International Journal of Pure and Applied Mathematical Sciences.
ISSN 0972-9828 Volume 10, Number 1 (2017), pp. 151-165
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MATHEMATICAL MODELING ON EFFECTS OF AEROSOL THROUGH OSCILLATING FLOW IN THE HUMAN RESPIRATORY TRACT

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Abstract

Inhaled particle pollutant have been implicate as a potential cause of respiratory diseases. The unsteady of an oscillatory laminar flow of a Newtonian fluid with uniform distribution of dust particles during inhalation in the trachea of the human respiratory tract have been investigated. The governing equations are composed of Navier-stokes equations and the continuity equations. The oscillating flow is described by setting one side of boundaries be a periodic function. Due to the complexity and requirement of high performance computing resources of numerical methods, we therefore present an efficient alternative method that is the method of analytical expression. The obtained solution is in a Bessel series form. The effects of dust are described by two parameters, the mass concentration of the dust f, ω is the frequency in trachea tube and τ is a relaxation time. Which measure the rate at which the velocity of the dust particles adjusts in the velocity of the clean air and depends upon the size of the individual particles. In the human trachea for different time,the velocity profiles,wall shear stress,and flow rate are drawn due to the effect of dust parameters f and τ and the results are compared with those of the clean air.

Keywords: Aerosol, Oscillating flow, Mass concentration, Relaxation time, Frequency, Number density.

1. INTRODUCTION

The natural atmosphere that we breathes contains not only its gaseous constraints but also large numbers of liquid and solid particles. These are known by the genetic name aerosols. They arise from a combination of natural and industrial sources including condensation, smokes, volcanic activity, soils and sands and micro flora. Dust is simply small particles of substances in a solid state.An aerosol is a complex system consisting of gas containing particles. Particles as aggregations of matter, either solid or liquid larger than individual molecules.

In general there are three classes of particles size delimited by source and composition those particles below 0.1µm those between 0.1µm and about 2µm and those larger than about 2µm.The particles size in the range between 0.1µm and 2µm are known as fine particles. The particle size range above 2µm is known as coarse particles [Saffman].

Some of the most common lung diseases caused by dust are non-allergic asthma, non-allergic rhinitis/mucous membrane irritations (MMI),chronic bronchitis, chronic airflow obstruction, organic dust toxic syndrome (ODTS),Asbestosis,Silicosis,Farmer’s lung, Humidifier fever, Cheese washers’ lung.

Nirmala P.Ratchagar and Chitra.M (2007) has studied effects of fine and coarse dust particle on the transport of air in the trachea.Vimala.C.S (2012) has provided flow of a dusty gas between two oscillating plates.Gupta and Gupta (1979)have discussed flow of a dusty gas through a channel with arbitrary time varying pressure gradient.Saffman.P.G (1961) studied on the stability of a laminar flow of a dusty gas.Kongunan.S and Pholuang.J (2012) studied a Fourier series-based analytical solution for the oscillating airflow in a human respiratory tract.

In this paper, we study the effects of unsteady phenomena together with fine dusty particles and clean air on oscillating flow of air through trachea [Chitra.M, and Radhakrishnan.S]. Since the trachea is usually of the geometry of a pipe and hence we consider the basic equations in cylindrical co-ordinates with approximation stated later. A viscous compressible dusty gas bounded by circular tube executing simple harmonic oscillations with a frequency ω in trachea tube radius $r = R_0$[Vimala.C.S and Kongunan.S].The oscillation airflow are important to aerosolized therapy. It defines particles trajectories, transportation and final locations.

In this paper, we have considered laminar flow of clean air and air with dust particles during inhalation in the symmetric form of uniform pipe in the human trachea [Nirmala P.Ratchagar and Chitra.M]. The flow is unsteady under the assumption of constant pressure gradient.

i) The dust particles are assumed to be spherical in shape and are uniformly distributed.

ii) The flow is fully developed

iii) The number density N of the dust particles are constant throughout the airways.

iv) The buoyancy force has been neglected (since $\frac{\rho}{\rho_1}$ is very small).

In this paper, we investigate the effects of velocity, flow rate and wall shear stress of clean air in the presence of fine dust particles for varying radial coordinates and for varying time considering the dust parameters f and τ.

The analytical solution of these basic equations are obtained incorporating the effects of fine dust particles suspended in clean air. These analytical solutions are numerically computed and the results are depicted in graphs.

2. MATHEMATICAL FORMULATION

We assume that the dust particles in the air and flow of clean air through the symmetric form of circular tube of trachea.

We denote the clean air and dusty air velocities by $u_a{}^*(x, y, t), u_d{}^*(x, y, t)$, respectively in the direction of the axis of the tube (i.e.x-axis).

