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COMPUTATIONAL ANALYSIS TO FIND SHEAR STRESS VALUES IN HORIZONTAL POROUS CHANNEL GEOMETRY BY USING THE NUMERICAL METHOD FEM

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ABSTRACT:

In this paper we analyzed the computational procedure to find Shear stress values by Mathematica 4.1 Software Commands in Horizontal Porous Channel Geometry by using the Numerical Method FEM.

KEY WORDS: Shear stress , Horizontal Porous Channel, FEM.

1. INTRODUCTION

Forced convection air-cooling is used if the heat flux from the component is more than 1000 W/m2. Even if the flow is a forced one, the effect of buoyancy is not negligible when the heat flux from the electronic chip is very high. Hence, the flow may be in the mixed convection regime. It is important to consider the effects of conjugateness and surface radiation when analyzing a mixed convection problem, to accurately predict the fluid flow and heat transfer characteristics, as for example the cooling of electronic components, when air is considered as a cooling medium. Degan et al [1] studied the forced convection in horizontal porous channels with hydrodynamic anisotropy. Dogan et al [2] studied the investigation of mixed convection heat transfer in a horizontal channel with discrete heat sources at the top and at the bottom.

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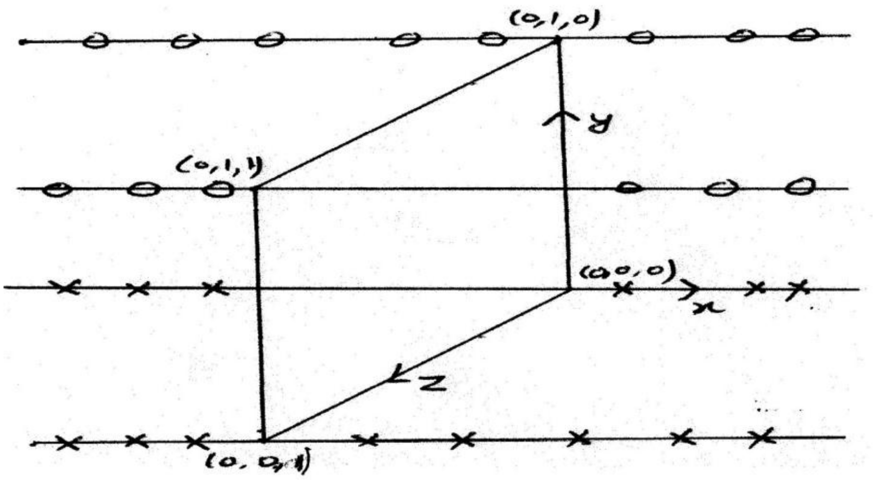

Fig.a Schematic diagram of the Horizontal Channel

2. FORMULATION OF THE PROBLEM

The equations governing the flow, heat, and mass transfer with Soret and dissipative effects are

$$\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial y^2} - \frac{u}{k} + v_o \frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{\partial p}{\partial x} \tag{1}$$

$$0 = \frac{\partial p}{\partial y} - \rho g \tag{2}$$

$$\rho_o C_p \left(u \frac{\partial T}{\partial z} - v_o \frac{\partial T}{\partial y}\right) = k_1 \left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{\rho_o v}{k}\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right] + \frac{\rho_o v}{k} u^2 \tag{3}$$

$$u \frac{\partial C}{\partial x} = D_1 \left(\frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right) + k_{21}\left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}\right) \tag{4}$$

$$\rho = \rho_0[1 - \beta(T - T_0) - \beta^*(C - C_0)] \tag{5}$$

The flow being unidirectional , in view of the equation of continuity $u = u(y, z)$.

