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Constrained Dynamics of Interacting Non-Abelian Anti-symmetric Tensor field Theories

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Abstract

We study the constraint structure of antisymmetric tensor fields interacting with non-Abelian vector fields using Dirac's procedure, and it is shown that both first class and second class constraints exist in the theory. The equations of motions and Dirac brackets of various fields are constructed using the second class constraints.

Keywords: Antisymmetric tensor field, Dirac Brackets.

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0.1 Introduction

Antisymmetric tensor fields [1] have come up in various areas of Physics, as mediating fields in String theories, in discussions on Black Hole theory. Also, these fields have been considered to play an important role in alternate mechanisms for generating masses for gauge fields. Antisymmetric tensor fields are also the subject of duality mechanisms, which explore their equivalences with other theories.

In this paper, we make a constraint analysis [2] of these interacting theories. We begin with a brief discussion of the Abelian theory of antisymmetric tensor field [3] interacting with a vector field. We next consider the non-Abelian theory [4], at its constraint structure. The presence of second class constraints leads to defining and construction of the Dirac brackets for the various field quantities.

1 Abelian Interaction Theory

The Abelian antisymmetric tensor field [3] $\mathsf{B}^{\mu\nu}$ interacting with a vector field A^{μ} is described by the Lagrangian density,

$$\mathcal{L} = -\frac{1}{4}\mathsf{F}_{\mu\nu}\mathsf{F}^{\mu\nu} + \frac{1}{12}\mathsf{H}_{\mu\nu\lambda}\mathsf{H}^{\mu\nu\lambda} + \frac{m}{4}\epsilon_{\mu\nu\lambda\sigma}\mathsf{F}^{\mu\nu}\mathsf{B}^{\lambda\sigma} \tag{1}$$

with $\mathsf{F}^{\mu\nu} = \partial^{\mu}\mathsf{A}^{\nu} - \partial^{\nu}\mathsf{A}^{\mu}$ and $\mathsf{H}^{\mu\nu\lambda} = \partial^{\mu}\mathsf{B}^{\nu\lambda} + \partial^{\nu}\mathsf{B}^{\lambda\mu} + \partial^{\lambda}\mathsf{B}^{\mu\nu}$. This Lagrangian is invariant under the separate gauge tranformations

$$\begin{aligned}\mathsf{A}^{\mu} &\longrightarrow \mathsf{A}'^{\mu} = \mathsf{A}^{\mu} + \partial_{\mu}\eta \qquad \texttt{and} \\ \mathsf{B}^{\mu\nu} &\longrightarrow \mathsf{B}'^{\mu\nu} = \mathsf{B}^{\mu\nu} + (\partial^{\mu}\xi^{\nu} - \partial^{\nu}\xi^{\mu})\end{aligned}$$

with η and ξ^{μ} as parameters of these transformations.

1.1 Constraint structure of Abelian Theory

Using the Lagrangian above, the canonical momentum fields are π_μ and $\pi_{\mu\nu}$ for the A^μ and $\mathsf{B}^{\mu\nu}$ respectively. Then the canonical Hamiltonian is

$$\begin{aligned}\mathcal{H}_c &= \frac{1}{2}\pi_1\pi_i + \frac{1}{4}\pi_{ij}\pi_{ij} + \frac{1}{12}\mathsf{H}_{ijk}\mathsf{H}_{ijk} + \frac{m^2}{4}\mathsf{B}_{ij}\mathsf{B}_{ij} + \frac{1}{4}\mathsf{F}_{ij}\mathsf{F}_{ij} - \frac{m}{2}\epsilon_{ijk}\pi_i\mathsf{B}_{jk} \\ &\quad + (\partial_i\mathsf{A}_0)\pi_i - \frac{1}{2}(\partial_j\mathsf{B}_{0i} - \partial_i\mathsf{B}_{0j})\pi_{ij} - \frac{m}{2}\epsilon_{ijk}\mathsf{B}_{oi}\mathsf{F}_{jk} \qquad (2)\end{aligned}$$