The equation of motion for dusty air and clean air in the human respiratory tract are;

$$\frac{\partial u_a{}^*}{\partial t^*} = -\frac{1}{\rho}\frac{\partial P^*}{\partial x} + \gamma_a\left(\frac{\partial^2 u_a{}^*}{\partial r^2} + \frac{1}{r}\frac{\partial u_a{}^*}{\partial r}\right) + \frac{KN_0}{\rho}(u_d{}^* - u_a{}^*) \quad --\to (1)$$

$$m\frac{\partial u_d{}^*}{\partial t^*} = K(u_a{}^* - u_d{}^*) \quad --\to (2)$$

The equation (1) and (2) are solved by using the following boundary and initial conditions.

$$\left.\begin{aligned} &\text{(i)Initial condition} \\ &\qquad u_a{}^* = \frac{c}{4L\omega}\left(1 - \frac{r^2}{R_0{}^2}\right) \text{ at } t^* = 0 \\ &\text{(ii)Boundary conditions} \\ &\qquad \frac{\partial u_a{}^*}{\partial r} = 0 \text{ at } r = 0 \\ &\qquad u_a{}^* = u_0{}^*\frac{\sin t^*}{L^2\omega} \text{ at } r = 1 \end{aligned}\right\} --\to (3)$$

In this model,

P^* is pressure gradient.It is assumed to be constant.

ρ and γ_a are density and kinematic viscosity of the clean air respectively.

m is the mass of a dust particle,

k is the stokes drag coefficient, which for spherical particles of radius r is $K = 6\pi r\mu$

N_0 is the number density of the dust particles.

ρ_1 is the density of the dust particles.

μ is the viscosity of the clean air and t^* is the time.

These equations are made dimensionless. Using.

$$R = \frac{r}{R_0};\ z = \frac{x}{L};\ P = \frac{P^* L\omega^2}{\rho};\ t = \frac{t^*}{L^2\omega};\ u_a = u_a{}^* L\omega;$$

$$\tau = \frac{m}{\omega k L^2};\ u_d = u_d{}^* L\omega;\ s = \frac{k N_0 L^2 \omega^2}{\rho};\ s\tau = \frac{m N_0}{\rho} = f;$$

Where

τ is the relaxation time scale,

f is the mass concentration of the dust particles,

ω is the frequency in trachea tube ,

R_0 and L are the radius and length of the trachea and other quantities are defined earlier. Substitute the above non-dimensional quantities into equations (1)-(3).we get

$$\frac{\partial u_a}{\partial t} = -\frac{\partial P}{\partial z} + \frac{L^2}{R_0{}^2}\left(\frac{\partial^2 u_a}{\partial R^2} + \frac{1}{R}\frac{\partial u_a}{\partial R}\right) + \beta(u_d - u_a) \qquad -\!-\!\!\rightarrow (4)$$

$$\tau\frac{\partial u_d}{\partial t} = u_a - u_d \qquad -\!-\!\!\rightarrow (5)$$

$$\left.\begin{array}{l}
\text{(i)Initial condition} \\
\qquad u_a = \frac{c}{4}(1 - R^2)\ \text{ at } t = 0 \\
\text{(ii)Boundary conditions} \\
\qquad \frac{\partial u_a}{\partial R} = 0 \text{ at } R = 0 \\
\qquad u_a = u_0 \sin t \text{ at } R = 1
\end{array}\right\} \qquad -\!-\!\!\rightarrow (6)$$

In this model the pressure gradient $-\frac{\partial p}{\partial z}$ is a constant.i.e.,

$$C = -\frac{\partial p}{\partial z} \qquad -\!-\!\!\rightarrow (7)$$

3. METHOD OF SOLUTION

It is known that the solution obtain by separation of variable will not satisfy the initial condition.

Therefore to find the transient solutions, we decompose the velocity into steady part and unsteady part as given below.

$$\left.\begin{aligned} u_a(R,t) &= u_{as}(R) + u_{at}(R,t) \\ u_d(R,t) &= u_{ds}(R) + u_{dt}(R,t) \end{aligned}\right\} \qquad --\rightarrow (8)$$

Where u_{as} and u_{ds} are the steady part of the clean air and dusty air respectively and u_{at} and u_{dt} unsteady (i.e., transient) state of the clean air and dusty air respectively.

Equations (4) and (5) using condition (8) and separating steady and unsteady part, we get

$$\frac{\partial^2 u_{as}}{\partial R^2} + \frac{1}{R}\frac{\partial u_{as}}{\partial R} + C = 0 \qquad --\rightarrow (9)$$

for steady state.