The heat mass flux being constant along the channel

$$\frac{dT}{dx} = A\ , \ \frac{dC}{dx} = B \text{ on the wall } y = b$$

where A and B are the uniform temperature and concentration gradients respectively. Hence the temperature and the concentration in the flow field may chosen to be

$$T = A\,x + T_1(y, z)$$

$$C = B\,x + C_1(y,z).e$$

The boundary conditions are

$$u = 0 \quad \text{on} \quad y = b$$

$$T = T_1 \quad , C = C_1 \text{ on } y = b \quad \text{at the entry } x = 0 \text{ and} \tag{6}$$

$$T = Ax + T_1 \ , C = B\,x + C_1 \ \text{ on } \ y = b\ , x \neq 0$$

In view of the symmetry w.r.t. the central line $y = 0$

$$\frac{\partial u}{\partial y} = 0 \quad , \quad \frac{\partial T}{\partial y} = 0 \quad and \quad \frac{\partial C}{\partial y} = 0 \qquad on \qquad y = 0 \tag{7}$$

We introduce the following non-dimensional variables as follows.

$$z = z^* b\ ,\ y = y^* b\ , \qquad T = T_0 + \theta^*(T_1 - T_0)$$

$$u^* = \frac{\nu\ u}{\beta\ gb^2 (T_1 - T_0)} \quad , C = C_0 + C^*(C_1 - C_0)$$

Substituting these non-dimensional variables in equations (1) – (4) and making use of the Boussinesque approximation , the governing dimensionless equations on elimination of p reduce to (dropping the asterisk).

$$\frac{\partial}{\partial y}\left(\frac{\partial^2 u}{\partial z^2}\right) + \frac{\partial^3 u}{\partial y^3} - D^{-1}\frac{\partial u}{\partial y} + S\frac{\partial^2 u}{\partial y^2} = N_1 + N_2 \tag{8}$$

$$\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} + PS\frac{\partial \theta}{\partial y} - \alpha\theta + PEcG(u_y^2 + u_z^2) + PEcG\,D^{-1}u^2 = N_1 PGU \tag{9}$$

$$Sc\left(-S\frac{\partial C}{\partial y}\right) + N_2 u = \left(\frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right) + \frac{S_0 Sc}{N}\left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}\right) \tag{10}$$

The corresponding boundary conditions in the non-dimensional form are

$u = 0$ on $y = 1$ $\theta = 1$, $C = 1$ at $x = 0$ on $y = 1$ (11)

$\theta = 1+N_1 x$, $C = 1+ N_2 x$, $x \neq 0$ on $y = 1$ (12)

$\frac{\partial u}{\partial y} = 0$, $\frac{\partial \theta}{\partial y} = 0$ *and* $\frac{\partial C}{\partial y} = 0$ on $y=0$ (13)

In view of the two dimensionality and symmetry of the flow w.r.t. the midplane of the channel we analyse the flow features in a domain in the upper half of the channel bounded by the impermeable wall lying between two parallel planes normal to the wall at unit distance apart. The finite element analysis with quadratic approximation functions is carried out using eight noded serendipity elements.

3. FINITE ELEMENT ANALYSIS OF THE PROBLEM

we now make a finite element analysis of the given problem governed by

(8) , (9) & (10)

subject to the conditions (11) , (12) and (13) .

We get the following equations

$$\int_{\Omega_i} [\sum_{k=1}^{8} u_k^i \{\frac{\partial^2 N_k^i}{\partial y^2}\frac{\partial N_j^i}{\partial z} + \frac{\partial^2 N_k^i}{\partial z^2}\frac{\partial N_j^i}{\partial y} + D^{-1} N_k^i \frac{\partial N_j^i}{\partial y} - SN_j^i \frac{\partial^2 N_k^i}{\partial y^2}\} + (N_1 + N_2) N_j^i] d\Omega_i = Q_j^i \quad (14)$$

$$\int_{\Omega_i} [\sum_{k=1}^{8} \theta_k^i \{\frac{\partial N_k^i}{\partial y}\frac{\partial N_j^i}{\partial y} + \frac{\partial N_k^i}{\partial z}\frac{\partial N_j^i}{\partial z} + PSN_j^i \frac{\partial N_k^i}{\partial y} - \alpha N_j^i N_k^i\}$$

$$+ \int_{\Omega_i} \sum_{k=1}^{8} u_k^i \{PEcG((\frac{\partial N_k^i}{\partial y})^2 + (\frac{\partial N_k^i}{\partial z})^2) + D^{-1}(N_k^i)^2 - N_1 PGN_k^i\} N_j^i] d\Omega_i = (Q^T)_j^i \quad (15)$$