The constraints of this system are

$$\begin{aligned}\pi_0 &\approx 0 & \pi_{0i} &\approx 0 \\ -\partial_i\pi_i &\approx 0 & \partial_j\pi_{ij} - \frac{m}{2}\epsilon_{ijk}\mathsf{F}_{jk} &\approx 0. \qquad (3)\end{aligned}$$

The above constraints are all of the first class; these are the generators of the gauge transformations given above. The theory can be handled further by standard methods such as gauge fixing, followed by canonical quantisation of path integral methods.

It has also been shown that this Abelian interaction theory is equivalent to the Proca model [5].

2 *The non-Abelian Interaction Theory*

The non - Abelian Antisymmetric tensor field [4] $(\mathsf{B}^a)^{\mu\nu}$ interacting with vector field $(\mathsf{A}^a)^\mu$ is described by,

$$\mathcal{L} = -\frac{1}{4}\mathsf{F}^a_{\mu\nu}\mathsf{F}^{a\mu\nu} + \frac{1}{12}\mathsf{H}^a_{\mu\nu\lambda}\mathsf{H}^{a\mu\nu\lambda} + \frac{m}{4}\epsilon_{\mu\nu\lambda\sigma}\mathsf{F}^{a\mu\nu}\mathsf{B}^{a\lambda\sigma}, \qquad (4)$$

where using the covariant derivative $(\mathcal{D}_\mu)^{ab} = \partial_\mu\delta^{ab} + gf^{acb}\mathsf{A}^c_\mu$, we have

$$\begin{aligned}(\mathsf{F}^a)^{\mu\nu} &= (\mathcal{D}^\mu\mathsf{A}^\nu)^a - (\mathcal{D}^\nu\mathsf{A}^\mu)^a \\ (\mathsf{H}^a)^{\mu\nu\lambda} &= (\mathcal{D}^\mu\mathsf{B}^{\nu\lambda})^a + (\mathcal{D}^\nu\mathsf{B}^{\lambda\mu})^a + (\mathcal{D}^\lambda\mathsf{B}^{\mu\nu})^a.\end{aligned}$$

Here a, b, c are group indices, and $\mu, \nu, \lambda = 0, 1, 2, 3$ are Lorentz indices.

The above Lagrangian is invariant under the gauge tranformations, $\mathsf{A}^a_\mu \longrightarrow \mathsf{A}'^a_\mu = \mathsf{A}^a_\mu + (\mathcal{D}_\mu\omega)^a$ and $B^a_{\mu\nu} \longrightarrow B'^a_{\mu\nu} = B^a_{\mu\nu} + gf^{abc}B^b_{\mu\nu}\omega^c$, with ω^a the transformation parameters.

However, the Lagrangian is not invariant under

$$\mathsf{B}^a_{\mu\nu} \longrightarrow \mathsf{B}'^a_{\mu\nu} = \mathsf{B}^a_{\mu\nu} + (\mathcal{D}_\mu\lambda_\nu - \mathcal{D}_\nu\lambda_\mu)^a.$$

This is unlike the Abelian case. This non-invariance is due to the non-Abelian nature of the fields.

The equations of motion for the fields are,

$$\begin{aligned} (\mathcal{D}^\nu \mathsf{F}_{\mu\nu})^a &= \frac{m}{6}\epsilon_{\mu\nu\rho\sigma}\mathsf{H}^{a\nu\tau\sigma} + \frac{1}{2}gf^{abc}\mathsf{H}^b_{\mu\nu\sigma}\mathsf{B}^{c\nu\sigma} \\ (\mathcal{D}^\sigma \mathsf{H}_{\mu\nu\sigma})^a &= \frac{m}{2}\epsilon_{\mu\nu\rho\sigma}\mathsf{F}^{a\rho\sigma} \end{aligned} \tag{5}$$

