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An Investigation of Some Common Fixed Point Theorems In Intuitionistic Fuzzy Metric Spaces

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Abstract

Some results regarding common fixed point theorems in intuitionistic fuzzy metric spaces are proved here in this paper. Our main result generalizes the outcomes of Turkoglu et al. [10] and other results which were proved in intuitionistic fuzzy metric spaces.

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INTRODUCTION

Park [7] defined intuitionistic metric space by using intuitionistic fuzzy set presented by Atanassov [3]. Manro et al. [6] established a common fixed point theorem for – chainable intuitionistic fuzzy metric spaces for weakly compatible maps. Samanta and Mohinta [8] defined (ε, λ) IF- uniformly locally contractive maps and proved that such maps possess a fixed point. Ionescu et al. [5] found some contractions in intuitionistic fuzzy metric spaces and proved fixed point results for these contraction classes. Chauhan et al. [4] produced common fixed point theorems for weakly compatible maps in modified intuitionistic fuzzy metric spaces satisfying φ contractive conditions.

Schweizer and Sklar [9] is referred in our paper for the definition of a binary operation * as continuous t-norm and another binary operation ◊ as continuous t-conorm. Alaca et al. [2] is used as a reference in our paper for the definition of intuitionistic fuzzy metric space, Cauchy sequence and completeness of a sequence in this space. Turkoglu et al. [10] defined weakly commuting and R-weakly commuting mappings in intuitionistic fuzzy metric space which are as follows:

Definition 1. [10] Let A and B be mappings from an intuitionistic fuzzy metric space (X, M, N, *, ◊) into itself. The mappings A and B are said to be weakly commuting if,

$M(ABx, BAx, t) \geq M(Ax, Bx, t)$, $N(ABx, BAx, t) \leq N(Ax, Bx, t)$ for all $x \in R$.

Definition 2. [10] Let A and B be mappings from an intuitionistic fuzzy metric space (X, M, N, *, ◊) into itself. The mappings A and B are said to be R-weakly commuting if there exists a positive real number R such that $M(ABx, BAx, t) \geq M(Ax, Bx, t/R)$, $N(ABx, BAx, t) \leq N(Ax, Bx, t/R)$ for all $x \in R$.

Turkoglu et al. [10] proved that weak commutativity implies R-weak commutativity in intuitionistic fuzzy metric space. However, R- weak commutativity implies weak commutativity only when $R \leq 1$.

Alaca et al. [1] proved the following two lemmas:

Lemma 1. [1] Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space. If there exists $k \in (0, 1)$ such that $M(x, y, kt) \geq M(x, y, t)$ and $N(x, y, kt) \leq N(x, y, t)$ for all x, $y \in X$. Then x = y.

Lemma 2. [1] Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space. If there exists a number k (0, 1) such that $M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t)$ ∈and $N(y_{n+2}, y_{n+1}, kt) \leq N(y_{n+1}, y_n, t)$

for all $t > 0$ and n = 1,2,…….. then $\{y_n\}$ is a Cauchy sequence in X.

Turkoglu et al. [10] proved the following result:

Let (X, M, N, *, ◊) be a complete intuitionistic fuzzy metric space and f and g be R - weakly commuting mappings of X satisfying the following conditions: (i) f(X) □ g(X) ; (ii) f or g is continuous; (iii) for all x, $y \in X$ and $0 < t < 1$, $M(f x, f y, t) \geq c(M(gx, gy, t))$, $N(f x, f y, t) \geq c'(N(gx, gy, t))$, where $c : [0,1] \to [0,1]$ and $c' : [0,1] \to [0,1]$ is a continuous function such that $c(t) > t$ and $c'(t) < t$. If the sequences $\{x_n\}$ and $\{y_n\}$ in X are such that, for all x, $y \in X$ and $t > 0$, $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$ implies $\lim_{n\to\infty} M(x_n, y_n, t) = M(x, y, t)$, $\lim_{n\to\infty} N(x_n, y_n, t) = N(x, y, t)$, then f and g have a unique common fixed point in X.

MAIN RESULT.

In this paper, a result regarding common fixed point theorem in intuitionistic fuzzy metric space is proved for R-weakly commuting mappings which is the generalization of the outcomes produced by Turkoglu et al. [10] and other results which were proved in intuitionistic fuzzy metric spaces.

