. There is only one change of batsman from the previous list based on $R1$: Border drops out and is replaced by Gavaskar. But there are several position changes. Sangakkara moves to the second position; Kallis drops to third; Ponting drops to fourth; Dravid drops to sixth; Lara moves up to fifth; Chanderpaul drops two positions to ninth; Steve Waugh drops to tenth; and Younis Khan moves up to seventh. However, Tendulkar easily retains the top position as the greatest post-war batsman.

Figure 2 is a scatterplot of the batting indices $R1$ and $R2$ against the batting average α. The blue dots represent $R1$ and the red dots represent $R2$. The ***trendlines*** are shown together with their ***fitted equations***. The ***mean values*** of α, $R1$ and $R2$ were 53.309, 114.475 and 54.056, respectively, which indicates that $R2$ was far more compatible with α than $R1$. The ***standard deviations*** of $R1$ and $R2$ data points were 15.527 and 9.229 respectively, signifying far greater spreads in the first case. Lastly, the ***correlation coefficient*** between α and $R1$ was $-.0125$, whereas that between α and $R2$ was .073. This, once again suggests better compatibility between the variables in the second case.

Interrestingly, the correlation coefficient between $R1$ and $R2$ was .997, which is not surprising, since both were derived from the same α values. In conclusion, the ellipsoidal scheme better represents the overall batting greatness than the vectorial scheme.

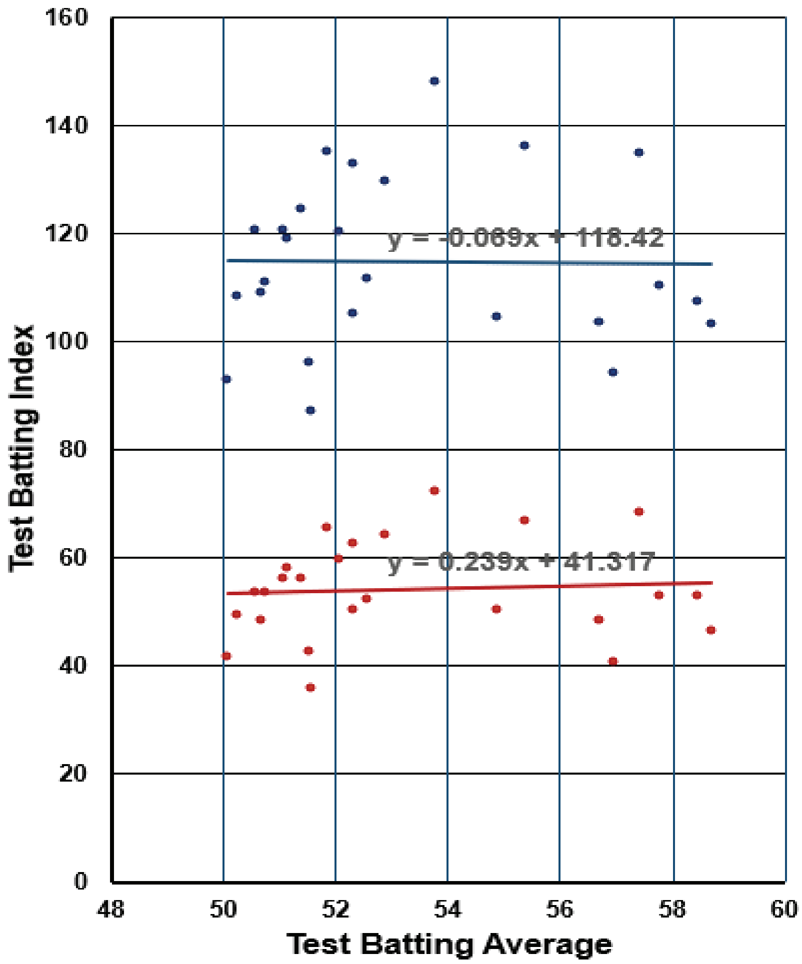

Figure 2

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