Equation (9) is solved using the boundary condition.

$$\left.\begin{aligned} \frac{du_{as}}{dR} &= 0 \ \text{ at } R = 0 \\ u_{as} &= u_0 \sin t \ \text{ at } R = 1 \end{aligned}\right\} \qquad --\rightarrow (10)$$

Then the solution of (9) satisfying the condition (10) is

$$u_{as}(R) = \frac{C}{4}(1 - R^2) + u_0 \sin t \qquad --\rightarrow (11)$$

Similarly,the unsteady state (ie.,transient solution) of clean air is obtained by solving the unsteady equations (4) and (5) using (8) becomes

$$\frac{\partial u_{at}}{\partial t} = \frac{\partial^2 u_{at}}{\partial R^2} + \frac{1}{R}\frac{\partial u_{at}}{\partial R} + \beta(u_{dt} - u_{at}) \qquad --\rightarrow (12)$$

Case i: Clean Air

We solve these equations using Laplace transform of the form

$$U(R,S) = L(u_{at}(R,t)) = \int_0^\infty e^{-st} u_{at}(R,t)dt$$

$$V(R,S) = L(u_{dt}(R,t)) = \int_0^\infty e^{-st} u_{dt}(R,t)dt$$

Where U and V are the Laplace transform of u_{at} and u_{dt} respectively and S is the Laplace parameter.

Applying Laplace transform to equations (12) and (5), we get

$$\left.\begin{array}{l} \text{(i)Initial condition} \\ \qquad U(R) = \frac{c}{4}(1-R^2)\ \text{ at } t=0 \\ \text{(ii)Boundary conditions} \\ \qquad \frac{\partial U}{\partial R} = 0 \text{ at } R=0 \\ \qquad U(R,t) = u_0 \sin t \ \text{ at } R=1 \end{array}\right\} \quad --\rightarrow (13)$$

$$\frac{d^2U}{dR} + \frac{1}{R}\frac{dU}{dR} - (\beta + S)U + \beta V = -\frac{C}{4}(1-R^2) \qquad --\rightarrow (14)$$

$$(\tau S + 1)V - \frac{\tau C}{4}(1-R^2) = U \qquad --\rightarrow (15)$$

Eliminating V between conditions (14) and (15)

$$\frac{d^2U}{dR} + \frac{1}{R}\frac{dU}{dR} - PU = -\frac{A}{4}(1-R^2) \qquad --\rightarrow (16)$$

Where

$$P = \frac{\tau S^2 + (1+\beta\tau)S}{1+\tau S} \text{ and } A = \frac{C(1+\tau S - \beta\tau)}{1+\tau S}$$

We solve (16) equation using Hankel transform of the form

$$\bar{U} = U(R) = \int_0^1 U(R)\, RJ_0(\varepsilon R) dR \qquad --\rightarrow (17)$$

Where the kernel $J_0(\varepsilon_n R)$ is the Bessel function of order zero,$J_o(\varepsilon_n)$ is a root of Bessel equation of order zero.

On solving equation (16)using the equation (17)

$$\bar{U} = \left[\frac{AJ_1(\varepsilon_n)}{2\varepsilon_n{}^3}\right]\left[\frac{1}{P+\varepsilon_n{}^2}\right] - \left[\frac{\varepsilon_n u_0 J'_0(\varepsilon_n)}{(S^2+1)(P+\varepsilon_n{}^2)}\right] \qquad --\rightarrow (18)$$

Taking inverse Hankel transform of (18), we get

$$L(u_a) = 2\sum_{n=1}^{5}\left(\frac{A}{2\varepsilon_n{}^3}\frac{1}{(P+\varepsilon_n{}^2)} + \frac{\varepsilon_n u_0}{(S^2+1)(P+\varepsilon_n{}^2)}\right)\frac{J_0(\varepsilon_n R)}{J_1(\varepsilon_n)} \qquad --\rightarrow (19)$$

For clean air the parameters $\tau = 0$ and $f = 0$

Therefore $A = C$ and $P = S$

Substitute these values of A and P in (19), we get

$$L(u_a) = 2\sum_{n=1}^{5}\left(\frac{C}{2\varepsilon_n{}^3}\frac{1}{(S+\varepsilon_n{}^2)} + \frac{\varepsilon_n u_0}{(S^2+1)(S+\varepsilon_n{}^2)}\right)\frac{J_0(\varepsilon_n R)}{J_1(\varepsilon_n)} \qquad --\rightarrow (20)$$

The inverse Laplace transform gives

$$u_{at}(R,t) = 2\sum_{n=1}^{5}\left[\frac{C\, e^{\varepsilon_n{}^2 t}}{2\varepsilon_n{}^3} + \frac{\varepsilon_n u_0\left(-\cos t + \varepsilon_n{}^2 \sin t + e^{-\varepsilon_n{}^2 t}\right)}{(1+\varepsilon_n{}^4)}\right]\frac{J_0(\varepsilon_n R)}{J_1(\varepsilon_n)} \qquad --\rightarrow (21)$$