Theorem. *Let P, Q, T and S be self-mappings of a complete intuitionistic fuzzy metric space (X, M, N, *, ◊) with continuous t-norm * defined by $a * b = \min\{a, b\}$, $a, b \in [0,1]$ and continuous t-conorm ◊ defined by $a \lozenge b = \max\{a, b\}$, $a, b \in [0,1]$, fulfilling the conditions:*

(1) Q(X) & S(X), P(X) & T(X);

(2) T and S are continuous;

(3) (P, S) and (Q, T) are R-weakly commuting self-maps of X;

(4) for all x, y in X and t > 0 ,there exists k ∈ (0, 1) such that

$M(Px, Qy, kt) \geq M(Sx, Ty, t) * M(Px, Sx, t) * M(Qy, Ty, t) * M(Sx, Qy, 2t) * M(Px, Ty, t)$ *and*

$N(Px, Qy, kt) \leq N(Sx, Ty, t) \Diamond N(Px, Sx, t) \Diamond N(Qy, Ty, t) \Diamond N(Sx, Qy, 2t) \Diamond N(Px, Ty, t)$;

for all x, y ∈ X and t > 0, sequences $\{x_n\}$ *and* $\{y_n\}$ *in X are such that*

(5) $\lim_{n\to\infty} x_n = x$, $\lim_{n\to\infty} y_n = y$ *gives*

$\lim_{n\to\infty} M(x_n, y_n, t) = M(x, y, t)$, $\lim_{n\to\infty} N(x_n, y_n, t) = N(x, y, t)$.

Then P, Q, T, and S have a unique common fixed point in X.

Proof. Assume that x_0 is an arbitrary point in X. Since Q(X) □ S(X) and P(X) □ T(X), there exists $x_1, x_2 \in X$ such that $Px_0 = Tx_1 = y_0$ and $Qx_1 = Sx_2 = y_1$. We form sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$Y_{2n} = Px_{2n} = Tx_{2n+1}$ and $y_{2n+1} = Qx_{2n+1} = Sx_{2n+2}$ where n = 0, 1, 2…

Firstly we proceed to show $\{y_n\}$ is a Cauchy sequence in X.

From (4), we have

$M(y_{2n}, y_{2n+1}, kt) = M(Px_{2n}, Qx_{2n+1}, kt)$

$\geq M(Sx_{2n},Tx_{2n+1},t)*M(Px_{2n},Sx_{2n},t)*M(Qx_{2n+1},Tx_{2n+1},t)*M(Sx_{2n},Qx_{2n+1},2t)*M(Px_{2n},Tx_{2n+1}, t)$.

$= M(y_{2n-1}, y_{2n}, t) * M(y_{2n}, y_{2n-1}, t) * M(y_{2n+1}, y_{2n}, t) * M(y_{2n-1}, y_{2n+1}, 2t) * M(y_{2n}, y_{2n}, t)$

From the definition of continuous t-norm by Schweizer and Sklar [9] and definition of intuitionistic fuzzy metric space by Alaca et al. [2], we get

$M(y_{2n}, y_{2n+1}, kt) \geq M(y_{2n-1},y_{2n}, t) * M(y_{2n+1}, y_{2n}, t)$ or $M(y_{2n}, y_{2n+1}, kt) \geq M(y_{2n-1}, y_{2n}, t)$

Similarly From the definition of continuous t-conorm by Schweizer and Sklar [9] and definition of intuitionistic fuzzy metric space by Alaca et al. [2], we get $N(y_{2n}, y_{2n+1}, kt) \leq N(y_{2n-1}, y_{2n}, t)$

So, we can conclude that

$M(y_{n+1}, y_{n+2}, kt) \geq M(y_n, y_{n+1}, t)$ and $N(y_{n+1}, y_{n+2}, kt) \leq N(y_n, y_{n+1}, t)$ for all $t > 0$. .

