

CONSTRUCTION OF STOCHASTIC MODEL FOR TIME TO DENGUE VIRUS TRANSMISSION WITH EXPONENTIAL DISTRIBUTION

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ABSTRACT

In this paper deals with the study of a stochastic model for predicting the time to Dengue virus transmission. As the immune capacities of an individual differ and also have its personal resistance, the antigenic diversity threshold is dissimilar for different person. We construct a stochastic model to study the damage process acting on the immune system that is non-linear. The mean time to Dengue virus transmission and its variance are derived with numerical example.

Keywords: Antigenic diversity threshold, Alpha-Poisson process, Mittag-Laffler distribution,

Dengue, Time to Dengue virus transmission.

1. Introduction

Dengue Fever (DF) and Dengue Hemorrhagic Fever (DHF), collectively known as "dengue," are mosquito-borne, potentially mortal, flu-like viral diseases that affect humans worldwide. Transmitted to humans by the bite of an infected mosquito *Aedes aegypti*, dengue is caused by any one of four serotypes are termed as DENV-1, DENV-2, DENV-3, and DENV-4, or antigen-specific viruses; dengue virus is part of the Flaviviridae family. Dengue is one of the most rapidly spreading mosquito-borne

viral diseases in the world and inflicts significant health, economic and social burdens on populations. The severe form of dengue is mediated by an increase in capillary permeability that can cause severe bleeding (e.g. Gastrointestinal bleeding) and plasma leakage resulting in ascites and pleural effusions. Dengue disease is a mosquito-borne condition that has become a major public health concern. Dengue severity can be classified into mild Dengue fever (DF) and severe Dengue or Dengue hemorrhagic fever (DHF).

Mathematical and statistical models describing the transmission of dengue viruses appeared in the study of observations related to pathogenesis of dengue hemorrhagic fever (Fischer, 1970) and providing a better understanding of the nature and dynamics of the transmission of dengue infection, as well as predict outbreaks and simulate the impact of control strategies in disease transmission (Rico-Hesse, 2010). The time to Dengue virus transmission by biting mosquito alone is the only mode of Dengue transmission. The bites of mosquitoes are assumed between a seropositive person who is labeled as index case and seropositive state takes place over an incubation period due to the contraction of Dengue to the partner from the index case by the bites of mosquitoes. In this research paper constructing a stochastic model to learn the damage process performing on the immune system that is non-linear and also mean and variance of time to Dengue virus transmission are derived with numerical example.

2. Objectives

Because of hardy associated with bites of mosquitoes, this work concentrates primarily on estimating the dynamics of the Dengue transmission by the bites of mosquitoes, as more than 95% of Dengue transmission accounts for bites of mosquitoes.

The objectives are:

- To realize the dynamics of Dengue in an infected individual so that control measures can be adopted in order to slow down the severity of the disease
- To learn the transmission dynamics of Dengue infection in a susceptible person so that the efficient strategies for controlling the spread of the epidemic can be implemented
- To categorize the statistical pattern of recognition of the time to Dengue virus transmission
- To derive the distribution of the time to Dengue virus transmission
- To grant the nature and extent of uncertainty in time to Dengue virus transmission with respect to the viral load, bite rate and antigenic diversity threshold

3. Assumptions and Notations

- Bites of mosquito are the only source of Dengue transmission.
- Damages to individuals are caused by transmission of Dengue at each bite and the inter arrival time between the bites are independent, identically distributed random variables.
- The damage process acting on the immune system of an infected individual is non-linear and cumulative.
- The total damage caused exceeds a threshold level, which itself is a random variable, the time to Dengue virus transmission occurs and the person is recognized as infected.
- The process that generates the bites, the sequence of damages and threshold are mutually independent.

and the model parameters are:

- Bites rate of the infected person (a)
- Intensity of the Dengue of the infected person (β)
- Antigenic diversity threshold (λ)

4. Stochastic model for time to Dengue virus transmission with one parameter

Exponential distribution

4.1 Distribution of time to Dengue virus transmission

Let us consider a susceptible population whose major mode of transmission is through bites of mosquitoes. Assume that at time $t=0$, a new member tested Dengue negative enters the population and makes bites of mosquito with members of the susceptible population.