2.1 *Hamiltonian and Constraint Structure*

In phase space, the canonical momentum fields π^a_μ and $\pi^a_{\mu\nu}$ conjugate to the $\mathsf{A}^{a\mu}$ and $\mathsf{B}^{a\mu\nu}$ respectively, are obtained from the Lagrangian in the standard way. Their fundamental Poisson brackets are,

$$\left\{\mathsf{A}^{a\mu}(\vec{x}), \pi^b_\nu(\vec{y})\right\} = \delta^{ab}\,\delta^\mu{}_\nu\delta(\vec{x}-\vec{y}) \tag{6}$$

$$\left\{\mathsf{B}^{a\mu\nu}(\vec{x}),\ \pi^b{}_{\rho\sigma}(\vec{y})\right\} = \delta^{ab}\left(\delta^\mu{}_\rho\delta^\nu{}_\sigma - \delta^\mu{}_\sigma\delta^\nu{}_\rho\right)\delta(\vec{x}-\vec{y}),$$

with all other Poisson brackets being zero. In the second line, the right hand side is due to the antisymmetric nature of the tensor fields $\mathsf{B}^{a\mu\nu},\ \pi^b{}_{\rho\sigma}$.

The primary field constraints obtained are,

$$\pi^a{}_0(\vec{x}) \approx 0, \qquad \pi^a{}_{0i}(\vec{x}) \approx 0 \qquad (i = 1, 2, 3). \tag{7}$$

The canonical Hamiltonian, directly from the Lagrangian is,

$$\mathsf{H}_c = \int d^3x\,\mathcal{H}_c = \int d^3x\left(\dot{\mathsf{A}}^{a\mu}\pi^a{}_\mu + \frac{1}{2}\dot{\mathsf{B}}^{a\mu\nu}\pi^a_{\mu\nu} - \mathcal{L}\right)$$

$$= \int d^3x \left[\frac{1}{4}\mathsf{F}^{aij}\mathsf{F}^a{}_{ij} + \frac{1}{4}\pi^{aij}\pi^a{}_{ij} - \frac{1}{12}\mathsf{H}^{aijk}\mathsf{H}^a{}_{ijk} - \mathsf{A}^a{}_0\left(\left(\mathcal{D}^i\pi_i\right)^a - \frac{gf^{abc}}{2}\pi^b_{ij}\mathsf{B}^{cij}\right)\right.$$

$$\left. + \mathsf{B}^a{}_{0i}\left(\left(\mathcal{D}_j\pi^{ij}\right)^a - \frac{m}{2}\epsilon^{ijk}\mathsf{F}^a_{jk}\right) - \frac{1}{2}\left(\pi^{ai} - \frac{m}{2}\epsilon^{ijk}\mathsf{B}^a_{jk}\right)\left(\pi^a{}_i - \frac{m}{2}\epsilon_{ijk}\mathsf{B}^{ajk}\right)\right] \quad (8)$$

The secondary and tertiary constraints are obtained by requiring time independence of already existing constraints with respect to the canonical Hamiltonian; we get,

$$\begin{aligned}
\Lambda^a(\vec{x}) &= \left(\mathcal{D}^i\pi_i\right)^a - gf^{abc}\pi^b{}_{ij}B^{cij} \quad \approx\ 0 \\
(\Lambda^a)^i(\vec{x}) &= -\left(\mathcal{D}_j\pi^{ij}\right)^a - \frac{m}{2}\epsilon^{ijk}\mathsf{F}^a_{jk} \quad \approx\ 0 \qquad (i = 1,2,3) \\
(\psi^a)_i(\vec{x}) &= \frac{1}{2}gf^{abc}(\mathsf{F}^b)^{jk}(\mathsf{H}^c)_{ijk} - gf^{abc}(\pi^b)_{ij}\left[(\pi^c)^j - \frac{m}{2}\epsilon^{jkl}(\mathsf{B}^c)_{kl}\right](\vec{x}) \ \approx\ 0.
\end{aligned} \quad (9)$$