$$\int_{\Omega_i} [\sum_{k=1}^{8} C_k^i \{\frac{\partial N_k^i}{\partial y}\frac{\partial N_j^i}{\partial y} + \frac{\partial N_k^i}{\partial z}\frac{\partial N_j^i}{\partial z} + S S_c N_j^i \frac{\partial N_k^i}{\partial y}\} d\Omega_i + \frac{ScS_o}{N}\int_{\Omega_i} \sum_{k=1}^{8} \theta_k^i \{\frac{\partial N_k^i}{\partial y}\frac{\partial N_j^i}{\partial y} + \frac{\partial N_k^i}{\partial z}\frac{\partial N_j^i}{\partial z}\} d\Omega_i$$

$$- \int_{\Omega_i} S_c N_2 \sum_{k=1}^{8} u_k^i (N_j^i N_k^i) d\Omega = (Q^C)_{j_i}^i \quad (16)$$

where

$$Q_j^i = \oint_{\Gamma_i} [N_j^i \frac{\partial^2 u^i}{\partial y^2} n_y + N_j^i \frac{\partial u^i}{\partial y} n_y + N_j^i \frac{\partial u^i}{\partial z} n_z] d\Gamma_i$$

$$(Q^T)_j^i = \oint_{\Gamma_i} [N_j^i \frac{\partial \theta^i}{\partial y} n_y + N_j^i \frac{\partial \theta^i}{\partial z} n_z] \, d\Gamma_i \qquad j = 1, 2, \ldots\ldots, 8.$$

$$(Q^{C})^{i}_{j} = \oint_{\Gamma_i} [N^{i}_{j}\{N \frac{\partial C^{i}}{\partial y} + S_0 Sc \frac{\partial \theta^{i}}{\partial y}\} n_y + N^{i}_{j}\{N \frac{\partial C^{i}}{\partial z} + + S_0 Sc \frac{\partial \theta^{i}}{\partial z}\} n_z] \, d\Gamma_i \qquad j = 1, 2, ..., 8.$$

Choosing different N^{i}_{k} 's corresponding to each element e_i the equation (14),(15) and (16) results in twenty four equations for three sets of unknown

$$(u^{i}_{k}), (\theta^{i}_{k}) \text{ and } (C^{i}_{k}) \text{ ,viz}$$

$$(a^{i}_{k\,j})\ (u^{i}_{k}) = (Q^{i}_{j}) \tag{17}$$

$$(b^{i}_{kj})(\theta^{i}_{k}) + (d^{i}_{k\,j})(u^{i}_{k}) = (Q^{T}_{j})^{i} \tag{18}$$

$$(m^{i}_{k\,j})(C^{i}_{k}) + (l^{i}_{k\,j})(u^{i}_{k}) = (n^{i}_{k\,j})(\theta^{i}_{k}) + (Q^{C}_{j})^{i} \quad (j, k = 1,2,.............,8) \tag{19}$$

where $(a^{i}_{k\,j}), (b^{i}_{k\,j}), (d^{i}_{k\,j}), (m^{i}_{k\,j}), (n^{i}_{k\,j})$ *and* $(l^{i}_{k\,j})$ are 8 × 8 stiffness matrices and (Q^{i}_{j}), $(Q^{T}_{j})^{i}$ and $(Q^{C}_{j})^{i}$ are 8 × 1 column matrices. Repeating the process with each of mn elements and making use of global coordinates and inter element continuity conditions as well as the boundary conditions to assemble the element matrices, we obtain global matrices for the unknown u , θ and C at the respective global nodes which are ultimately determined on solving the matrix equation.