Let the bites of mosquito occur at random time points which is assumed to follow the Alpha-Poisson distribution (2001) with parameters 'a' and ' β ' which is given as

$$p_{a,\beta}(n,t) = \sum_{k=0}^{\infty} (-1)^k \binom{k+n}{k} \frac{(at)^{\beta(k+n)}}{\Gamma(\beta(k+n)+1)}, \quad a > 0, 0 < \beta \leq 1, n = 0, 1, 2, \dots$$

$$= \frac{e^{(-at)\beta} (at)^{\beta k}}{\Gamma(\beta k + 1)}, \quad a > 0, t > 0, 0 < \beta \leq 1,$$

Where a = No. of bites , β = intensity of Dengue infected person

Let $G(t)$ be the distribution function of the inter bite between the bites which follows Mittag-Leffler distribution. The distribution function of Mittag-Leffler distribution (2001) is given by

$$G_{a,\beta}(t) = \sum_{k=1}^{\infty} \frac{(-1)^{(k-1)}}{\Gamma(\beta k + 1)} (at)^{\beta k}, t \geq 0, a > 0 \text{ and } 0 < \beta \leq 1$$

Let the time to Dengue virus transmission of the individual be represented by the random variable. We obtain the distribution of time to Dengue virus transmission by a stochastic model based on the assumptions with linear damage process acting on the immune system, we have the following theorem.

If the number of bites is an Alpha-Poisson process with parameters ‘ a ’ and ‘ β ’ and inter bite time is a Mittag-Leffler distribution while the threshold level is an exponential distribution with parameter ‘ λ ’, then the probability density function of time to Dengue virus transmission is a three parameter Weibull distribution.

The density function of the distribution of time to Dengue virus transmission:

Consider $S(t) = P\{\text{no infection in } (0, t)\}$

$$\begin{aligned} &= P\{T > t\} \\ &= \sum_{k=1}^{\infty} P\{\text{Number of Seroconversion before } t \text{ given exactly } k \text{ bite in } (0, t) \\ &\quad \text{with intensity } \beta\} \times P\{\text{exactly } k \text{ bites in } (0, t) \text{ with intensity } \beta\} \\ &= \left\{ \sum_{k=1}^{\infty} V_k(t) \right\} \left\{ \sum_{k=1}^{\infty} X_i < Y \right\} \end{aligned}$$

where $V_k(t) =$ Probability of k bites in $(0, t)$ with intensity β

(i.e., the Alpha Poisson distribution with parameter ‘ a ’ and ‘ β ’)

$$= \frac{(at)^{\beta k} e^{-(at)^\beta}}{\Gamma(\beta k + 1)}, a > 0, 0 < \beta < 1, k = 1, 2, 3, \dots$$

$$p(x) = \lambda e^{-\lambda x}$$

$$P(X < Y) = \int_{\beta}^{\infty} G(x) \lambda e^{-\lambda x} dx = \lambda G^*(\lambda) = \lambda \frac{g^*(\lambda)}{\lambda} = g^*(\lambda), \text{ where } G^*(\lambda) = \int_{\beta}^{\infty} G(x) \lambda e^{-\lambda x} dx$$

$$P(X < Y) = g^*(\lambda)$$

$$P\{X_1 + X_2 + \dots + X_k < Y\} = \int_0^{\infty} g_k(x) e^{-\lambda x} dx = g^*(\lambda) \times g^*(\lambda) \times g^*(\lambda) \times \dots \times g^*(\lambda) = [g^*(\lambda)]^k$$

where $g^*(\lambda)$ is the Laplace Transformation of $g(x)$ and $g_x(x) = p.d.f.$ of $\sum_{k=0}^{\infty} X_i$