The required velocity of the clean air from(8) using (11) and (21) take the form

$$\begin{aligned} u_a(R,t) = {} & \frac{C}{4}(1-R^2) + u_0 \sin t \\ & + 2\sum_{n=1}^{5}\left[\frac{C\, e^{\varepsilon_n{}^2 t}}{2\varepsilon_n{}^3} + \frac{\varepsilon_n u_0\left(-\cos t + \varepsilon_n{}^2 \sin t + e^{-\varepsilon_n{}^2 t}\right)}{(1+\varepsilon_n{}^4)}\right]\frac{J_0(\varepsilon_n R)}{J_1(\varepsilon_n)} \qquad --\rightarrow (22) \end{aligned}$$

Case ii: Fine Dust Particles

$\tau \ll \frac{L}{U^*}$ and it follows that $V_a = U_a$ for disturbance with length scale L or Larger.Then equation (5) becomes

$$u_a - u_d = \left[\frac{\partial u_a}{\partial t}\right]\tau \qquad --\rightarrow (23)$$

Substitute equation (23) in equation (4), we get

$$\frac{\partial u_a}{\partial t} = -\frac{\partial P}{\partial Z} + \left[\frac{\partial^2 u_a}{\partial R^2} + \frac{1}{R}\frac{\partial u_a}{\partial R}\right] - \beta\tau\frac{\partial u_a}{\partial t}$$

$$\frac{\partial u_a}{\partial t} + \beta\tau\frac{\partial u_a}{\partial t} = -\frac{\partial P}{\partial Z} + \left[\frac{\partial^2 u_a}{\partial R^2} + \frac{1}{R}\frac{\partial u_a}{\partial R}\right]$$

$$(1+\beta\tau)\frac{\partial u_a}{\partial t} = -\frac{\partial P}{\partial Z} + \left[\frac{\partial^2 u_a}{\partial R^2} + \frac{1}{R}\frac{\partial u_a}{\partial R}\right] \qquad --\rightarrow (24)$$

Here $\beta = \frac{f}{\tau}$ i.e., $\beta\tau = f$.

$$(1+f)\frac{\partial u_a}{\partial t} = C + \left[\frac{\partial^2 u_a}{\partial R^2} + \frac{1}{R}\frac{\partial u_a}{\partial R}\right]$$

Put $u_a = u_f$ takes the form

$$(1+\mathrm{f})\frac{\partial \mathrm{u_f}}{\partial \mathrm{t}} = \mathrm{C} + \left[\frac{\partial^2 \mathrm{u_f}}{\partial \mathrm{R}^2} + \frac{1}{\mathrm{R}}\frac{\partial \mathrm{u_f}}{\partial \mathrm{R}}\right] \qquad --\rightarrow (25)$$

Where $\mathrm{u_f}$ is the velocity of fine dust particles, equation (25) is is solved using the following initial and boundary conditions

$$\left.\begin{array}{l} \text{(i)Initial condition} \\ \qquad \mathrm{u_f} = \frac{\mathrm{c}}{4}(1-\mathrm{R}^2) \ \ \text{at } \mathrm{t}=0 \\ \text{(ii)Boundary conditions} \\ \qquad \frac{\partial \mathrm{u_f}}{\partial \mathrm{R}} = 0 \text{ at } \mathrm{R}=0 \\ \qquad \mathrm{u_f} = \mathrm{u_0}\sin \mathrm{t} \ \text{ at } \mathrm{R}=1 \end{array}\right\} \qquad --\rightarrow (26)$$

To find the solution in this case, we follow the same procedure used in finding solution for clean air. That is we decompose the velocity with steady and unsteady part on in (8) with suffixes $\mathrm{f_s}$ and $\mathrm{f_t}$ denoting steady and unsteady part of fine dust particles. This procedure gives,

$$\frac{\partial^2 \mathrm{u_{fs}}}{\partial \mathrm{R}^2} + \frac{1}{\mathrm{R}}\frac{\partial \mathrm{u_{fs}}}{\partial \mathrm{R}} + \mathrm{C} = 0 \qquad --\rightarrow (27)$$

In this case, the boundary conditions for the steady state using (26) become

$$\left.\begin{array}{ll} \frac{\partial \mathrm{u_{fs}}}{\partial \mathrm{R}} = 0 & \text{at } \ \mathrm{R}=0 \\ \mathrm{u_{fs}}(\mathrm{R}) = \mathrm{u_0}\sin \mathrm{t} & \text{at } \ \mathrm{R}=1 \end{array}\right\} \qquad --\rightarrow (28)$$

The solution of (27) is

$$\mathrm{u_{fs}}(\mathrm{R}) = \frac{\mathrm{C}}{4}(1-\mathrm{R}^2) + \mathrm{u_0}\sin \mathrm{t} \qquad --\rightarrow (29)$$