For computational purpose ,we choose a serendipity element with (0,0) , (0,1) , (1,0) and (1, 1) as its vertices. The eight nodes of the element are as shown in Fig .(b) and the quadratic interpolation function at these nodes are

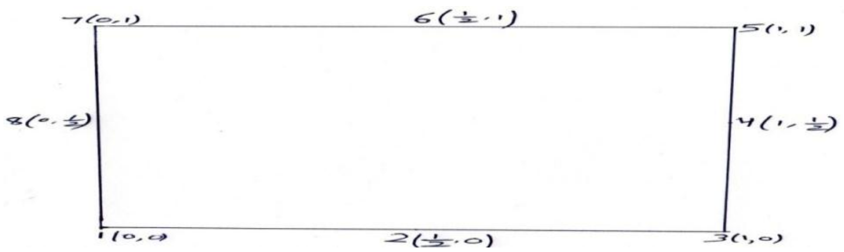

Fig. b Eight noded Rectangular Serendiptiy element

$N_1 = -2\,(y-1)(z-1)(z + y - ½)$; $N_2 = 4\,(z-1)(y-1)(z)$: $N_3 = -2\,(y-1)(z)(z - y - ½)$

$N_4 = -4\,(y-1)(z)(y)$; $N_5 = 2\,z\,y\,(z + y - 3/2)$; $N_6 = -4\,z\,y(z-1)$

$N_7 = 2\,y\,(z-1)(z - y + ½)$; $N_8 = 4\,y\,(z-1)(y-1)$

Substituting these shape functions in (17) and integrating over the element domain the matrix for the global nodes of u viz., U_i (i = 1 ,2…….,8) reduces to a 8×8 matrix equations and we write in the partitioned form.

The 8×8 matrix equation for θ_j (j = 1, 2 ,………..,8) and we write in the partitioned form.

Similarly the 8 × 8 matrices equations for C_j (j = 1, 2,……….,8) and we write in the partitioned form.

The boundary conditions (essential boundary conditions on the primary variables) are

$U_5 = U_6 = U_7 = 0$, $\theta_5 = \theta_6 = \theta_7 = 1$ and $C_5 = C_6 = C_7 = 1$ on y = 1

In view of the symmetry conditions we obtain.

$$Q_1 = Q_2 = Q_3 = Q_4 = Q_8 = 0 \quad Q_1^T = Q_2^T = Q_3^T = Q_4^T = Q_8^T = 0 \quad Q_1^C = Q_2^C = Q_3^C = Q_4^C = Q_8^C = 0 \Big| \quad on \quad y = 0$$

Solving the ultimate 8 × 8 matrix we determine the unknown global nodal values of U_i , θ_i and C_i (i = 1,2,…….,8).

The solution for u , θ and C may now be represented as

$$u = \sum_{i=1}^{8} U_i N_i, \qquad \theta = \sum_{i=1}^{8} \theta_i N_i, \qquad C = \sum_{i=1}^{8} C_i N_i$$

The rate of Shear stress in the non-dimensional form on the boundary is

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=1}$$

The Shear stress values evaluated computationally for different variations in the governing parameters.

4. NUMERICAL COMPUTATION

To find out Shear stress values using mathematica 4.1 software commands are given below

Shape functions

$$s_1 := -4X(-1+y)X\left(\frac{y}{2}+\frac{1}{2}\left(-\frac{1}{2}+z\right)\right)X(-1+z) \qquad s_2 := 4X(-1+y)X(-1+z)Xz$$

$$s_3 := -4X(-1+y)X\left(-\frac{y}{2}+\frac{1}{2}\left(-\frac{1}{2}+z\right)\right)Xz \qquad s_4 := -4X(-1+y)XyXz$$

$s_5 := 4XyX(\frac{1}{2}(-\frac{1}{2}+y)+\frac{1}{2}(-1+z)Xz \qquad s_6 := -4XyX(-1+z)Xz \qquad s_7 := -4XyX(\frac{1}{2}(-1+y)+\frac{1}{2}(\frac{1}{2}-z))X(-1+z)$

$s_8 := 4X(-1+y)XyX(-1+z)$

FEM equation for Momentum

$$Do[m_i = \sum_{j=1}^{8} u_j X \int_0^1\int_0^1 (\partial_y s_i X \partial_{y,y} s_j + \partial_z s_i X \partial_{z,z} s_j + (ad + M \wedge 2) X s_i X \partial_y s_j - SX s_i X \partial_{y,y} s_j) dydz /.$$

$$u_5 -> 0 /. u_6 -> 0 /. u_7 -> 0 + (n_1 + n_2) X \int_0^1\int_0^1 (s_i) dydz, \{i,1,8\}]$$

