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MINIMIZING THE MAKESPAN AND RESOURCE IDLENESS FOR OSSP WITH RELEASE DATES

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ABSTRACT

Scheduling has its origin in manufacturing industries. Scheduling is a decision making process which is used on regular basis in many manufacturing industries. In this paper we are considering the open shop scheduling problem with release dates for the objective of minimizing makespan and resource idleness of machines, when pre-emption is not allowed. In [15], we developed an algorithm for the OSSP problem with release dates to minimize makespan and now we tested weather the same algorithm performs well for the objective of resource idleness of the machines also. Computations was made in two cases namely, number of jobs greater than number of machines and number of jobs equal to number of machines. In both the cases, it was found that the proposed algorithm performs better than the existing algorithm with respect to makespan and resource idleness.

Key Words: Heuristic, Open Shop Scheduling Problem, Makespan, Resource idleness.

1. INTRODUCTION

Shop scheduling problems are widely used in the modelling of industrial production process and are receiving an increasing amount of attention from researchers.

Scheduling is the allocation of resources over a period of time to perform a collection of tasks. Scheduling problem exists almost everywhere in real industrial world situations. In the theory of scheduling, there are three basic types, namely Flow-shop, Job-shop and Open-shop scheduling problems. If the jobs have different process sequence, the problem is known as Job-shop scheduling problem. If the jobs have same process sequence, the problem is known as Flow-shop scheduling problem. In Open shop scheduling problem (OSSP), jobs have no predetermined processing sequence. i.e., jobs can be processed in any conceivable order. The OSSP is similar to the job shop scheduling problem with the exemption that there are no precedence relations between the operations of each job. The OSSP has considerably larger solution space than the other scheduling problems (Flow- shop & Job –shop) and seems to receive less attention in the literature, although it is an important and universal problem.

An open shop model finds numerous applications in the real world situations. For example, Satellite communications, Optical networks, multi message multicasting and unicasting, mesh computing, automobile repair, quality control centres, semiconductor manufacturing, teacher-classes assignments and examination scheduling, railway reservation etc.,

A schedule is preemptive, if the execution of any operation may arbitrarily often be interrupted and resumed at a later time. Otherwise it is known as non-preemptive schedule. i.e., each operation is executed continuously from start to completion without interrupted. In this paper, we consider open shop scheduling problem with release dates for the objective of minimizing the makespan and resource idleness of machines, when pre-emption not allowed. The rest of the paper is organised as follows: In section 2, we gave the statement of the problem; section 3 is devoted for literature review; section 4 details the scope of the objective; in section 5, we gave our proposed algorithm followed by an example in section 6; results and discussion was given in section 7.

2. STATEMENT OF THE PROBLEM

In an OSSP, a set of n jobs $J_1, J_2, \dots, J_n$ has to be processed on a set of m machines $M_1, M_2, \dots, M_m$. Every job consists of m operations each of which must be processed on a different machine for a given processing time. The operations of each job can be processed in any order. At any time one operation can be processed on each machine, and almost one operation of each job can be processed. There is no precedence relation between the operations. All jobs are independent and continuously available for their process at time zero. All machines are continuously available. The process of a job cannot be interrupted. There are infinite buffer between machines (i.e., a job needs a machine that is occupied it can wait indefinitely until the machine becomes idle again). There are no transportation times between machines. It is assumed that the processing times of all operations are assumed to be given in advance. $O(i,j)$ denotes operation of job j on machine i . The processing time of job j on machine i, $i = 1, 2, \dots m$ is denoted by $p(i,j)$. It is assumed that the processing times are bounded by $P_{max}$and $i.i.d.$ (independently and identically distributed) random variables. $R(i,j)$is the starting time of operation $O(i,j)$ and the completion time of job j on machine i is denoted by $C(i,j)$. For each job j_i, there may be given a release date $r_j \geq 0$ which is the earliest possible time when the first operation of this job may start. The maximum completion time of all the jobs is known as makespan of the schedule and is denoted as C_{max}. The objective is to find a sequence of jobs with the given processing times on each machine to minimize the makespan and idleness of machines as well.

3. LITREATURE REVIEW

In the literature of OSSP, most of the attention has been paid to the minimization of makespan without considering release dates or due dates. Most of the researchers focus on the problem with the assumption that all jobs are available at time zero, whereas we consider the release dates also for our problem.

Graham R L, Lawler E L, Lenstra J K & Rinnooy kan A H G [13] provided the standard notation for scheduling problem. With that the general OSSP problem can be described as $Om//C_{max}$,where m is the number of machines. Pinedo M [18] presented a priority rule, Longest Alternate processing time first (LAPT) for problem $O2//C_{max}$, with which the optimal schedule can be found in polynomial time. For the case $m = 2$ or $n = 2$, a polynomial time algorithms is provided by Gonzalez T & Sahni S [12], to the OSSP with arbitrary number of jobs and machines and preemption allowed. Also they proved that the problem $Om/2/C_{max}$ is strongly NP-hard.