The unsteady state equation using (25) takes the form

$$(1+\mathrm{f})\frac{\partial \mathrm{u_{ft}}}{\partial \mathrm{t}} = \frac{\partial^2 \mathrm{u_{ft}}}{\partial \mathrm{R}^2} + \frac{1}{\mathrm{R}}\frac{\partial \mathrm{u_{ft}}}{\partial \mathrm{R}} \qquad --\rightarrow (30)$$

The initial and boundary conditions (26) and using (29) takes the form

$$\left.\begin{array}{ll} \mathrm{u_{fs}}(\mathrm{R}) = \mathrm{u_{ft}}(\mathrm{R}) = \frac{\mathrm{c}}{4}(1-\mathrm{R}^2) + \mathrm{u_0}\sin \mathrm{t} & \text{at } \ \mathrm{t}=0 \\ \frac{\partial \mathrm{u_{ft}}}{\partial \mathrm{R}} = 0 & \text{at } \mathrm{R}=0 \\ \mathrm{u_{ft}}(\mathrm{R},\mathrm{t}) = \mathrm{u_0}\sin \mathrm{t} & \text{at } \mathrm{R}=1 \end{array}\right\} \qquad --\rightarrow (31)$$

We apply Laplace transform on (30) together with the condition (31)

The Laplace transform of two Variables (R, t) for dusty air is

$$\mathrm{u_{ft}(R,S) = L[u_{ft}(R,t)]} = \int_0^\infty \mathrm{e^{-st}u_{ft}(R,t)dt} \qquad --\rightarrow (32)$$

We get,

$$\frac{\partial^2 \mathrm{U_{ft}}}{\partial \mathrm{R}^2} + \frac{1}{\mathrm{R}}\frac{\partial \mathrm{U_{ft}}}{\partial \mathrm{R}} - \mathrm{S(1+f)U_{ft}} + \frac{\mathrm{C}}{4}\mathrm{(1+f)(1-R^2)} = 0 \qquad --\rightarrow (33)$$

Solve this equation using Hankel transform ie.,

$$\overline{\mathrm{U}}_{\mathrm{ft}} = \int_0^1 \mathrm{u_{ft}(R,t)RJ_0(\varepsilon_n,R)\,dR} \qquad --\rightarrow (34)$$

On solving equation (33) using equation (34), we get

$$\overline{\mathrm{U}}_{\mathrm{ft}}\,(\mathrm{R}) = \frac{\mathrm{CJ_1(\varepsilon_n)}}{2\varepsilon_n{}^3}\frac{1}{\left(\mathrm{S}+\frac{\varepsilon_n{}^2}{1+\mathrm{f}}\right)} - \frac{\varepsilon_n \mathrm{u_0 J_0{}'(\varepsilon_n)}}{(1+\mathrm{f})}\frac{1}{(\mathrm{S}^2+1)\left(\mathrm{S}+\frac{\varepsilon_n{}^2}{1+\mathrm{f}}\right)} \qquad --\rightarrow (35)$$

Taking inverse Hankel transform of (35), we get

$$\mathrm{L}\big(\mathrm{u_{ft}(R)}\big) = \overline{\mathrm{U}}_{\mathrm{ft}}\,(\mathrm{S})$$

$$= 2\sum_{\mathrm{n}=1}^{5}\left[\frac{\mathrm{C}}{2\varepsilon_n{}^3}\frac{1}{\left(\mathrm{S}+\frac{\varepsilon_n{}^2}{1+\mathrm{f}}\right)} + \frac{\varepsilon_n \mathrm{u_0}}{(1+\mathrm{f})(\mathrm{S}^2+1)\left(\mathrm{S}+\frac{\varepsilon_n{}^2}{1+\mathrm{f}}\right)}\right]\frac{\mathrm{J_0(\varepsilon_n R)}}{\mathrm{J_1(\varepsilon_n)}} \qquad -\rightarrow (36)$$

On taking Inverse Laplace transform of (36), we get

$$\mathrm{u_f(R,t)}$$
$$= 2\sum_{\mathrm{n}=1}^{5}\left[\frac{\mathrm{C}}{2\varepsilon_n{}^3}\ \mathrm{e}^{-\frac{\varepsilon_n{}^2}{(1+\mathrm{f})}\mathrm{t}} + \frac{\varepsilon_n \mathrm{u_0}}{(1+\mathrm{f})}\,\frac{\left(-(1+\mathrm{f})^2\cos \mathrm{t} + (1+\mathrm{f})\varepsilon_n{}^2 \sin \mathrm{t} + (1+\mathrm{f})^2 \mathrm{e}^{-\frac{\varepsilon_n{}^2}{(1+\mathrm{f})}\mathrm{t}}\right)}{(1+\mathrm{f})^2+\varepsilon_n{}^2}\right]\frac{\mathrm{J_0(\varepsilon_n R)}}{\mathrm{J_1(\varepsilon_n)}} \qquad -\rightarrow (37)$$