$$S(t) = \left\{ \sum_{k=0}^{\infty} V_k(t) \right\} [g^*(\lambda)]^k$$

$$= e^{-(at)^\beta} \left\{ \frac{(at)^0 [g^*(\lambda)]^0}{\Gamma 1} + \frac{(at)^\beta [g^*(\lambda)]^1}{\Gamma 2} + \frac{(at)^{2\beta} [g^*(\lambda)]^2}{\Gamma 3} + \dots + \frac{(at)^{k\beta} [g^*(\lambda)]^k}{\Gamma(k\beta + 1)} \right\}$$

since $0 < \beta < 1$, hence $\beta = 1$

$$= e^{-(at)^\beta} \left\{ \frac{(at)^{0\beta} [g^*(\lambda)]^{0\beta}}{\Gamma 1} + \frac{(at)^\beta [g^*(\lambda)]^\beta}{\Gamma 2} + \frac{(at)^{2\beta} [g^*(\lambda)]^{2\beta}}{\Gamma 3} + \dots + \frac{(at)^{k\beta} [g^*(\lambda)]^{k\beta}}{\Gamma(k + 1)} \right\}$$

$$S(t) = e^{-(at)^\beta [1-g^*(\lambda)]}$$

$$L(t) = 1 - S(t) \text{ is called Prevalence function} = 1 - e^{-(at)^\beta [1-g^*(\lambda)]}$$

The probability density function of time to Dengue virus transmission is

$$f(t) = \frac{d}{dt} L(t)$$

$$f(t) = \frac{d}{dt} \left[1 - e^{-(at)^\beta [1-g^*(\lambda)]} \right] = 0 + \beta a^\beta t^{\beta-1} [1-g^*(\lambda)] e^{-(at)^\beta [1-g^*(\lambda)]}$$

The probability density function of time to Dengue virus transmission t is

$$f(t) = \begin{cases} \beta a^\beta t^{\beta-1} [1-g^*(\lambda)] e^{-(at)^\beta [1-g^*(\lambda)]}, & t > 0, \lambda > 0, a > 0, 0 < \beta < 1 \\ 0, & \text{Otherwise} \end{cases}$$

Using Mittag Leffler distribution

$$g^*(\lambda) = \frac{a^\beta}{a^\beta + \lambda^\beta} \Rightarrow 1 - g^*(\lambda) = 1 - \frac{a^\beta}{a^\beta + \lambda^\beta} \Rightarrow 1 - g^*(\lambda) = \frac{\lambda^\beta}{a^\beta + \lambda^\beta}$$

The above function is in the form of three parameter Weibull distribution. Furthermore, we noticed that if the number of bites is a Alpha-Poisson process with parameters 'a' and 'β' the inter bite time is a Mittag-Leffler distribution while the threshold level is also an exponential distribution with parameter (λ), then the probability density function of time to Dengue virus transmission is a three parameter Weibull distribution.

4.2 Probability of time to Dengue virus transmission

The probability of time to Dengue virus transmission is calculated for various intervals by defining

$$p_i = \int_{t_i}^{t_{i+1}} f(t)dt \text{ for } i = 1, 2, 3, \dots$$

$$\text{where } f(t) = \frac{\lambda^\beta a^\beta \beta t^{\beta-1}}{a^\beta + \lambda^\beta} e^{-\left[\frac{\lambda^\beta a^\beta t^\beta}{a^\beta + \lambda^\beta}\right]}, t > 0, \lambda > 0, a > 0 \text{ and } 0 < \beta < 1$$

$$p_i = \int_{t_i}^{t_{i+1}} \frac{\lambda^\beta a^\beta \beta t^{\beta-1}}{a^\beta + \lambda^\beta} e^{-\left[\frac{\lambda^\beta a^\beta t^\beta}{a^\beta + \lambda^\beta}\right]} dt$$

$$= \frac{\lambda^\beta a^\beta}{a^\beta + \lambda^\beta} \int_{t_i}^{t_{i+1}} \beta t^{\beta-1} e^{-\left[\frac{\lambda^\beta a^\beta t^\beta}{a^\beta + \lambda^\beta}\right]} dt \text{ for } i = 1, 2, 3, \dots$$

