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Magneto Hydrodynamic effects on a transient nanofluid past over an vertical plate

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ABSTRACT

In this paper, the characteristics of heat transfer with magneto hydrodynamic effects on an unsteady nanofluid flow over a vertical plate are considered. Three different types of water based nanofluids (Cu, Al_2O_3, TiO_2) are considered to derive the exact solution for the problem. The dimensionless governing equations are solved by laplace transform technique. Solutions for temperature and velocity of the plate are derived and the graphs are also obtained for the different parameters like thermal Grashof number, time, magnetic field parameter, Prandtl number and solid volume fraction. The effects of these parameters on the velocity and temperature of the plate are discussed in a detailed manner.

KEYWORDS: Vertical Plate, MHD, Nanofluid, Laplace transform.

INTRODUCTION

In recent years, considerable attention has been devoted to the study of magnetohydrodynamics (MHD) flow and heat. In addition to useful features of MHD flows, such studies can be helpful in prediction of the effects of magnetic intrusions. Magnetohydrodynamics is attracting the attention of the many authors due to its applications in geophysics; it is applied to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere etc. In engineering in MHD pumps, MHD bearings etc. at high temperatures attained in some engineering devices, gas, for example, can be ionized and so becomes an electrical conductor. The ionized gas or plasma can be made to interact with the magnetic and alter heat transfer and friction characteristic.

Raptis and Singh [1] studied MHD free convection flow past an accelerated vertical plate.Md. Jashim Uddin et al [2] explained with Scaling Group Transformation for MHD Boundary Layer Slip Flow of a Nanofluid over a Convectively Heated Stretching Sheet with Heat Generation. Farhad Ali et al [3] studied Heat and Mass Transfer with Free Convection MHD Flow Past a Vertical Plate Embedded in a Porous Medium. Alam and Rahman [4] studied Dufour and Soret effects on MHD free convective heat and mass transfer flow past a vertical porous plate embedded in a porous medium. Ahmed et al [5] studied analytical Study on Unsteady MHD Free Convection and Mass Transfer Flow Past a Vertical Porous Plate. Das et al [6] analyzed Unsteady Free Convection Flow past a Vertical Plate with Heat and Mass Fluxes in the Presence of Thermal Radiation. Xiang-Qi Wang et al. [7] studied Experiments and applications of nanofluids. Rajesh vemula et al [8] studied the unsteady MHD free convection flow of nanofluid past an exponentially accelerated vertical plate with variable temperature and thermal radiation. The topic of heat transfer in nanofluids has been surveyed in review articles by Das and Choi [9], Kakac and Pramuanjaroenkij[10], Wang and Mazumdar [11], Makinde [12] analysed a hydromagnetic mixed convection flow and mass transfer over a vertical porous plate with constant heat flux embedded in a porous medium. Makinde and Ogulu [13] analysed the effect of thermal radiation on the heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate permeated by a transverse magnetic field. Natural convective magneto-nanofluid flow and radiative heat transfer past a moving vertical plate was discussed by Das and Jana [14].

It is noticed that from the above literature survey no authors have been studied the transient incompressible flow of a nanofluid with magnetohydrodynamic effects on a vertical plate. .Here the dimensionless governing equations are solved by laplace transform technique. Cu-water combination is analyzed for the different parameters like time, thermal Grashof number, Magnetic field parameter and solid volume fraction on the velocity and temperature of the plate and the graphs are also drawn and the effects of the above parameters are also discussed in a detailed manner. The graphs are also obtained for the different nanofluids (Cu, Al_2O_3, TiO_2) on the velocity and the temperature profile.

MATHEMATICAL ANALYSIS

The transient free convective viscous flow of a nanofluid through aa infinite vertical plate with a magnetohydrodynamic effect has been considered. Further it is an unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate maintaining temperature T_∞. The *x*-axis is considered along the plate in the vertically upward direction and the *y*-axis is assigned normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature T_∞. At time $t' > 0$, the plate is accelerated with a velocity $\frac{u_0^3}{v_f} t'$ its own plane against gravitational field and the temperature from the plate is raised to T_w. The transverse magnetic field B_0 is applied normal to the plate. The fluid is a water based nanofluid containing three types' nanoparticles Cu, Al_2O_3 and TiO_2. It is further assumed that the base fluid and the suspended nanoparticles are in thermal equilibrium. The thermophysical properties of the nanofluids are given in Table 1[14].