The required velocity of fine dust by summing equations (29) and (37) is

$$u_f(R,t) = \frac{C}{4}(1-R^2) + u_0 \sin t + 2\sum_{n=1}^{5}\left[\frac{C}{2{\varepsilon_n}^3} \quad e^{-\frac{{\varepsilon_n}^2}{(1+f)}t} + \frac{\varepsilon_n u_0}{(1+f)} \frac{\left(-(1+f)^2\cos t + (1+f){\varepsilon_n}^2 \sin t + (1+f)^2 e^{-\frac{{\varepsilon_n}^2}{(1+f)}t}\right)}{(1+f)^2 + {\varepsilon_n}^2}\right] \frac{J_0(\varepsilon_n R)}{J_1(\varepsilon_n)} \quad \longrightarrow (38)$$

Now to find the quantitative impact of dust on a transport of air in trachea it is advantages to calculate the mass flow rate and wall shear stress which is the skin friction.

The mass flow rate of clean air is

$$Q_a(t) = \frac{2\pi\gamma_a {R_0}^2}{L}\int_0^1 u_a \, R \, dR$$

$$Q_a(t) = \frac{2\pi {R_0}^2 \gamma_a}{L}\int_0^1 \left[\frac{C}{4}(1-R^2) + u_0 \sin t + 2\sum_{n=1}^{5}\left(\frac{C\, e^{-{\varepsilon_n}^2 t}}{2{\varepsilon_n}^3} + \frac{\varepsilon_n u_0\left(-\cos t + {\varepsilon_n}^2 \sin t + e^{-{\varepsilon_n}^2 t}\right)}{(1+{\varepsilon_n}^4)}\right)\frac{J_0(\varepsilon_n R)}{J_1(\varepsilon_n)}\right] R \, dR$$

After integration takes the form

$$Q_a(t) = \frac{2\pi {R_0}^2 \gamma_a}{L}\left[\frac{C}{16} + \frac{u_0 \ \sin t}{2} + 2\sum_{n=1}^{5}\left(\frac{C\, e^{-{\varepsilon_n}^2 t}}{2{\varepsilon_n}^4} + \frac{u_0\left(-\cos t + {\varepsilon_n}^2 \sin t + e^{-{\varepsilon_n}^2 t}\right)}{(1+{\varepsilon_n}^4)}\right)\right]$$

Similarly, the mass flow rate of fine dusty air is

$$Q_f(t) = \frac{2\pi\gamma_a {R_0}^2}{L}\int_0^1 u_f \, R \, dR$$

$$Q_f(t) = \frac{2\pi R_0{}^2\gamma_a}{L}\int_0^1 \left[\frac{C}{4}(1-R^2) + u_0\mathrm{sint} + 2\sum_{n=1}^{5}\left(\frac{C\, e^{\frac{-\varepsilon_n{}^2 t}{(1+f)}}}{2\varepsilon_n{}^3} + \frac{\varepsilon_n u_0\left(-(1+f)\mathrm{cost} + \varepsilon_n{}^2 \sin t + (1+f)\, e^{\frac{-\varepsilon_n{}^2 t}{(1+f)}}\right)}{(1+f)^2\varepsilon_n{}^4}\right)\frac{J_0(\varepsilon_n R)}{J_1(\varepsilon_n)}\right] R\, dR$$

After integration takes the form

$$Q_f(t) = \frac{2\pi R_0{}^2\gamma_a}{L}\left[\frac{C}{16} + \frac{u_0\ \sin t}{2} + 2\sum_{n=1}^{5}\left(\frac{C\, e^{\frac{-\varepsilon_n{}^2 t}{(1+f)}}}{2\varepsilon_n{}^4} + \frac{u_0\left(-(1+f)\mathrm{cost} + \varepsilon_n{}^2 \sin t + (1+f)\, e^{\frac{-\varepsilon_n{}^2 t}{(1+f)}}\right)}{(1+f)^2 + \varepsilon_n{}^4}\right)\right]$$