$$\text{Let } z = \frac{\lambda^\beta a^\beta t^\beta}{a^\beta + \lambda^\beta} \text{ then } dz = \frac{\beta \lambda^\beta a^\beta t^{\beta-1}}{a^\beta + \lambda^\beta} dt \text{ and } \frac{dz}{\frac{\lambda^\beta a^\beta}{a^\beta + \lambda^\beta}} = \beta t^{\beta-1} dt$$

$$\text{where } C = \frac{\lambda^\beta a^\beta}{a^\beta + \lambda^\beta} \text{ then } \frac{dz}{C} = \beta t^{\beta-1} dt, \text{ limit } \begin{bmatrix} t \rightarrow t_i & t_{i+1} \\ z \rightarrow Ct_i^\beta & Ct_{i+1}^\beta \end{bmatrix}$$

$$p_i = \int_{Ct_i^\beta}^{Ct_{i+1}^\beta} e^{-z} dz = \left[\frac{e^{-z}}{-1} \right]_{Ct_i^\beta}^{Ct_{i+1}^\beta} \text{ for } i = 1, 2, 3, \dots$$

The probability of time to Dengue virus transmission is calculated for various intervals is given as

$$p_i = \left[e^{-Ct_i^\beta} - e^{-Ct_{i+1}^\beta} \right] = \left[e^{-\left[\frac{\lambda^\beta a^\beta}{a^\beta + \lambda^\beta}\right] t_i^\beta} - e^{-\left[\frac{\lambda^\beta a^\beta}{a^\beta + \lambda^\beta}\right] t_{i+1}^\beta} \right]$$

where t_i and t_{i+1} has one width of class interval width

4.3 Performance of measures of time to Dengue virus transmission

The expected time to Dengue virus transmission and its variance are obtained below:

$$\text{Mean} = E(T) = \mu_1 = \int_0^{\infty} t f(t) dt$$

$$\mu_1 = \int_0^{\infty} t \frac{\lambda^\beta a^\beta \beta t^{\beta-1}}{a^\beta + \lambda^\beta} e^{-\left[\frac{\lambda^\beta a^\beta t^\beta}{a^\beta + \lambda^\beta}\right]} dt \Rightarrow \mu_1 = \frac{\lambda^\beta a^\beta \beta}{a^\beta + \lambda^\beta} \int_0^{\infty} t^\beta e^{-\left[\frac{\lambda^\beta a^\beta t^\beta}{a^\beta + \lambda^\beta}\right]} dt$$

$$\text{Let } y = \frac{\lambda^\beta a^\beta t^\beta}{a^\beta + \lambda^\beta} \text{ where } C = \frac{\lambda^\beta a^\beta}{a^\beta + \lambda^\beta} \Rightarrow y = Ct^\beta \Rightarrow \frac{y}{C} = t^\beta$$

$$\frac{dy}{C} = \beta t^{\beta-1} dt \Rightarrow \frac{dy}{C} = \beta \frac{t^\beta}{t} dt \Rightarrow t \frac{dy}{C} = \beta t^\beta dt \Rightarrow \left(\frac{y}{C}\right)^{\left(\frac{1}{\beta}\right)} = \beta t^\beta dt, \text{ limit } \begin{cases} t=0 \rightarrow y=0 \\ t=\infty \rightarrow y=\infty \end{cases}$$

$$\mu_1 = \frac{1}{C^{\left(\frac{1}{\beta}\right)}} \Gamma\left(\frac{1}{\beta} + 1\right)$$