Table 1 Thermo physical properties of water and nanoparticles

Physical properties	Water/Base fluid	Cu(Copper)	Al_2O_3 (Alumina)	TiO_2
ρ (kg / m^3)	997.1	8933	3970	4250
c_ρ (J / kgK)	4179	385	765	686.2
K(W / mK)	0.613	401	40	8.9538
β x 10^5 (K^{-1})	21	1.67	0.85	0.90
ϕ	0.0	0.05	0.15	0.2
σ (S / m)	5.5 x 10^{-6}	59.6 x 10^6	35 x 10^6	2.6 x 10^6

.

Then under usual Boussinesq's approximation, the unsteady viscous flow of a nanofluid is governed by the following equations:

$$\rho_{nf} \frac{\partial u}{\partial t'} = g(\rho\beta)_{nf}(T - T_\infty) + \mu_{nf} \frac{\partial^2 u}{\partial y^2} - \sigma B_0^2 u \quad (1)$$

$$(\rho c_p)_{nf} \frac{\partial T}{\partial t'} = k_{nf} \frac{\partial^2 T}{\partial y^2} \quad (2)$$

where u is the velocity components along the x-direction, T the temperature of the nanofluid, μ_{nf} the dynamic viscosity of the nanofluid, β_{nf} the thermal expansion coefficient of the nanofluid, ρ_{nf} the density of the nanofluid, k_{nf} the thermal conductivity of the nanofluid, g the acceleration due to gravity, q_r the radiative heat flux and $(\rho c_p)_{nf}$ the heat capacitance of the nanofluid which are given by

$$\begin{aligned} &\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s \\ &(\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s \\ &(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s \end{aligned} \tag{3}$$

where ϕ is the solid volume fraction of the nanoparticle, ρ_f the density of the base fluid, ρ_s the density of the nanoparticle, μ_f the viscosity of the base fluid, $(\rho c_p)_f$ the heat capacitance of the base fluid and $(\rho c_p)_s$ the heat capacitance of the nanoparticle. It is worth mentioning that the expressions (1) are restricted to spherical nanoparticles, where it does not count for other shapes of nanoparticles. The effective thermal conductivity of the nanofluid given by Hamilton and Crosser model followed by Kakac and Pramuanjaroenkij [9], and Oztop and Abu-Nada [14] is given by

$$k_{nf} = k_f \left[\frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \right] \tag{4}$$

where k_f is the thermal conductivity of the base fluid and k_s the thermal conductivity of the nanoparticle. In Eqs. (1) – (4), the subscripts nf, f and s denote the thermophysical properties ofthe nanofluid, base fluid and nanoparticles, respectively.

The initial and boundary conditions of the proposed problem are given by:

$$\begin{aligned} &u = 0, \quad T = T_\infty \quad for\ all \quad y, t' \le 0 \\ t' > 0: \quad &u = \frac{u_0^{\,3}}{\nu_f} t', \quad T = T_\omega \quad at \quad y = 0 \\ &u \to 0 \quad T \to T_\infty \quad as \quad y \to \infty \end{aligned} \tag{5}$$

On introducing the following non dimensional quantities are:

$$U = \frac{u}{u_0}\ ,\ t = \frac{t' {u_0}^2}{v_f},\ Y = \frac{y u_0}{v_f}, \tag{6}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty},\ Gr = \frac{g\, v_f \beta (T_w - T_\infty)}{{u_0}^3},\quad M = \frac{\sigma {B_0}^2 V}{\rho {u_0}^2}, \Pr = \frac{\mu c_p}{k_f}$$

By using equations (3), (4) and (6), equations (1) and (2) leads to,

$$L_1 \frac{\partial U}{\partial t} = L_3 \frac{\partial^2 U}{\partial Y^2} + L_2 Gr\theta - MU$$
$$L_5 \frac{\partial \theta}{\partial t} = L_6 \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial Y^2} \tag{7}$$

Where $L_1 = (1-\phi) + \phi\left(\frac{\rho_s}{\rho_f}\right), L_2 = (1-\phi) + \phi\left(\frac{(\rho\beta)_s}{(\rho\beta)_f}\right), L_3 = \frac{1}{(1-\phi)^{2.5}}$

$$L_5 = (1-\phi) + \phi\left(\frac{(\rho c_p)_s}{(\rho c_p)_f}\right), L_6 = \left[\frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}\right] \tag{8}$$

Where Pr is the Prandtl number, Gr is the thermal Grashof number, Gr approximates the ratio of the buoyancy force to the viscous force acting.