Next we should find Coefficient matrix for momentum equation by executing the below command

$Table[Coefficient[m_i, u_j], \{i,1,8\}, \{j,1,8\}]$

Equate this coefficient to M and execute this command

ReplacePart[M,-1,{{5,5},{6,6},{7,7}}] for getting non singular matrix and this matrix we can take as coefficient matrix

Next we should find constant matrix for momentum equation by executing below two commands

$Do[u_i = 0, \{i,1,8\}]; \qquad Table[m_i, \{i,1,8\}]$

FEM equation for temperature

$$Do[t_i = \sum_{j=1}^{8} \theta_j X \int_0^1\int_0^1 (\partial_y s_i X \partial_y s_j + \partial_z s_i X \partial_z s_j - pXsX s_i + (ad + M \wedge 2) X s_i X \partial_y s_j + \alpha X s_i X s_j) dydz -$$

$$\sum_{j=1}^{8} u_j X \int_0^1\int_0^1 (pXkXGX s_i X((\partial_y s_j) \wedge 2 + (\partial_z s_j) \wedge 2) - pXkXGX(ad + M \wedge 2) X s_i X((s_j) \wedge 2) +$$

$$n_1 XpXGX s_i X s_j) dydz /. u_5 -> 0 /. u_6 -> 0 /. u_7 -> 0 /. \theta_5 -> 1 /. \theta_6 -> 1 /. \theta_7 -> 1, \{i,1,8\}];$$

Next we should find Coefficient matrix for temperature equation by executing the below command

$Table[Coefficient[t_i, \theta_j], \{i,1,8\}, \{j,1,8\}]$

Equate this coefficient to M and execute this command

ReplacePart[M,-1,{{5,5},{6,6},{7,7}}] for getting non singular matrix and this matrix we can take as coefficient matrix

Next we should find constant matrix for temperature equation by executing below two commands

$$Do[\theta_i = 0,\{i,1,8\}]; \quad Table[t_i,\{i,1,8\}]$$

FEM equation for Concentration

$$Do[con_i = \sum_{j=1}^{8}\int_0^1\int_0^1 (\partial_y s_i X \partial_y s_j + \partial_z s_j) X c_j + (\frac{s_o X s_c}{an}) X (-pXsX s_i + (ad + M \wedge 2) X s_i X \partial_y s_j + \alpha X s_i X s_j) dydz - (\partial_y s_i X \partial_y s_j + \partial_z s_j) X \theta_j - ((ScXsX s_i X \partial_y s_j) X c_j) + (ScX n_2 X s_i X s_j) X u_j) dydz /. u_5 -> 0 /. u_6 -> 0 /. u_7 -> 0 /. \theta_5 -> 1 /. \theta_6 -> 1 /. \theta_7 -> 1 /. c_5 -> 1 /. c_6 -> 1 /. c_7 -> 1, \{i, 1, 8\}]$$

Next we should find Coefficient matrix for diffusion equation by executing the below command

$$Table[Coefficient[con_i, c_j], \{i, 1, 8\}, \{j, 1, 8\}]$$

Equate this coefficient to M and execute this command

ReplacePart[M,-1,{{5,5},{6,6},{7,7}}] for getting non singular matrix and this matrix we can take as coefficient matrix