Lawler E L, Lenstra J K, Rinnooy kan A H G & Shmoys D B [17] proved that the problem $Om/2/C_{max}$, is strongly NP- hard, which means that the optimal solution of the problem cannot be obtained in polynomial time. For small scale problem, branch and bound algorithms are used to solve it [Brucker P, Hurink J, Jurisch B & Wostmann B [5], Dorndorf U, Pesch E & Phan-Huy T [11]). For large scale problems, constructing heuristic algorithms is an effective way to obtain the approximately optimal solution. Jayakumar S [14] focus attention in OSSP with the objective of minimizing the makespan and resource idleness and invented that in OSSP, a heuristic approach with longest processing time (LPT) perform better than the shortest processing time (SPT).

A feasible schedule for the open shop problem is called dense when any machine is idle if and only if there is no job which currently could be processed on that machine. This concept was introduced by Racsmany (cf Barany I, & Fiala T [3]) and it has been shown that the makespan of any dense schedule is almost twice the optimum makespan. This result can also be derived as a corollary from a more general result by Aksjonov V A [1]. Blazewicz J, Pesch E, Sterna M & Wesner F [4] considered OSSP with a common due date where the goal is to minimize total weighted late work.

Chen B & Strusevich V A [6] conjecture that for every $m \geq 2$, dense sschedules are almost a factor of $2 - \frac{1}{m}$ away from the optimum, and they proved this conjecture for $m = 3$. Strusevich V A [21] proved that when jobs are pre-ordered, the DS is improved and the new algorithm is almost $2 - \frac{1}{m+1}$ times of the optimal solution for problem $Om//C_{max}$. It is also noted that there is no polynomial time approximation algorithm with the worst- case performance ratio strictly less than $\frac{5}{4}$ unless P = NP by Williamson D P, Hall L A , Hoogeveen H A, Hurkens C A J , Lenstra J K, Sevast'janov S V & Shmoys D B [22].

The jobs are available only after its arrival in real world problems. The OSSP which seeks minimizing makespan occurs with release dates. If $r_j$be the release date for job j, then the problem can be described as $Om/r_j/C_{max}$. Lawler E L, Lenstra J K & Rinnooy kan A H G [16] pointed out that the problem $O2/r_j/C_{max}$ is strongly NP-hard. Chen R [9] proved that the worst-case performance ratio of DS is $\frac{3}{2}$ for the problem $O2/r_j/C_{max}$ and conjectured that the ratio is bounded by $2 - \frac{1}{m}$ where m is arbitrary. Chen R, Huang W & Tang G [10] proved that the worst-case performance ratio of DS is bounded by 2 for problem $O3/r_j/C_{max}$ and showed that the ratio can reach $\frac{5}{3}$ for some special case. A survey of algorithms of on-line scheduling problems was provided by Sgall J [19]. Chen B, Vestjens A P A & Woeginger G J [7] generalized the DS to schedule the on-line version of the problem $Om/r_j/C_{max}$ and proved that the conjecture that the worst competitive ratio of DS for the two- machine case is bounded by $2 - \frac{1}{m}$.

4. OBJECTIVE AND SCOPE OF THE PROBLEM

In 2013, D Bai and L Tang [2] had developed the DSPT – DS (Dynamic Shortest processing time-Dense schedule) algorithm to the open shop scheduling problems with release dates. In 2014, we focus our attention to the same type of problems and developed DLPT – DS (Dynamic longest processing time-Dense schedule) algorithm which provides better makespan value than DSPT – DS algorithm. In this paper we test the effectiveness of DLPT-DS heuristic algorithm by comparing its makespan value and idleness time of machines with those values obtained by DSPT-DS heuristic algorithm.

Definition [10] An idle interval [b, e) on machine i, $i = 1, 2, \ldots m$ for a given schedule S is called reasonable if one of the following conditions holds for job j, $j = 1, 2, \ldots n$,

(1) Job j has been finished on machine i before time b i.e., $C(i, j) \leq b$; or

(2) Job j is being processed on a machine other than i at any time t in [b, e),

i.e., [b, e) $\subseteq \bigcup_{i' \neq i} \left[R(i', j), C(i', j) \right)$;

(3) Job j released after time e, i.e., $r_j \geq e$.

A schedule is dense if all its idle intervals are reasonable. It is supposed that any idle interval does not traverse any release date, i.e., if there is an idle interval [b ,e) in which there is a release date $b < r_j < e$, then we denote $[b, e)$ by two idle intervals$[b, r_j)$ and $[r_j, e)$.