The nondimensionalform of corresponding initial and boundary conditions are,

$$\begin{array}{lllll} & U = 0, & \theta = 0, & for\ all & Y, t \le 0 \\ t > 0: & U = gt, & \theta = 1, & at & Y = 0 \\ & U \to 0, & \theta \to 0, & as & Y \to \infty \end{array} \tag{9}$$

Where $g = \frac{L_1}{L_3}$

METHODS OF SOLUTIONS

Equation (7) is solved subject to the initial and boundary conditions (9) with the help of Laplace transform analytically. The Exact solutions are expressed in terms of exponential function and complementary error function.

$$\theta = erfc(\eta\sqrt{L7})$$

$$U = \frac{gt}{2}[\exp(2\eta\sqrt{dt})erfc(\eta\sqrt{g} + \sqrt{dt}) + \exp(-2\eta\sqrt{dt})erfc(\eta\sqrt{g} - \sqrt{dt})]$$

$$-\frac{c}{b}[erfc(\eta\sqrt{g})] + \frac{c}{b}[erfc(\eta\sqrt{a})]$$

$$+\frac{c(\exp(bt))}{2b}[\exp(2\eta\sqrt{g(d+b)t})erfc(\eta\sqrt{g} + \sqrt{(d+b)t}))$$

$$+\exp(-2\eta\sqrt{g(d+b)t})erfc(\eta\sqrt{g} - \sqrt{(d+b)t})]$$

$$-\frac{c(\exp(bt))}{2b}[\exp(2\eta\sqrt{abt})erfc(\eta\sqrt{a} + \sqrt{bt})$$

$$+\exp(-2\eta\sqrt{abt})erfc(\eta\sqrt{a} - \sqrt{bt})]$$

$$where, a = \frac{L5\Pr}{L6}; c = \frac{GrL2}{L7L3 - L1}; d = \frac{M}{L9}; g = \frac{L1}{L3}; \eta = \frac{1}{2\sqrt{t}}$$

RESULTS AND DISCUSSION

In order to get the physical insight into the problem, the computed values of different parameters like thermal Grashof number, Prandtl number, time , volume solid fraction and magnetic field parameter on the temperature and velocity of the plate are considered and the effects are explained in a detailed manner through graph. By changing the values of various parameters for the above expression, the numerical values are computed until they converge to free stream boundary conditions. For that the Prandtl number Pr = 7.1 is considered which corresponds to water, time t = 0.5, thermal Grashof number Gr = 5 which represents cooling of the plate. We have considered three different types of nanofluids containing copper (Cu), Aluminium oxide (Al_2O_3) and Titanium oxide (TiO_2) with water as a base fluid. The nanoparticle volume fraction is considered in the range of $0 \le \phi \le 0.2$. In this study, we have considered spherical nanoparticles with thermal conductivity and dynamic viscosity [15] shown in model I in Table 2.

Table 2 Thermal conductivity and dynamic viscosity for various shapes of nanoparticles

Model	Shape of nanoparticles	Thermal conductivity	Dynamic viscosity
I	Spherical	$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}$	$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}$

II	Spherical (nanotubes)	$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}$	$\mu_{nf} = \mu_f(1 + 7.3\phi + 12.3\phi^2)$
III	Cylindrical	$\frac{k_{nf}}{k_f} = \frac{k_s + \frac{1}{2}k_f - \frac{1}{2}\phi(k_f - k_s)}{k_s + \frac{1}{2}k_f + \phi(k_f - k_s)}$	$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}$
IV	Cylindrical(nanotubes)	$\frac{k_{nf}}{k_f} = \frac{k_s + \frac{1}{2}k_f - \frac{1}{2}\phi(k_f - k_s)}{k_s + \frac{1}{2}k_f + \phi(k_f - k_s)}$	$\mu_{nf} = \mu_f(1 + 7.3\phi + 12.3\phi^2)$