There are no more constraints. The full set of constraints are,

$$\begin{aligned}
\pi^a{}_0(\vec{x}) &\approx 0 \\
\pi^a{}_{0i}(\vec{x}) &\approx 0 \\
\Lambda^a(\vec{x}) &= \left(\mathcal{D}^i\pi_i\right)^a - gf^{abc}\pi^b{}_{ij}B^{cij} \quad \approx\ 0 \\
(\Lambda^a)^i(\vec{x}) &= -\left(\mathcal{D}_j\pi^{ij}\right)^a - \frac{m}{2}\epsilon^{ijk}\mathsf{F}^a_{jk} \quad \approx\ 0 \\
(\psi^a)_i(\vec{x}) &= \frac{1}{2}gf^{abc}(\mathsf{F}^b)^{jk}(\mathsf{H}^c)_{ijk} - gf^{abc}(\pi^b)_{ij}\left[(\pi^c)^j - \frac{m}{2}\epsilon^{jkl}(\mathsf{B}^c)_{kl}\right](\vec{x}) \ \approx\ 0.
\end{aligned} \quad (10)$$

This set of constraints given earlier define a constraint surface Γ in the phase space.

Following Dirac's procedure, we can classify all these constraints as below:

The Poisson brackets of the constraints $\pi^a{}_0 \approx 0$, $\pi^a{}_{0i} \approx 0$ with all the constraints vanish. Among the remaining constraints we have,

$$\begin{aligned}
\{\Lambda^a(\vec{x}), \Lambda^b(\vec{y})\} &= -gf^{abc}\Lambda^c\delta(\vec{x}-\vec{y}), \qquad \{\Lambda^a(\vec{x}), \psi^b_i(\vec{y})\} = gf^{abc}\psi^c{}_i\delta(\vec{x}-\vec{y}) \\
\{\Lambda^a(\vec{x}), \Lambda^{bi}(\vec{y})\} &= gf^{abc}\Lambda^{ci}\delta(\vec{x}-\vec{y}) \\
\{\Lambda^{ai}(\vec{x}), \Lambda^{bj}(\vec{y})\} &= 0,
\end{aligned}$$

$$\{\Lambda^{ai}(\vec{x}),\ \psi^{b}{}_{j}(\vec{y})\} \ = \ gf^{acd}gf^{bce}\left[(\pi^{d})^{ik}(\pi^{e})_{jk} + \frac{\delta^{i}{}_{j}}{2}(\mathsf{F}^{d})_{kl}(\mathsf{F}^{e})^{kl} - (\mathsf{F}^{d})_{jk}(\mathsf{F}^{e})^{ik}\right]\delta(\vec{x}-\vec{y})$$

We see that π^a_0, π^a_{0i} and Λ^a are of the first class and Λ^{ai} and ψ^a_i are of the second class. Further, the constraints ψ^a_i have non-zero Poisson bracket among themselves,