If the jobs are indexed according to their arriving sequence, i.e., $r_1 \leq r_2 \leq \cdots \leq r_n$, then the following lower bound for problem $Om/r_j/C_{max}$ can be easily obtained by observation.

$$C_{LB} = Max \left\{ \max_{1 \leq j \leq n,\ 1 \leq i \leq m} \left\{ r_j + \sum_{g=j}^{n} p(i, g) \right\}, \max_{1 \leq j \leq n} \left\{ r_j + \sum_{i=1}^{m} p(i, j) \right\} \right\}$$

Theorem 1[2] Let release date r_j be nonnegative random variables, $j = 1, 2, \ldots n$, and the processing time $p(i, j)$ of job j, $j = 1, 2, \ldots n$, $i = 1, 2, \ldots m$, be independent random variables and have the same continuous distribution with nonzero bounded density $\varphi(.)$. Then, for a series of randomly generated instances of problem $Om/r_j/C_{max}$, with probability one, we have

$\lim_{n \to \infty} \frac{C_{LB}}{n} = \lim_{n \to \infty} \frac{C_{max}(S^*)}{n} = \lim_{n \to \infty} \frac{C_{max}(DS)}{n}$, where $C_{max}(S^*)$ and $C_{max}(DS)$ denote the objective

values obtained by the optimal schedule and the DS, respectively.

Theorem 2 [2] The sequence of operations in a DS does not influence the asymptotic optimality.

5. PROPOSED ALGORITHM

In general, the idle time in the problem $Om/r_j/C_{max}$ is mainly arisen by the arriving jobs. As the sequence of operations in a DS does not influence the asymptotic optimality (Theorem 2), we process the available operation with longest processing time forward to reduce the waiting time of the sequence arrivals. Based on this idea, an algorithm called DLPT- DS heuristic is constructed. We describe our heuristic as follows.

5.1 DLPT-DS HEURISTIC [15]

Let $B = O(i,j)_{\ m \times j}, 1 \leq j \leq n$, denote the operations that are available at time $t, t \geq 0$ and $R(i,j)$ be the starting time of operation$O(i,j)$.

Step 1. At time $t, t \geq 0$ process the operation with the longest processing time, say $O(i_1, j_1)$ among all the available ones in matrix B. If some operations simultaneously satisfy the condition, give preference to the operation with smallest $O(i_1, j_1)$ index. Update the starting times of the operations, which are at the same column and row with $O(i_1, j_1)$, to $t + p(i_1, j_1)$in matrix B. Remove operation from matrix B.

Step 2. If some jobs arrive, go to step3; if matrix B becomes empty, go to step4.

Step 3. Sort the operations of the arrivals into matrix B, and update the starting time of each new operation to the longest starting time of its row in matrix B. Then go to Step1.

Step 4. Let the machines remain idle until a job arrives, and go to step 3 .If the scheduling is completed, terminate the program.

6. ILLUSTRATION

We illustrate the comparison of DSPT- DS and DLPT – DS by considering the problem of scheduling four jobs on four machines. The processing times of job $J_j, j = 1, 2, 3, 4$ on machine $M_i, i = 1, 2, 3, 4$ and the release dates r_j are given below.

	J_1	J_2	J_3	J_4
M_1	3	5	2	6
M_2	5	7	8	4
M_3	7	5	3	4
M_4	3	2	2	4
r_j	5	3	2	8

If we schedule the operations by using DSPT-DS, we obtain the makespan value as 29 units of time and idle time for machine 1, machine 2, machine 3, and machine 4 as 10, 5, 5 and 9 units of time respectively, totally 29 units (see figure 6.1.1). Where as if we schedule the operations by using DLPT-DS, we obtain the makespan value as 27 units of time and idle time for machine 1, machine 2, machine 3, and machine 4 as 4, 3, 5 and 12 units of time respectively, totally 24 units (see figure 6.1.2).

