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Effect of the Porous Medium through a Constricted Channel under the Influence of Transverse Magnetic Field

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Abstract

The steady flow of a viscous incompressible fluid in a constricted channel with slip at the permeable boundaries with the effect of porous medium under the influence of transverse magnetic field is investigated. Analytical solutions are constructed for velocity profile, volumetric flow rate, shear stress. The results are depicted graphically.

Keywords: Newtonian fluid, Porous medium, Magnetic flied, incompressible fluid, Pressure gradient, Slip parameter, Hartman number.

INTRODUCTION

The requirements of modern technology have stimulated the interest in fluid flow studies, which involve the interaction of several phenomenon. One such study is presented, when a viscous fluid flow over a porous surface, because of its importance in many engineering problems such as flow of liquid in a porous bearing , in the field of water in river beds, in petroleum technology to study the movement of natural gas, oil and water through the oil reservoirs , in chemical engineering for filtration and purification, in physiology and biomedical engineering to study the blood flow in the alveolar sheet, dialysis of blood flow through artifical kidneys and human airways etc.

The study of flow of electrically conducting fluid, the so called Magnetohydrodynamics (MHD) has a lot of attention due to its diverse application. In astrophysics and geophysics, it is applied to the study of stellar and solar structures, interstellar matter, and radio propagation through ionosphere.

Various mathematical models have been investigated by several researchers to explore the effects of porous medium on flow through the constricted channel under the influence of transverse magnetic field. A survery of MHD studies in the technological field can be found in Moreau 1990 [1]. Makinde and Sibanda 2006 [2] discussed the MHD steady flow in a channel with slip at the permeable boundaries. Majumder and Deka 2007 [3] studied the MHD flow past an impulsively started infinite vertical plate in the presence of thermal radiation. M.Syamala et al 2011 [4] studied the MHD flow of a couple stress fluid through a porous medium in a parallel plate channel in presence of effect of inclined magnetic field . MHD flow of radiating and chemically reacting viscoelastic fluid through a porous medium in porous vertical channel with constant suction studied by Khem Chand et al 2013 [5]. Daniel et al 2014 [6] discussed the Slip effect on MHD oscillatory flow of fluid in a porous medium with heat and mass transfer and chemical Reaction.

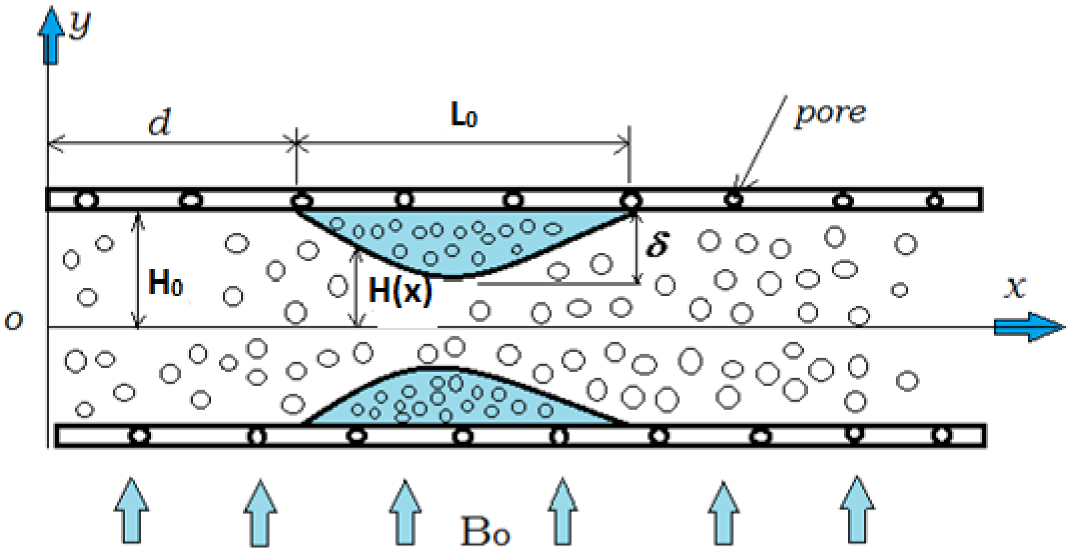
The main objective of the present paper is to study the effect of porous medium with transverse magnetic field over the flow characteristics of the fluid in a single axially symmetric constricted channel. Fluid is considered to be Newtonian and viscosity is constant. The governing motion and continuity equation is reduced to a differential equation and analytical techniques are utilized for its solution. The relative velocity profile, volumetric flow rate, and wall shear stress are shown graphically with the effect of porous medium through the constricted channel.

MATHEMATICAL FORMULATION

Consider the axially symmetric laminar steady flow of the fluid through the channel with constriction under the influence of uniform transverse magnetic field with porous medium and electromagnetic force is very small. The constricted portion of the channel is termed as stenosis.

$$\frac{H(x)}{H_0} = \begin{cases} 1 - \frac{\delta}{2H_0}\left(1 + \cos\left(\frac{2\pi}{L_0}\right)\left(x - d - \frac{L_0}{2}\right)\right); & d \le x \le L_0 + d \\ 1 & ; \text{ otherwise} \end{cases} \tag{1}$$

Where H_0 is the radius of the unconstricted channel, H(x) is the radius of the constricted channel , L_0 is the length of the stenosis , d is the location of the stenosis, δ is the maximum height of stenotic growth.