Next we should find constant matrix for concentration equation by executing below two commands

$$Do[c_i = 0,\{i,1,8\}]; \quad Table[con_i,\{i,1,8\}]$$

We get Coefficient Matrices mom, T, on and Constant matrices A,B,F

By Serendipity element the velocity, temperature and concentration equations are given below

$$vel = u_1 X s_1 + u_2 X s_2 + u_3 X s_3 + u_4 X s_4 + u_8 X s_8;$$

$$temp = \theta_1 X s_1 + \theta_2 X s_2 + \theta_3 X s_3 + \theta_4 X s_4 + \theta_8 X s_8 + s_5 + s_6 + s_7;$$

$$concen = c_1 X s_1 + c_2 X s_2 + c_3 X s_3 + c_4 X s_4 + c_8 X s_8 + s_5 + s_6 + s_7;$$

$$Shearstress := -4(\frac{1}{2} + \frac{1}{2}(-\frac{1}{2} + z))(-1 + z)\, u_1 + 4(-1 + z) z\, u_2 - 4(-\frac{1}{2} + \frac{1}{2}(-\frac{1}{2} + z)) z\, u_3 - 4z\, u_4 + 4(-1 + z)\, u_8$$

Iterations for finding Shear stress values

$ad = 5*10^2; G = 3*10^3 : S = 0.2; M = 5 : n_1 = 0.5, n_2 = 1.5; an = 1; p = 0.7; k = 0.3; Sc = 1.3; s_o = 0.5; \alpha = 2; z = 0.5; x = 1;$

$\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\} = Inverse[mom].A;$ $\{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8\} = Inverse[T].B;$

$\{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8\} = Inverse[on].F;$

$TableForm[Table[Shearstress]]$

$ad = 10^3; G = 3*10^3 : S = 0.2; M = 5 : n_1 = 0.5, n_2 = 1.5; an = 1; p = 0.7; k = 0.3; Sc = 1.3; s_o = 0.5; \alpha = 2; z = 0.5; x = 1;$

$\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\} = Inverse[mom].A;$ $\{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8\} = Inverse[T].B;$

$\{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8\} = Inverse[on].F;$

$TableForm[Table[Shearstress]]$

$ad = 2*10^3; G = 3*10^3 : S = 0.2; M = 5 : n_1 = 0.5, n_2 = 1.5; an = 1; p = 0.7; k = 0.3; Sc = 1.3; s_o = 0.5; \alpha = 2; z = 0.5; x = 1;$

$\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\} = Inverse[mom].A;$ $\{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8\} = Inverse[T].B;$

$\{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8\} = Inverse[on].F;$

$TableForm[Table[Shearstress]]$

$ad = 3*10^3; G = 3*10^3 : S = 0.2; M = 5 : n_1 = 0.5, n_2 = 1.5; an = 1; p = 0.7; k = 0.3; Sc = 1.3; s_o = 0.5; \alpha = 2; z = 0.5; x = 1;$

$\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\} = Inverse[mom].A;$ $\{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8\} = Inverse[T].B;$

$\{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8\} = Inverse[on].F;$

$TableForm[Table[Shearstress]]$

Output

-0.672405

-0.646892

-0.633761

-0.629326

We should calculate Shear stress values at z=0, 0.5 and 1 levels. At every iteration changing one parameter and remaining other parameter values fixed we get all parameter values at z=0, 0.5, and 1 levels.

5. DISCUSSION OF THE NUMERICAL RESULTS

The flow is unidirectional and the behaviour of the shear stress is discussed for the Darcy parameter D^{-1},s,N_1&N_2. The pressure gradient is chosen positive so that negative u indicates the actual flow while positive u corresponds to reversal flow.

The variation of u in the horizontal level y = 0 w.r.t D^{-1} show that u changes from positive to negative as we move along the horizontal level ($0 \le z \le 1$), there by indicating the reversal flow in the unidirection and near the boundary z = 1 and the region of reversal flow enlarges in the mid-region and shrinks in the vicinity of the boundary for an increase in the porous permeability parameter D^{-1}. For an increase in D^{-1} the fluid moves with faster velocity in the mid-region and lesser velocity near the boundary with maximum u attained at z = 0 (fig. 1(a)). At the higher horizontal levels y = 1/2 we notice a reversal flow in the mid-region and in the vicinity of the boundary. The region of reversal flow at y = 1/2 level is more than that at y = 0 level. Also the axial velocity u increases with D^{-1} everywhere in the fluid region except in the vicinity of the boundary where we notice an inhibition in u (fig 1(b)). The variation of u at the different vertical levels z = 0, ½ & 1 is shown in (fig.2(a)-fig.3(a)). These profiles are asymmetric curves exhibiting a reversal flow in the entire fluid region at z = 0 & z = 1 levels and at z = 1/2 level the reversal flow is noticed in a narrow region adjacent to y = 1. The axial velocity falls gradually from maximum at y = 0 to attain the prescribed value zero at y = 1. An increase in D^{-1} increases u at z = 0 level and decreases it at higher vertical levels z = 1/2 & 1. Thus lesser the permeability of the porous medium larger the magnitude of u at z = 0 level and smaller $|u|$ at z = ½ and 1 levels. (fig.2(a)-fig.3(a)).