Figure 1 shows that the effect of thermal grashof number Gr on the velocity of the plate for different values (Gr = 3, 5,7). Here Prandtl number Pr = 7.1, time t = 0.5, Magnetic field parameter M = 1 and solid volume fraction ϕ =0.1are considered. Since Grashof number is the ratio of the buoyancy force to viscous force. Grashof number Gr decreases, the velocity of the plate increases due to the buoyancy force for the nanofluid Cu –Water. The effect of Magnetic field parameter M (M = 2,5,7) on the velocity of the plate for the nanofluid Cu-Water is shown in Figure 2. Here Prandtl number Pr = 7.1, time t = 0.5, Thermal grashof number Gr = 5 and solid volume fraction ϕ =0.1are kept to be constant. Since Lorentz force causes due to the magnetic field and it acts against the plate. Whenever magnetic field parameter increases, the velocity of the plate decreases due to the Lorentz force and it is shown in Figure 2.

Figure 3 reveals the effect of time t(t = 0.2, 0.4, 0.6)on the velocity of the plate when Prandtl number Pr = 6.1, Thermal Grashof number Gr = 5, Magnetic field parameter M=2 and solid volume fraction ϕ =0.1are taken. This graph shows that the time enhances the velocity of the plate. Figure 4 shows the effect of the solid volume fraction on the velocity profile for different values (ϕ =0.1, 0.15, 0.2). Here the constants are Prandtl number Pr = 7.1, Thermal