Geometry of the constricted channel

The equation of motion and continuity for the steady state incompressible laminar flow in the constricted channel is given by:

$$-\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{\mu}{\rho}\frac{\partial^2 V}{\partial y^2}-\frac{B_0^2\sigma_e}{\rho}V+\frac{\mu}{K_0}V=0 \qquad (2)$$

$$\frac{\partial P}{\partial y}=0 \qquad (3)$$

$$\frac{\partial V}{\partial x}=0 \qquad (4)$$

Where ,V is the velocity of the fluid,P is the fluid pressure, $\frac{\partial P}{\partial x}$ is the pressure gradient, ρ is the constant density of the fluid, μ is the co-efficeint of viscosity.

$B_0=\frac{\mu_e}{H}$ is the electromagnetic induction, μ_e is the magnetic permeability, H is the intensity of magnetic field and □$_e$ is the conductivity of fluid.

The boundary conditions are

$$V=-\beta\frac{\partial V}{\partial y} \qquad at \quad y=H(x) \qquad (5)$$

$$\frac{\partial V}{\partial y}=0 \qquad at \quad y=0 \qquad (6)$$

Where $-\beta$ is the slip parameter .

Using the transformation $\eta = \frac{y}{H_0}$ in the equation (2), we get

$$\frac{\partial^2 V}{\partial \eta^2} - \left[M^2 + \frac{1}{K}\right] V = P \quad (7)$$

Where, $M = B_0 H \left(\sqrt{\frac{\sigma_e}{\mu}}\right)$ is the Hartman number and $P = \frac{H_0^2}{\mu} \frac{\partial P}{\partial x}$

Then from conditions (5) and (6) transformed to

$$V = -\beta \frac{\partial V}{\partial \eta} \quad at \quad \eta = \frac{H(x)}{H_0} \quad (8)$$

$$\frac{\partial V}{\partial \eta} = 0 \quad at \quad \eta = 0 \quad (9)$$

solving the equation (7) by using the boundary conditions (8) and (9). we get the velocity profile

$$V = \frac{H_0^2}{\mu w}\left(\frac{\partial P}{\partial x}\right)\left[1 - \frac{cosh\sqrt{w}\eta}{\cosh(\sqrt{w}\frac{H(x)}{H_0}) + \beta\sqrt{w}\, sinh\left(\sqrt{w}\frac{H(x)}{H_0}\right)}\right] \quad (10)$$

Where $w = \left(M^2 + \frac{1}{K}\right)$

If V_0 is the velocity profile for the uniform cross-section in the unconstricted channel in the absence of magnetic field and porous medium then we have

$$V_0 = \frac{H_0^2}{\mu}\left(\frac{\partial P}{\partial x}\right)_0 (1 - \eta^2) \quad (11)$$

And the relative velocity as:

$$\bar{V} = \frac{V}{V_0} = \left[\frac{\left(\frac{\partial P}{\partial x}\right)}{\left(\frac{\partial P}{\partial x}\right)_0}\right] \left[\frac{\left(\frac{1}{w}\right)\left(1 - \frac{cosh\sqrt{w}\eta}{cosh\sqrt{w}\frac{H(x)}{H_0} + \beta\sqrt{w}\, sinh\left(\sqrt{w}\frac{H(x)}{H_0}\right)}\right)}{1 - \eta^2}\right] \quad (12)$$

The volumetric flow rate Q of fluid in stenotic region is given by

$$Q = \int_0^{\frac{H(x)}{H_0}} V\eta d\eta \quad (13)$$

We obtain the volumetric flow rate Q

$$Q = \frac{H_0^2}{w}\left(\frac{\partial P}{\partial x}\right)\left(\frac{(H(x))^2}{2H_0^2} - \frac{1}{\cosh\sqrt{w}\frac{H(x)}{H_0} + \beta\sqrt{w}\,\sinh\left(\sqrt{w}\frac{H(x)}{H_0}\right)}\left(\frac{H(x)}{H_0}\right)\sinh\left(\sqrt{w}\,\frac{H(x)}{H_0}\right) + \cosh\left(\sqrt{w}\,\frac{H(x)}{H_0}\right) - \left(\frac{1}{w}\right)\right) \qquad (14)$$