$$\begin{aligned}
\left\{\psi^{a}{}_{i}(\vec{x}),\psi^{b}{}_{j}(\vec{y})\right\} \ &= \ g^2 f^{acd}f^{bed}m\epsilon_{kmn}(\pi^{f})_{l}{}^{m}(\pi'^{\,g})^{n}\left(\delta_{i}{}^{k}\delta_{j}{}^{l}\ \delta^{ef}\delta^{cg} - \delta_{j}{}^{k}\delta_{i}{}^{l}\ \delta^{eg}\delta^{cf}\right)\delta(\vec{x}-\vec{y}) \\
&+g^2 f^{acd}f^{bef}\left[\frac{gf^{dfg}}{2}\ (\mathsf{B}^{g})^{mn}\epsilon^{ecl}\epsilon_{hkl}(\pi^{h})_{ij}(\mathsf{F}^{k})_{mn}\right. \\
&+\ gf^{dfg}\left[\delta_{i}{}^{k}\delta_{j}{}^{l}\ \delta^{cc'}\delta^{ee'} + \delta_{j}{}^{k}\delta_{i}{}^{l}\ \delta^{ce'}\delta^{ec'}\right](\pi^{c'})_{km}(\mathsf{F}^{e'})^{mn}(\mathsf{B}^{g})_{ln} \\
&+\ \left[(\pi^{c})_{im}(\vec{x})(\mathsf{H}^{f})_{j}{}^{mn}(\vec{y})(\mathcal{D}^{de}_{n\vec{x}}) - (\pi^{e})_{jm}(\vec{y})(\mathsf{H}^{d})_{i}{}^{mn}(\vec{x})(\mathcal{D}^{fc}_{n\vec{y}})\right] \\
&+\ g_{ij}\left[(\mathsf{F}^{e})^{mn}(\vec{y})(\pi')^{d}{}_{m}(\vec{x})(\mathcal{D}^{cf}_{n\vec{y}}) - (\mathsf{F}^{c})^{mn}(\vec{x})(\pi')^{f}{}_{m}(\vec{y})(\mathcal{D}^{de}_{n\vec{x}})\right] \\
&\left.+\epsilon^{rkl}\left[\mathsf{F}^{em}_{\ i}(\vec{y})\epsilon_{mjr}(\pi'^{\,d})_{k}(\vec{x})(\mathcal{D}^{cf}_{l\vec{y}}) - \mathsf{F}^{cm}_{\ j}(\vec{x})\epsilon_{mir}(\pi'^{\,f})_{k}(\vec{y})(\mathcal{D}^{de}_{l\vec{x}})\right]\right]\delta(\vec{x}-\vec{y})
\end{aligned}$$

where $(\pi'^{g})^{n} = \left[\pi^{gn} - \frac{m}{2}\epsilon^{npq}B^{g}{}_{pq}\right]$

In Dirac's procedure, the total Hamiltonian includes a combination of all constraints,

$$\mathsf{H_T} = \mathsf{H}_c \ + \ \int d^3x\ \left(\mu^{a}{}_{1}\pi^{a}{}_{0} + \mu^{ai}_{\ 2}\pi^{a}{}_{0i} + \mu^{a}{}_{3}\Lambda^{a} + \mu^{a}{}_{4i}\Lambda^{ai} + \mu^{ai}_{\ 5}\psi^{a}{}_{i}\right), \tag{11}$$

with the μ^a_1, μ^{ai}_2, μ^a_3, $\mu^a{}_{4i}$, μ^{ai}_5 being the Lagrange multipliers. The multipliers $\mu^a{}_{4i}$, μ^{ai}_5 corresponding to the second class constraints can be determined (on the surface Γ)

$$\mu^{a}{}_{4i}(\vec{x}) \ \approx \ \int d^3y(\mathcal{E}^{-1})^{ab}{}_{i}{}^{j}(\vec{x},\vec{y})\{\psi^{b}{}_{j}(\vec{y}),\mathsf{H}_c\} \qquad \texttt{and} \qquad \mu^{ai}_{\ 5}(\vec{x}) \ \approx \ 0$$

The $\mu^a{}_1, \mu^{ai}_2, \mu^{ai}_3$ corresponding to the first class constraints, cannot be undetermined, indicating the presence of gauge invariance in the theory. The Hamiltonian is,

$$\begin{aligned}
\mathsf{H_T} \ &= \ \mathsf{H}_c \ + \ \int d^3x\left(\mu^{a}{}_{1}\pi^{a}{}_{0} + \mu^{ai}_{\ 2}\pi^{a}{}_{0i} + \mu^{a}{}_{3}\Lambda^{a}\right) \\
&\quad + \ \int d^3(x,y)(\mathcal{E}^{-1})^{ab}{}_{i}{}^{j}(\vec{x},\vec{y})\left\{\psi^{b}{}_{j}(\vec{y}),\mathsf{H}_c\right\}\Lambda^{ai}(\vec{x})
\end{aligned} \tag{12}$$