The wall shear stress also called skin friction for clean air is

$$\tau_a(t) = \left[-\mu_a\rho\gamma_0{}^2\frac{\partial u_a}{\partial R}\right]_{R=1}$$

$$= -\mu_a\rho\gamma_0{}^2\frac{\partial}{\partial R}\left[\frac{C}{4}(1-R^2) + u_0\mathrm{sint} + 2\sum_{n=1}^{5}\left(\frac{C\, e^{-\varepsilon_n{}^2 t}}{2\varepsilon_n{}^3} + \frac{\varepsilon_n u_0\left(-\mathrm{cost} + \varepsilon_n{}^2\sin t + e^{-\varepsilon_n{}^2 t}\right)}{(1+\varepsilon_n{}^4)}\right)\frac{J_0(\varepsilon_n R)}{J_1(\varepsilon_n)}\right]_{R=1}$$

$$\tau_a(t) = \mu_a\rho\gamma_0{}^2\ \left[\frac{C}{2} + 2\sum_{n=1}^{5}\left(\frac{\varepsilon_n{}^2 u_0\left(-\mathrm{cost} + \varepsilon_n{}^2\sin t + e^{-\varepsilon_n{}^2 t}\right)}{(1+\varepsilon_n{}^4)}\right) - \frac{e^{-\varepsilon_n{}^2 t}}{2\varepsilon_n{}^2}\right]$$

Similarly for fine dusty air, we get

$$\tau_f(t) = \left[-\mu_f \rho \gamma_0{}^2 \frac{\partial u_f}{\partial R}\right]_{R=1}$$

$$= -\mu_f \rho \gamma_0{}^2 \frac{\partial}{\partial R}\left[\frac{C}{4}(1-R^2) + u_0 \mathrm{sint} + 2\sum_{n=1}^{5}\left(\frac{C\, e^{\frac{-\varepsilon_n{}^2 t}{(1+f)}}}{2\varepsilon_n{}^3} + \frac{\varepsilon_n u_0\left(-(1+f)\mathrm{cost} + \varepsilon_n{}^2 \sin t + (1+f)e^{\frac{-\varepsilon_n{}^2 t}{(1+f)}}\right)}{(1+f)^2 + \varepsilon_n{}^4}\right)\frac{J_0(\varepsilon_n R)}{J_1(\varepsilon_n)}\right]_{R=1}$$

$$\tau_f(t) = \mu_f \rho \gamma_0{}^2\left[\frac{C}{2} + 2\sum_{n=1}^{5}\frac{\varepsilon_n{}^2 u_0\left(-(1+f)\mathrm{cost} + \varepsilon_n{}^2 \sin t + (1+f)e^{\frac{-\varepsilon_n{}^2 t}{(1+f)}}\right)}{(1+f)^2 + \varepsilon_n{}^4} - \frac{Ce^{-\varepsilon_n{}^2 t}}{2\varepsilon_n{}^2}\right]$$

4. CONCLUSION

The velocity, flow rate, wall shear stress are depend more on the mass concentration and frequency 'ω'of the dust particles than on their size.The changes in the velocity, flow rate, wall shear stress of fine dust particles with varying time 't' due to the dust parameters 'f' ,'τ' and frequency 'ω' are computed. The result are depicted in graph(only in graph $n = \mu$).

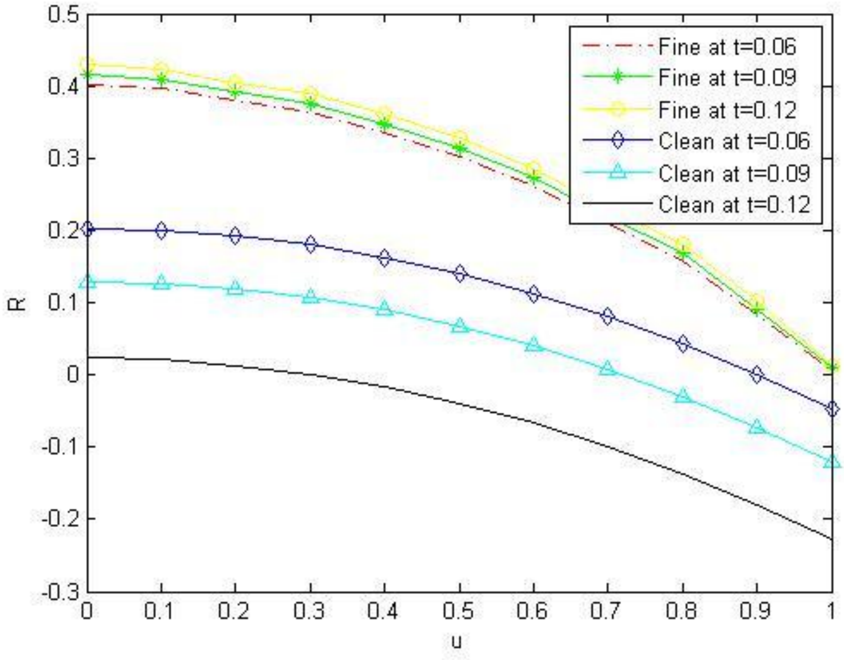

FIGURE 1. Variation of axial velocity u with radial position R for different values of time t (f=0.2,τ=0.1)