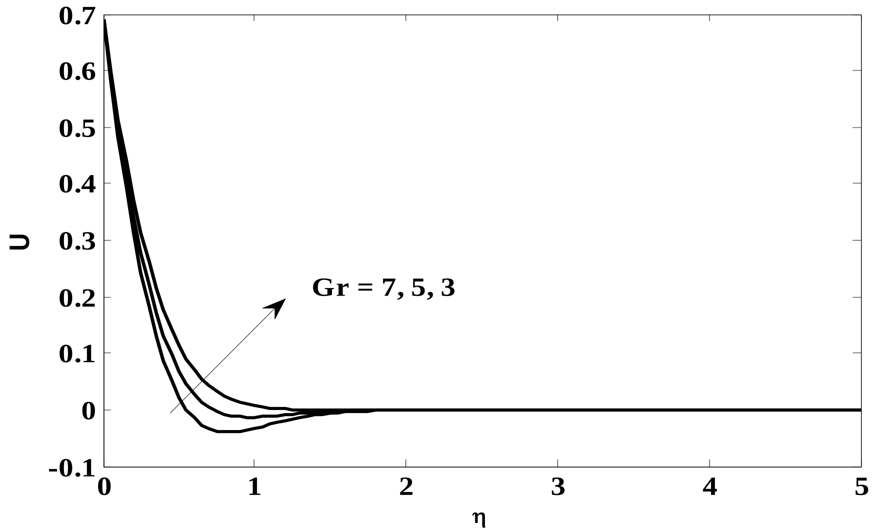

Figure 1. Velocity profile for different values of Gr

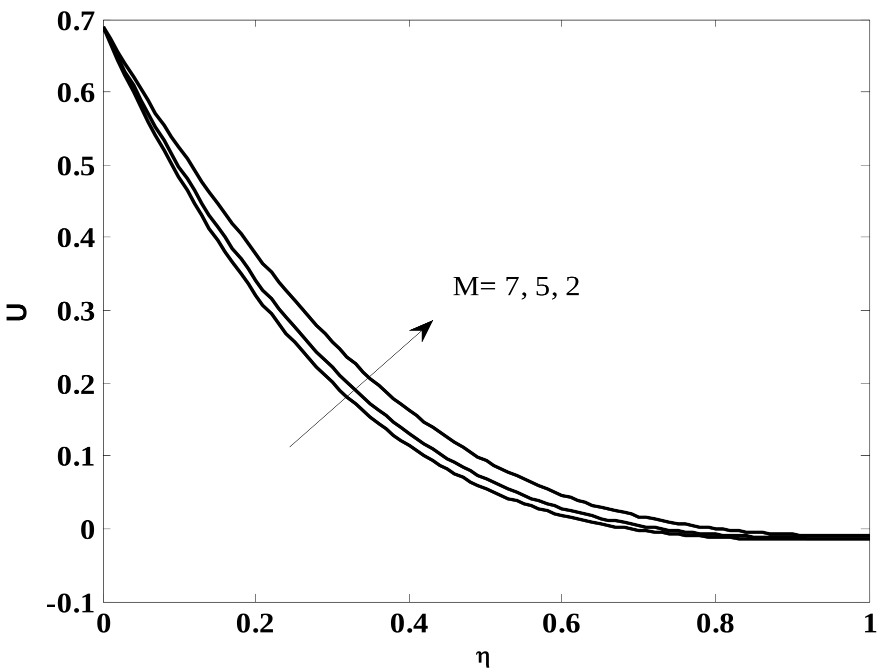

Figure 2 .Velocity profile for different values of M

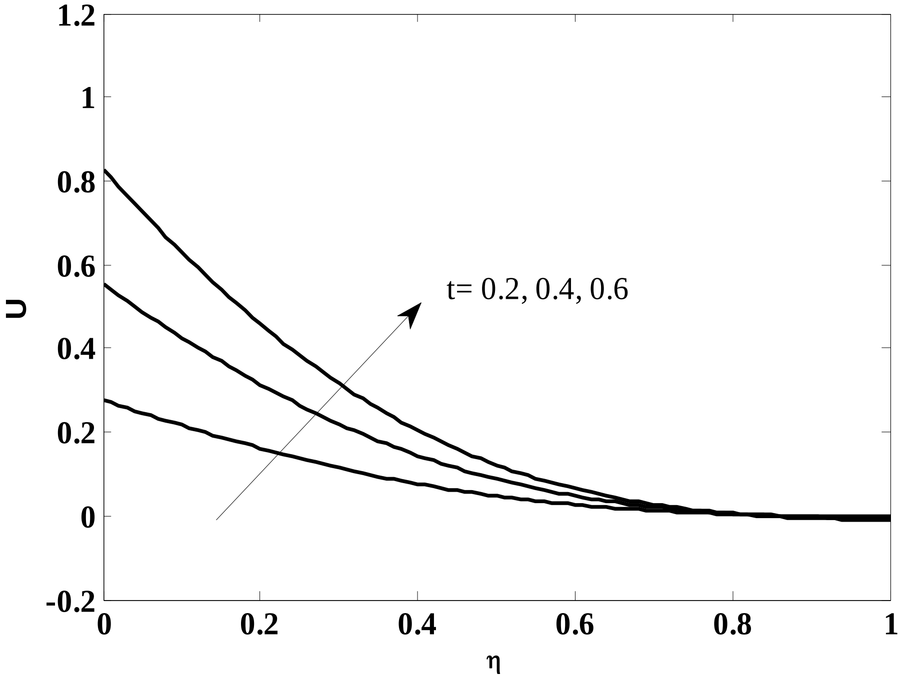

Figure 3. Velocity profile for different values of t

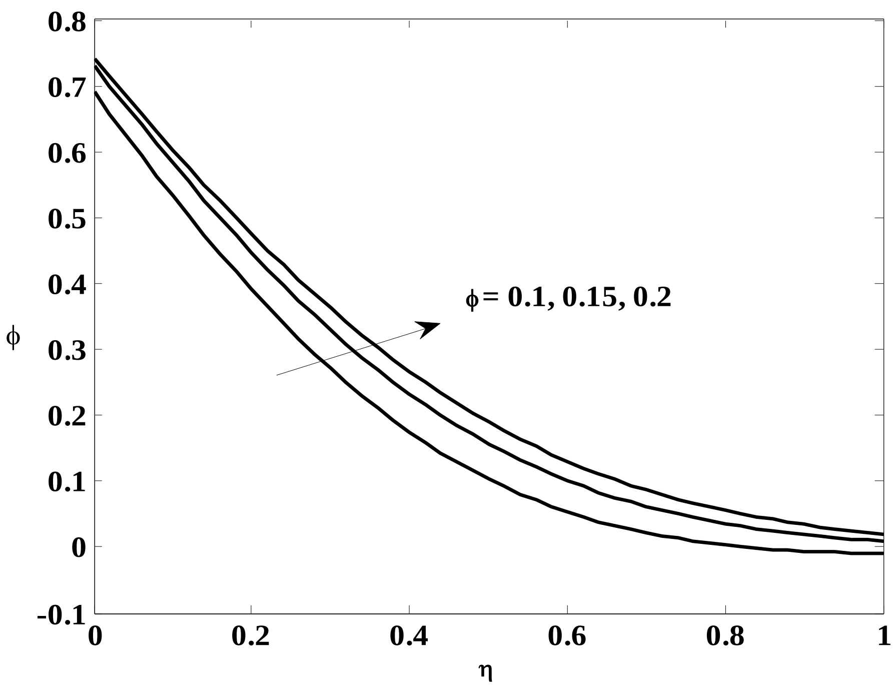

Figure 4. Velocity profile for different values of ϕ

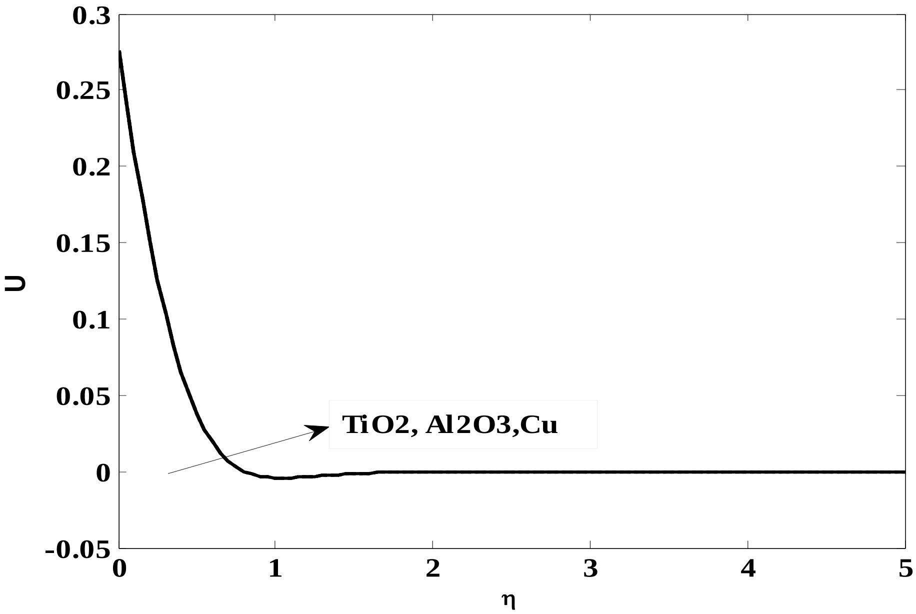

Figure 5. Velocity profile for different values of nano fluid

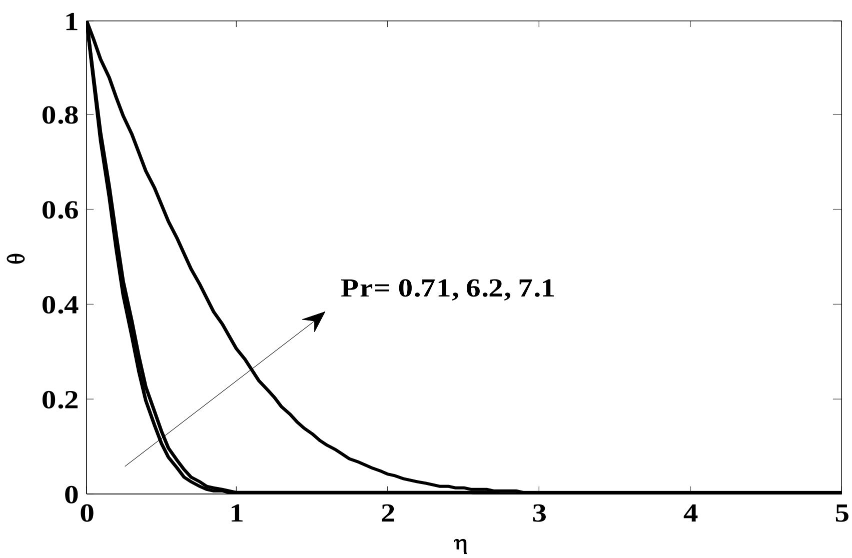

Figure 6. Temperature profile for different values of Pr

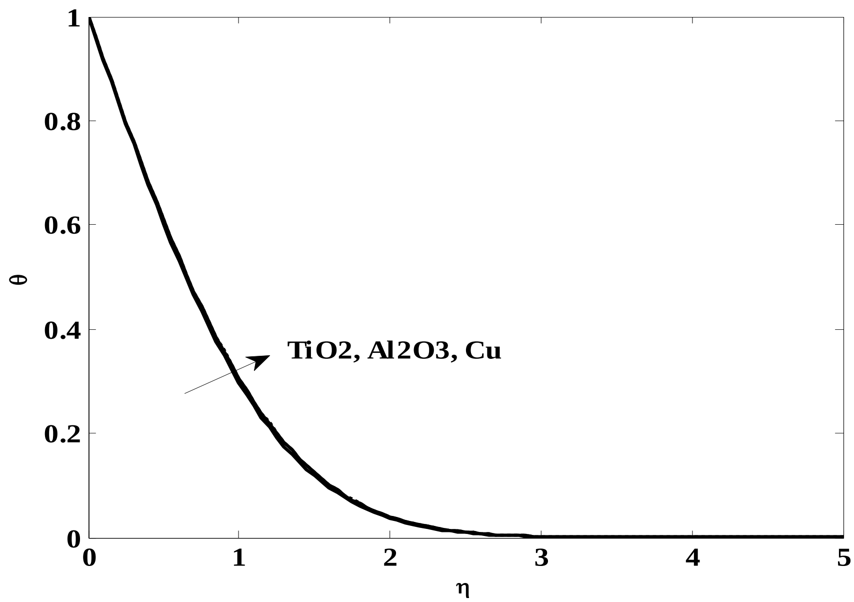

Figure 7. Temperature profile for different values of nano fluid

Grashof number Gr = 5, Magnetic field parameter M = 1and time t = 0.5. The velocity of the plate increases for the increasing values of solid volume fraction.

Figure.5. reveals that the plate velocity variations for the three types of water based nanofluids Cu-Water, Al_2O_3- water and TiO_2-Water when Pr=7.1, M = 5 and Gr=5 at