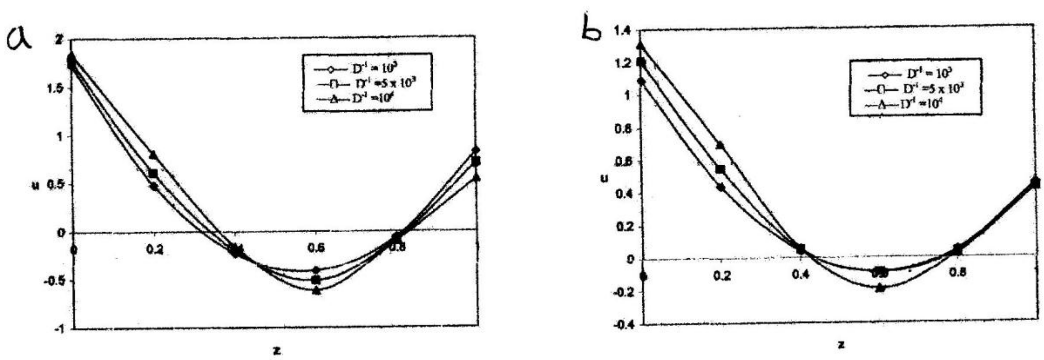

Fig. 1 Variation of u with D^{-1} at y = 0 level. G = 10^3 , N = 1, Ec = 0.1, Sc = 1.3 ,So = 0.5, □□□S = 0.3

Fig. 2Variation of u with D^{-1} at y = 0.5 level.G=10^3 , N=1, Ec = 0.1, Sc = 1.3, So = 0.5, □□□ S = 0.3

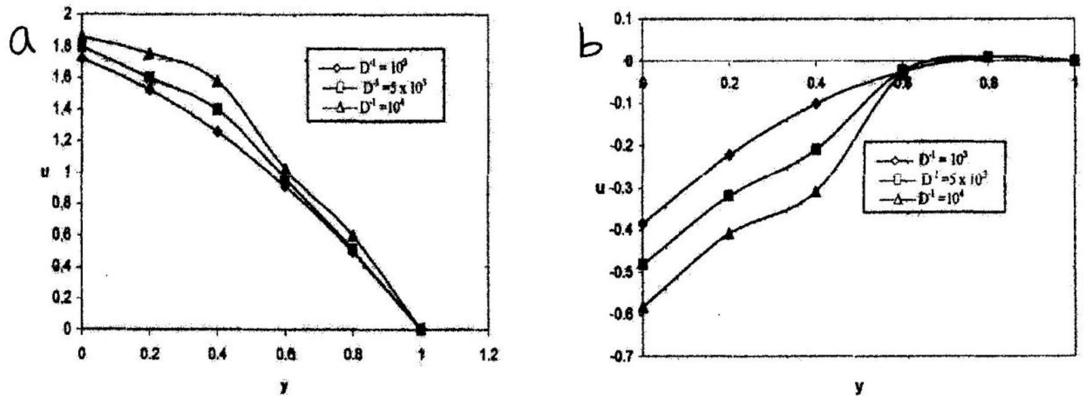

Fig. 3 Variation of u with D^{-1} at z = 0 level. G = 10^3 , N = 1, Ec = 0.1, Sc 1.3 ,So = 0.5, □□□S = 0.3

Fig. 4 Variation of u with D^{-1} at z = 0.5 level. G = 10^3 , N = 1, Ec = 0.1, Sc 1.3 ,So = 0.5, □□□S = 0.3