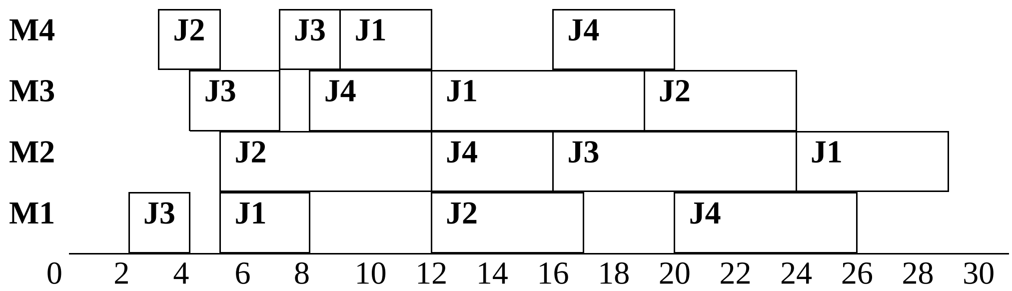
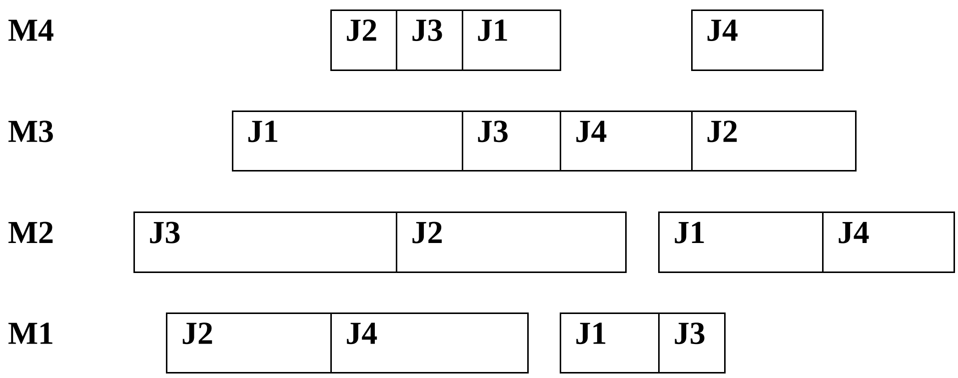


Fig. 6. 1. 1. DSPT-DS Schedule for general OSSP



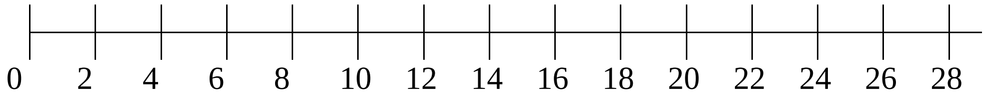


Fig. 6. 1. 2. DLPT-DS Schedule for general OSSP

7. RESULT AND DISCUSSION

In order to know the efficiency of the algorithm developed by us for the objective of minimizing the makespan and resource idleness of machines, we have arbitrarily generated 40 instances to check the validity of the algorithm. It was found that, our algorithm performs better than the algorithm found in the literature for the two cases namely, number of jobs greater than the number of machines and number of jobs equal to number of machines. It was found that for solving OSSP with release dates with the objective of minimizing makespan and resource idleness, one may choose our algorithm for better results.

8. APPENDIX

List of sample problems and corresponding results are given in Appendix.

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8.APPENDIXfea

TABLE 1: Number of Jobs Greater than Number of Machines

Sl. No	No. Of		DSPT-DS		DLPT- DS	
	Jobs	M/c's	TOTAL IDLE TIME OF MACHINES	MAKESPAN	TOTAL IDLE TIME OF MACHINES	MAKESPAN
1	4	3	11	37	6	36
2	4	3	14	24	12	23
3	4	3	10	37	17	35
4	5	3	11	39	8	36
5	5	3	9	39	8	38
6	5	3	5	19	4	18
7	6	3	9	47	6	46
8	6	3	7	37	6	35
9	6	3	12	36	6	31
10	5	4	36	38	25	37
11	5	4	16	36	18	35
12	5	4	21	32	25	31
13	6	4	10	34	16	33
14	6	4	14	33	9	32
15	7	4	21	48	15	40
16	7	4	9	43	16	40
17	6	5	26	37	26	33
18	6	5	43	46	30	43
19	7	5	18	40	20	40
20	7	5	18	47	19	47

TABLE 2: Number of Jobs Equal to Number of Machines

Sl. No	No. of		DSPT-DS		DLPT- DS	
	Jobs	M/c's	TOTAL IDLE TIME OF MACHINES	MAKESPAN	TOTAL IDLE TIME OF MACHINES	MAKESPAN
1	3	3	21	29	18	27
2	3	3	12	14	11	14
3	3	3	22	27	15	23
4	3	3	27	32	20	27
5	4	4	29	29	24	27
6	4	4	21	24	25	22
7	4	4	39	37	31	30
8	4	4	25	26	26	26
9	5	5	42	41	48	38
10	5	5	29	34	34	34
11	5	5	37	40	23	37
12	5	5	28	37	21	37
13	6	6	42	34	43	32
14	6	6	31	29	31	25
15	6	6	35	32	43	32
16	6	6	50	40	51	40
17	7	7	63	44	59	45
18	7	7	42	50	62	46
19	7	7	61	50	59	45
20	7	7	53	49	50	46