t=0.2. Here the velocity of the plate is found to be similar for Al_2O_3, TiO_2 and Cu. In Figure.6. temperature profile for different values of Prandtl number (Pr= 0.71, 6.2, 7.1) is presented when t=0.5. The size of the thermal boundary layer increases with decreasing Prandtl number because the Boussinesq's approximation in the momentum equation consists of assuming that density of the fluid varies with temperature linearly. Here temperature profile gradually reduces to reach the free stream temperature. Figure.7. reveals that the plate velocity variations for the three types of water based nanofluids Cu-Water, Al_2O_3- water and TiO_2-Water when Pr=07.1, and t=0.5. Due to higher thermal conductivity of Cu–water nanofluids, the temperature of Cu–water nanofluid is found to be higher than Al_2O_3-–water and TiO_2 - water nanofluids. It is also seen that the thermal boundary layer thickness is more for Cu–water than Al_2O_3–water and TiO_2–water nanofluids.

CONCLUSION

The Unsteady flow of a viscous incompressible nanofluid flow over an infinite vertical plate with the effect of Magnetichydrodynamic was considered. Here the dimensional governing equations are converted into a nondimensional form using dimensionless quantities. These dimensionless equations with the nondimensional boundary conditions are solved by Laplace transform technique. The solutions are obtained in terms of exponential and complementary error functions. The graphs are plotted for different values of thermal Grashof number, Prandtl number, time, volume solid fraction and magnetic field parameter on the temperature and velocity of the plate and discussed in a detailed manner. In this study the conclusion is as follows:

(i). The velocity of the plate enhances for the increasing values of time and solid fraction fraction but the trend is reversed in the case of thermal Grashof number and magnetic field parameter.

(ii). The temperature of the plate increases for the increases values of Prandtl number.

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