Therefore, from lemma 2, $\{y_n\}$ is a Cauchy sequence in X. Since X is complete, therefore $\{y_n\}$ converges to some point z in X. Consequently, the subsequences $\{Px_{2n-2}\}$, $\{Tx_{2n-1}\}$, $\{Qx_{2n-1}\}$ and $\{Sx_{2n}\}$ converges to z in X. Since S is continuous, $\lim_{n\to\infty} SPx_{2n} = Sz$ and $\lim_{n\to\infty} SSx_{2n} = Sz$. The pair (P, S) is R-weakly commuting, therefore

$$M(PSx_{2n}, SPx_{2n}, t) \geq M(Px_{2n}, Sx_{2n}, t/R) \qquad \text{(i)}$$

$$N(PSx_{2n}, SPx_{2n}, t) \leq N(Px_{2n}, Sx_{2n}, t/R) \qquad \text{(ii)}$$

Take $n\to\infty$ in (i) and (ii), we obtain $\lim_{n\to\infty} PSx_{2n} = Sz$.

Put $x = Sx_{2n}$ and $y = x_{2n-1}$ in (4), let $n\to\infty$ and using above results, we get

$M(Sz, z, kt) \geq M(Sz, z, t)$ and $N(Sz, z, kt) \leq N(Sz, z, t)$

Now by Lemma 1, we get $Sz = z$.

As T is continuous, we have $\lim_{n\to\infty} TQx_{2n-1} = Tz$ and $\lim_{n\to\infty} TTx_{2n-1} = Tz$

As (Q, T) is R-weakly commuting

$$M(QTx_{2n-1}, TQx_{2n-1}, t) \geq M(Qx_{2n-1}, Tx_{2n-1}, t/R) \qquad \text{(iii)}$$

$$N(QTx_{2n-1}, TQx_{2n-1}, t) \leq N(Qx_{2n-1}, Tx_{2n-1}, t/R) \qquad \text{(iv)}$$

Take $n\to\infty$ in (iii) and (iv), we obtain $\lim_{n\to\infty} QTx_{2n-1} = Tz$.

Put $x = Sx_{2n}$ and $y = Tx_{2n-1}$ in (4), let $n\to\infty$ and use above results, we obtain $M(Sz, Tz, kt) \geq M(Sz, Tz, t)$ and $N(Sz, Tz, kt) \leq N(Sz, Tz, t)$. Now using Lemma 1, we obtain $Sz = Tz$. Hence, $Sz = Tz = z$.

Put $x = z$ and $y = Tx_{2n-1}$ in (4), let $n\to\infty$ and use above results, we obtain $M(Pz, Tz, kt) \geq M(Pz, Tz, t)$and $N(Pz, Tz, kt) \leq N(Pz, Tz, t)$. Now using Lemma 1, we obtain $Pz = Tz$. Hence $Sz = Pz = Tz = z$.

Again Put $x = z$ and $y = z$ in (4) and use above results, we obtain $M(z, Qz, kt) \geq M(z, Qz, t)$ and $N(z, Qz, kt) \leq N(z, Qz, t)$. We obtain $Qz = z$. Hence $Pz = Qz = Tz = Sz = z$.

Which shows that common fixed point of P, Q, T and S is z.

Now we prove Uniqueness of z.

Assume that the mappings P, Q, T and S have u' as another common fixed point. So, we can write $u' = Pu' = Qu' = Tu' = Su'$. Put $x = z$ and $y = u'$ in (4) and use above results, we get $M(z, u', kt) \geq M(z, u', t)$ and $N(z, u', kt) \leq N(z, u', t)$

Hence $z = u'$. Thus, z is the unique common fixed point of P, Q, S and T.

Corollary. *Let P, Q, T and S be self-mappings of a complete intuitionistic fuzzy metric space (X, M, N, *, ◊) with continuous t-norm * defined by a * b= min{a, b}, a, b∈ [0,1] and continuous t-conorm ◊ defined by a ◊ b = max{a, b}, a, b∈ [0,1], satisfying the conditions (1), (2), (4), (5) of the above theorem and the pairs (P, S) and (Q, T) are weakly commuting self-mappings of X. Then P, Q, T and S have a unique common fixed point in X as weak commutativity implies R-weak commutativity.*

CONCLUSION

In this work, we established a common fixed point theorem for R-weakly commuting mappings in intuitionistic fuzzy metric space which is the generalization the outcome of Turkoglu et al. [10] and other results which were proved in intuitionistic fuzzy metric spaces.

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