2.2 Dirac Brackets

The presence of second class contraints in this theory indicates Dirac brackets can be defined. To do so, we first rearrange all the second class constraints in the following way,

$$\xi^a_{\ m}(\vec{x}) \equiv \left(\Lambda^{ai},\ \psi^a_{\ i}\right)(\vec{x}), \qquad i = 1,2,3. \tag{13}$$

Then we construct the Poisson brackets,

$$(\mathcal{A}^{ab})_{mn}(\vec{x},\vec{y}) = \left\{\xi^a_{\ m}(\vec{x}),\ \xi^b_{\ n}(\vec{y})\right\}. \tag{14}$$

from which we have a block-diagonal matrix $\mathcal{A}$,

$$\mathcal{A} = \begin{bmatrix} 0 & \mathcal{E} \\ -\mathcal{E} & \mathcal{B} \end{bmatrix}$$

with the sub-matrices $\mathcal{E}$ and $\mathcal{B}$ given by

$$\mathcal{E}^{abi}_{\ \ \ j}(\vec{x},\vec{y}) = \left\{\Lambda^{ai}(\vec{x}), \psi^b_{\ j}(\vec{y})\right\} \qquad and \qquad \mathcal{B}^{abi}_{ij}(\vec{x},\vec{y}) = \left\{\psi^a_{\ i}(\vec{x}), \psi^b_{\ j}(\vec{y})\right\}$$

The inverse of this $\mathcal{A}$ matrix is,

$$\mathcal{A}^{-1} = \begin{bmatrix} \mathcal{E}^{-1}\mathcal{B}\mathcal{E}^{-1} & -\mathcal{E}^{-1} \\ \mathcal{E}^{-1} & 0 \end{bmatrix} \tag{15}$$

The Dirac bracket for any two variables C and D is,

$$\begin{aligned} \{\mathsf{C}(\vec{x}), \mathsf{D}(\vec{y})\}_{\mathsf{DB}} &= \{\mathsf{C}(\vec{x}), \mathsf{D}(\vec{y})\} - \int d^3z d^3z' \ \{\mathsf{C}(\vec{x}), \xi^a_{\ m}(\vec{z})\} (\mathcal{A}^{-1})^{ab\ mn}(\vec{z},\vec{z}\,') \left\{\xi^b_{\ n}(\vec{z}\,'), \mathsf{D}(\vec{y})\right\} \\ &= \{\mathsf{C}(\vec{x}), \mathsf{D}(\vec{y})\} - \int d^3z d^3z' \left[\left\{\mathsf{C}(\vec{x}), \Lambda^{ai}(\vec{z})\right\} (\mathcal{E}^{-1}\mathcal{B}\mathcal{E}^{-1})^{ab}_{\ \ ij} \left\{\Lambda^{bj}(\vec{z}\,'), \mathsf{D}(\vec{y})\right\}\right. \\ &\quad - \left\{\mathsf{C}(\vec{x}), \Lambda^{ai}(\vec{z})\right\} (\mathcal{E}^{-1})^{ab}_{\ \ ij} \left\{(\psi^b)_j(\vec{z}\,'), \mathsf{D}(\vec{y})\right\} \\ &\quad \left. + \{\mathsf{C}(\vec{x}), \psi^a_{\ i}(\vec{z})\} (\mathcal{E}^{-1})^{abi}_{\ \ \ j} \left\{\Lambda^{bj}(\vec{z}\,'), \mathsf{D}(\vec{y})\right\}\right] \end{aligned}$$