$$E(T^2) = \mu_2 = \int_0^{\infty} t^2 f(t) dt$$

$$\mu_2 = \int_0^{\infty} t^2 \frac{\lambda^\beta a^\beta \beta t^{\beta-1}}{a^\beta + \lambda^\beta} e^{-\left[\frac{\lambda^\beta a^\beta t^\beta}{a^\beta + \lambda^\beta}\right]} dt = \frac{\lambda^\beta a^\beta \beta}{a^\beta + \lambda^\beta} \int_0^{\infty} t^2 t^{\beta-1} e^{-\left[\frac{\lambda^\beta a^\beta t^\beta}{a^\beta + \lambda^\beta}\right]} dt$$

$$\mu_2 = \frac{\lambda^\beta a^\beta \beta}{a^\beta + \lambda^\beta} \int_0^{\infty} t^{\beta+1} e^{-\left[\frac{\lambda^\beta a^\beta t^\beta}{a^\beta + \lambda^\beta}\right]} dt$$

$$\text{Let } y = \frac{\lambda^\beta a^\beta t^\beta}{a^\beta + \lambda^\beta} \text{ where } C = \frac{\lambda^\beta a^\beta}{a^\beta + \lambda^\beta} \Rightarrow y = Ct^\beta \Rightarrow \left(\frac{y}{C}\right)^{\left(\frac{1}{\beta}\right)} = t \Rightarrow \left(\frac{y}{C}\right)^{\left(\frac{2}{\beta}\right)} = t^2$$

$$\frac{dy}{C} = \beta t^{\beta-1} dt \Rightarrow \frac{dy}{C} = \beta \frac{t^\beta}{t} dt \Rightarrow t \frac{dy}{C} = \beta t^\beta dt \Rightarrow \left(\frac{y}{C}\right)^{\left(\frac{2}{\beta}\right)} \frac{dy}{C} = \beta t^{\beta+1} dt, \text{ limit } \begin{cases} t=0 \rightarrow y=0 \\ t=\infty \rightarrow y=\infty \end{cases}$$

$$\mu_2 = C \int_0^{\infty} e^{-\lambda \left(\frac{y}{C}\right)^{\left(\frac{2}{\beta}\right)}} \frac{dy}{C}$$

$$= \frac{C}{C^{\left(\frac{2}{\beta}\right)+1}} \int_0^{\infty} e^{-\lambda y^{\left(\frac{2}{\beta}\right)}} dy \Rightarrow \mu_2 = \frac{1}{C^{\left(\frac{2}{\beta}\right)}} \Gamma\left(\frac{2}{\beta} + 1\right)$$

Variance of time to Dengue virus transmission is

$$V(T) = E(T^2) - [E(T)]^2$$

$$V(T) = \mu_2 = \frac{1}{C^{\left(\frac{2}{\beta}\right)}} \left\{ \Gamma\left(\frac{2}{\beta} + 1\right) - \left[\frac{1}{C^{\left(\frac{1}{\beta}\right)}} \Gamma\left(\frac{1}{\beta} + 1\right) \right]^2 \right\}$$

Particular case if $\beta = 1$ then

$$E(T) = \mu_1 = \frac{1}{C^{\left(\frac{1}{\beta}\right)}} \Gamma\left(\frac{1}{\beta} + 1\right) = \frac{a^\beta + \lambda^\beta}{a^\beta \lambda^\beta}$$

$$V(T) = \mu_2 = \frac{1}{C^{\left(\frac{2}{\beta}\right)}} \left\{ \Gamma\left(\frac{2}{\beta} + 1\right) - \left[\Gamma\left(\frac{1}{\beta} + 1\right) \right]^2 \right\}$$

$$= \frac{1}{C^2} \left\{ \Gamma(2+1) - [\Gamma(1+1)]^2 \right\}$$

$$V(T) = \left(\frac{a^\beta + \lambda^\beta}{a^\beta \lambda^\beta} \right)^2$$

5. Conclusion

In the study of Dengue epidemic the seroconversion time of Dengue transmission is an inevitable component. Since the bite rate is non observable in most cases one mosquito would expect that the spread of Dengue would have an impact on the human life. In this research article we constructed a stochastic model for time to Dengue transmission and it is noted that mean time to Dengue transmission to is less than the variance of time to Dengue transmission.

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