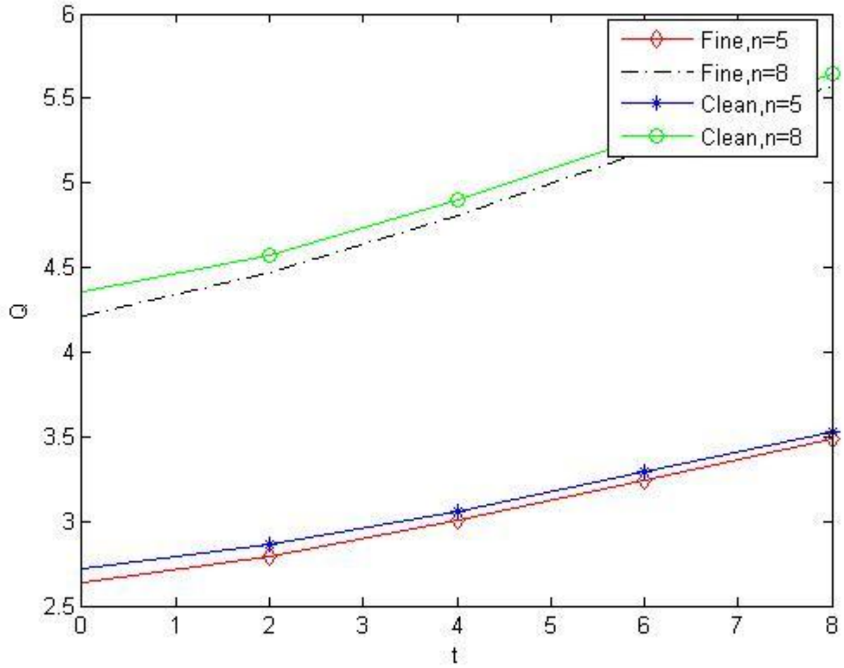

FIGURE 2. Variation of flow rate Q with time t for different values of viscosity n=μ (f=0.2, τ=0.1)

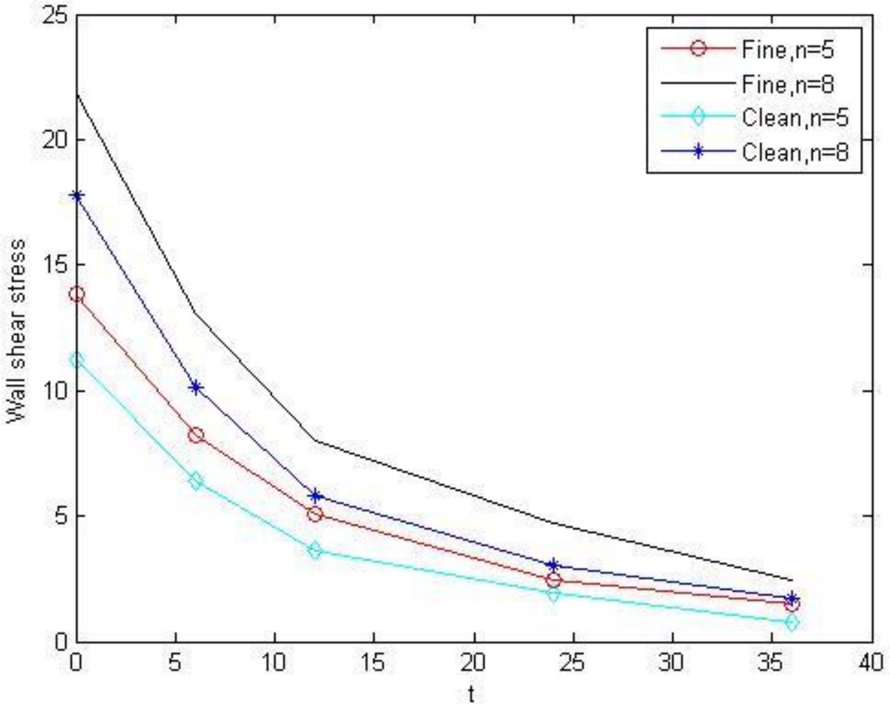

FIGURE 3. Variation of wall shear stress τ with axial position time t for different values of viscosity n=μ (f=0.2, τ=0.1).

5. RESULT

The velocity profiles for the clean air and fine dust particles varying time are drawn in fig.1.This figure reveals that an increase of velocity with increase of‘t’. It is clear that the velocity of fine dust particles is more advanced when compared to the clean air in the human trachea.

The flow rate and wall shear stress for the clean air and fine dust particles of the air ways varying with time‘t’ for different values of viscosity of the air are also computed and the results are depicted in fig.2 and fig.3.From these two figures shows that flow rate decreases with increase of viscosity of air and wall shear stress increases with increase of viscosity of air. It also clear that the flow rate and wall shear stress of fine dust particles is more advanced when compared to the clean air in the human trachea.

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