The non-zero Dirac Brackets are

$$\left\{\mathsf{A}^{a0}(\vec{x}), \pi^b_{\ 0}(\vec{y})\right\}_{\mathsf{DB}} = \delta^{ab}\delta(\vec{x}-\vec{y}),$$

$$
\begin{aligned}
\left\{\mathsf{A}^{ai}(\vec{x}),\ \pi^{b}{}_{j}(\vec{y})\right\}_{\mathsf{DB}} &= \delta^{ab}\delta^{i}{}_{j}\delta(\vec{x}-\vec{y}) + gf^{ace}(\pi^{e})_{k}{}^{i}(\vec{x})\left[gf^{bdf}(\pi^{f})_{jl} - m\epsilon_{jlm}(\mathcal{D}^{bd}_{\vec{y}})^{m}\right](\mathcal{A}^{-1})^{cd\,kl}(\vec{x},\vec{y}) \\
\left\{\mathsf{A}^{ai}(\vec{x}),(\mathsf{B}^{b})^{jk}(\vec{y})\right\}_{\mathsf{DB}} &= -\,gf^{ace}(\pi^{e})^{i}{}_{l}(\vec{x})\epsilon^{jkm}\epsilon_{mnp}\left[(\mathcal{D}^{p}_{\vec{y}})^{bd}(\mathcal{A}^{-1})^{cd\,ln}\right]\delta(\vec{x}-\vec{y}) \\
\left\{\mathsf{A}^{a\,i}(\vec{x}),(\pi^{b})_{jk}(\vec{y})\right\}_{\mathsf{DB}} &= 0 \\
\left\{\pi^{a}_{i}(\vec{x}),\pi^{b}_{j}(\vec{y})\right\}_{\mathsf{DB}} &= [gf^{ace}\pi^{e}_{ik} - m\epsilon_{ikp}(\mathcal{D}^{ac}_{\vec{x}})^{p}](gf^{bdh}\pi^{h}_{jl} - m\epsilon_{jlm}(\mathcal{D}^{bd}_{\vec{y}})^{m})(\mathcal{A}^{-1}\mathcal{B}\mathcal{A}^{-1})^{cd\,kl} \\
&\quad -[gf^{dgh}(\mathcal{D}^{bg}_{\vec{y}})^{m} + \frac{1}{2}g^{2}f^{dgh}f^{bhT}B^{T}_{[mn}(\vec{y})\delta_{l]j}(\mathcal{A}^{-1})^{cd\,lk}]\delta(\vec{x}-\vec{y}) \\
&\quad -[gf^{aef}(\mathcal{D}^{ae}_{\vec{y}})^{p}H^{f}_{ikp}(\vec{y}) - \frac{1}{2}g^{2}f^{cef}f^{afh}B^{h}_{[pq}(\vec{x})\delta_{k]i}] \\
&\quad [gf^{bdg}\pi^{g}_{jl}(\vec{y}) - m\epsilon_{jlm}(\mathcal{D}^{bd}_{\vec{y}})^{m}](\mathcal{A}^{-1})^{cd\,kl}\delta(\vec{x}-\vec{y}) \\
\left\{\mathsf{B}^{aoi}(\vec{x}),\pi^{b}{}_{0j}(\vec{y})\right\}_{\mathsf{DB}} &= \delta^{ab}\delta^{i}{}_{j}\delta(\vec{x}-\vec{y}) \\
\left\{\mathsf{B}^{aij}(\vec{x}),\pi^{b}{}_{kl}(\vec{y})\right\}_{\mathsf{DB}} &= \delta^{ab}\left(\delta^{i}{}_{k}\delta^{j}{}_{l} - \delta^{i}{}_{l}\delta^{j}{}_{k}\right)\delta(\vec{x}-\vec{y}) \\
&\quad +\epsilon^{ijr}\epsilon_{rmp}(\mathcal{D}^{ac}_{\vec{x}})^{p}[\frac{1}{2}gf^{deh}(\mathcal{D}^{bh}_{\vec{y}})^{[n}\epsilon^{rs]t}\epsilon_{tkl}F^{e}_{rs}(\vec{y}(\mathcal{A}^{-1})^{cdm}_{n} \\
&\quad -mgf^{bde}\epsilon_{tkl}(\mathcal{A}^{-1})^{cdm}_{n}\pi^{ent}(\vec{y})]\delta(\vec{x}-\vec{y})
\end{aligned}
$$