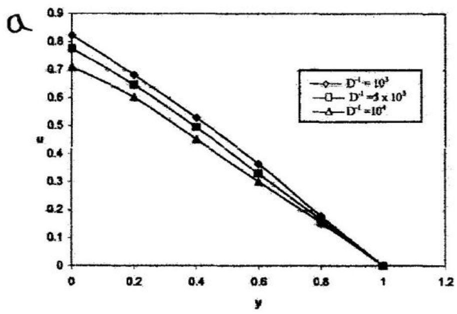

Fig. 5 Variation of u with D^{-1} at Z = 1 level. G = 10^3 , N = 1, Ec = 0.1, Sc 1.3 ,So = 0.5, □□□S = 0.3

The shear stress (τ) at the boundary y = 1 at different levels z = 0 , ½ & 1 is shown in tables 1-3 for variation in the governing parameters D^{-1}, S, N_1 and N_2 . We find that the shear stress at z = 0 , ½ and 1levels is negative. We find that at all levels the shear stress increases with increase in D^{-1} or S. Also an increase in the temperature/Concentation gradient decreases τ at all levels (tables 1-3).

Table – 1 Shear Stress (τ) at Z = 0 Level

G = 3 x 10^3, N = 1, Ec = 0.3, Sc = 1.3, So = 0.5, □ = 2,p=0.7,x=1

D^{-1}	I	II	III	IV	V
5 x 10^2	-1.05364	-1.05269	-1.05174	-1.93168	-1.75607
10^3	-1.06279	-1.0623	-1.06181	-1.94845	-1.77132
2 x 10^3	-1.06747	-1.06722	-1.06697	-1.95703	-1.77912
3 x 10^3	-1.06905	-1.06888	-1.06871	-1.95992	-1.78175

	I	II	III	IV	V
S	0.2	0.4	0.6	0.2	0.2
N_1	0.5	0.5	0.5	1.5	0.5
N_2	0.7	0.7	0.7	0.7	1.5

Table – 2 Shear Stress (τ) at Z = 0.5 Level

G = 3 x 10^3, N = 1, Ec = 0.3, Sc = 1.3, So = 0.5, □ = 2,p=0.7,x=1

D^{-1}	I	II	III	IV	V
5 x 10^2	-0.0616375	-0.0621889	-0.0627408	-0.113002	-0.102729
10^3	-0.0627398	-0.063024	-0.0633084	-0.115023	-0.104566
2 x 10^3	-0.0633083	-0.0634527	-0.0635971	-0.116065	-0.105514
3 x 10^3	-0.0635005	-0.0635972	-0.063694	-0.116418	-0.105834

	I	II	III	IV	V
S	0.2	0.4	0.6	0.2	0.2
N_1	0.5	0.5	0.5	1.5	0.5
N_2	0.7	0.7	0.7	0.7	1.5

Table – 3 Shear Stress (τ) at Z = 1 Level

G = 3 x 10^3, N = 1, Ec = 0.3, Sc = 1.3, So = 0.5, □ = 2,p=0.7,x=1

D^{-1}	I	II	III	IV	V
5 x 10^2	-0.403443	-0.403593	-0.403745	-0.740199	-0.672405
10^3	-0.388135	-0.388219	-0.388303	-0.711889	-0.646892
2 x 10^3	-0.380256	-0.380301	-0.380345	-0.697299	-0.633761
3 x 10^3	-0.377596	-0.377626	-0.377656	-0.692369	-0.629326

	I	II	III	IV	V
S	0.2	0.4	0.6	0.2	0.2
N_1	0.5	0.5	0.5	1.5	0.5
N_2	0.7	0.7	0.7	0.7	1.5

6. CONCLUSIONS

1. By using Mathematica 4.1 commands we executed and calculated Shear stress values for

different parameters.

2. We observe that for an increase in D^{-1} the fluid moves with faster velocity in the mid-region

and lesser velocity near the boundary with maximum u attained at z = 0.

3. We find that the shear stress at z = 0 , ½ and 1 levels is negative.

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