$$
\begin{aligned}
\left\{\mathsf{B}^{aij}(\vec{x}),\mathsf{B}^{bmn}(\vec{y})\right\}_{\mathsf{DB}} &= \epsilon^{ijk}\epsilon_{kpq}\epsilon^{mnl}\epsilon_{lrs}\left[\left(\mathcal{D}^{bd}_{\vec{y}}\right)^{r}(\mathcal{D}^{ac}_{\vec{x}})^{p}\left(\mathcal{A}^{-1}\mathcal{B}\mathcal{A}^{-1}\right)^{cd\,qs}(\vec{x},\vec{y})\right. \\
&\quad +(\mathcal{D}^{bd}_{\vec{y}})^{r}(\mathcal{D}^{ac}_{\vec{x}})^{p}(\mathcal{A}^{-1})^{cd\,qs} \\
&\quad \left.+gf^{ace}\left(\pi^{ep} - \frac{m}{2}\epsilon^{ptu}\mathsf{B}^{e}_{tu}\right)(\vec{x})\left(\mathcal{D}^{bd}_{\vec{y}}\right)^{r}\left(\mathcal{A}^{-1}\right)^{cd\,qs}\right] \\
\left\{\pi^{a}{}_{i}(\vec{x}),\mathsf{B}^{b\,jk}(\vec{y})\right\}_{\mathsf{DB}} &= \epsilon^{jkr}\epsilon_{rnq}\left[gf^{ace}\pi^{e}{}_{im}(\vec{x}) - m\epsilon_{imp}(\mathcal{D}^{ac}_{\vec{x}})^{p}\right] \\
&\quad \left\{(\mathcal{D}^{bd}_{\vec{y}})^{q}\left(\mathcal{A}^{-1}\mathcal{B}\mathcal{A}^{-1}\right)^{cd\,mn} + (\mathcal{A}^{-1})^{cd\,mn}gf^{bdh}(\pi'^{\,h})^{q}\vec{y}\right\} \\
&\quad +\epsilon^{jkr}\epsilon_{rnq}gf^{cgh}(\mathcal{D}^{ag}_{\vec{x}})^{p}\left\{(\mathsf{H}^{h})_{imp}(\vec{x})(\mathcal{D}^{bd}_{\vec{y}})^{q}(\mathcal{A}^{-1})^{cd\,mn}(\vec{x},\vec{y})\right\} \\
&\quad +\frac{1}{2}\epsilon^{jkr}\epsilon_{rnq}gf^{cgh}gf^{ahe}F^{g}{}_{rs}\ \delta_{i}{}^{[m}(\mathsf{B}^{e})^{rs]}(\mathcal{D}^{bd}_{\vec{y}})^{q}(\mathcal{E}^{-1})^{cd\,mn}(\vec{x},\vec{y})
\end{aligned}
$$

All other Dirac brackets are Zero.

These Dirac brackets replace the corresponding Poisson brackets in variou calculations, so that the second class constraints are no longer considered. For quantisation, the Dirac brackets are considered to be taken over to be commutators, so that the quantised theory also has no second class constraint operators.

3 Conclusion

We have considered two interacting systems, one involving Abelian fields, and the other with non-Abelian fields. Both theories display gauge invariance, implying the presence of first class constraints. But the constraint structure of the non-Abelian theory is more complicated. Second class constraints are also present. These are removed by constructing Dirac brackets, which here are found to have complicated expressions. The resulting quantum theory is also expected to be complicated. One possible way to avoid this is to convert the second class constraints into first class constraints. This may result in an enlarged gauge symmetry in the theory. Work is proceeding in this direction.

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