If Q_0 is the volumetric flow rate for the uniform cross-section in the unconstructed channel in the absence of magnetic field and porous medium then we have

$$Q_0 = H_0^2\left(\frac{\partial P}{\partial x}\right)_0 \qquad (15)$$

Where $\left(\frac{\partial P}{\partial x}\right)_0$ is the pressure gradient of flow in the unconstructed uniform channel in the absence of magnetic field and porous medium. Now for steady flow if Q and Q_0 occurs in the same closed system then $Q/Q_0 = 1$. Then from equation (14) and (15), we have the relative pressure gradient as:

$$\frac{(\partial P/\partial x)}{(\partial P/\partial x)_0} = \frac{1}{\left(\frac{(H(x))^2}{2H_0^2} - \frac{1}{\cosh\sqrt{w}\frac{H(x)}{H_0} + \beta\sqrt{w}\,\sinh\left(\sqrt{w}\frac{H(x)}{H_0}\right)}\left(\frac{H(x)}{H_0}\right)\sinh\left(\sqrt{w}\,\frac{H(x)}{H_0}\right) + \cosh\left(\sqrt{w}\,\frac{H(x)}{H_0}\right) - \left(\frac{1}{w}\right)\right)} \qquad (16)$$

The wall shear stress is defined as :

$$\tau_w = \left[\mu\frac{dV}{d\eta}\right]_{\eta=\frac{H(x)}{H_0}} \qquad (17)$$

Using velocity profile (10) in (17), we have

$$\tau_w = \frac{H_0^2}{w}\left(\frac{\partial P}{\partial x}\right)\left[\frac{H(x)}{H_0} - \frac{\sqrt{w}\,\sinh\left(\sqrt{w}\frac{H(x)}{H_0}\right)}{\cosh\left(\sqrt{w}\frac{H(x)}{H_0}\right) + \beta\sqrt{w}\sinh\left(\sqrt{w}\frac{H(x)}{H_0}\right)}\right] \qquad (18)$$

If τ_{w_0} is the shear at the wall in the absence of stenosis and magnetic field , then the non- dimensional form of (17) is

$$\tau_{w_0} = H_0^2\left(\frac{\partial P}{\partial x}\right)_0 \qquad (19)$$

The non-dimensional form of (18) is

$$\bar{\tau}_w = \frac{\tau_w}{\tau_{w_0}} \qquad (20)$$

Using (18) and (19) in (20) we get

$$\bar{\tau}_w = \frac{1}{w}\left[\frac{\left(\frac{\partial P}{\partial x}\right)}{\left(\frac{\partial P}{\partial x}\right)_0}\right]\left[\frac{H(x)}{H_0} - \frac{\sqrt{w}\,sinh\left(\sqrt{w}\frac{H(x)}{H_0}\right)}{cosh\left(\sqrt{w}\frac{H(x)}{H_0}\right) + \beta\sqrt{w}sinh\left(\sqrt{w}\frac{H(x)}{H_0}\right)}\right] \qquad (21)$$

RESULTS AND DISCUSSIONS

Theoretical result for velocity profile, volumetric flow rate, and wall shear stress for the porous medium with magnetic effect on steady flow through a constricted channel have been obtained.

The relative velocity profile, volumetric flow rate, and wall shear stress are shown graphically for different values of porous medium *(K)* .

Figure -1 shows the velocity profile for different value of *K*.. It is clear from figure that Velocity profile slightly decreases as permeability of porous medium *K* increases. Figure-2 shows the variation of volumetric flow rate along the length of the constricted channel for different value of permeability of porous medium *K*. It is clear from the figure that the flow rate slightly increases due to permeability of porous medium increases... Figure -3 shows that wall shear stress increase along the length of the constricted channel and attain maximum value and then it falls slowly. It is clear from figure that wall shear stress slightly increases as permeability of porous medium *K* increases. Hence, we conclude that the porous medium with transverse magnetic field through the constricted channel affects the flow behavior of the fluid.

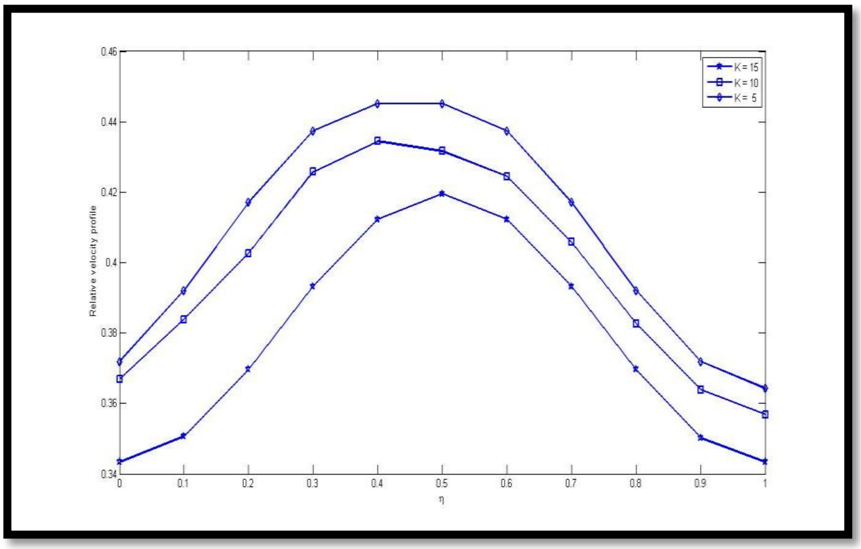


Fig 1: Variation of relative velocity profile ($\bar{V}$) with longitudinal constricted region H(x) for different values of permeability K for fixed Hartman number (M=2) and $\delta = 0.2$

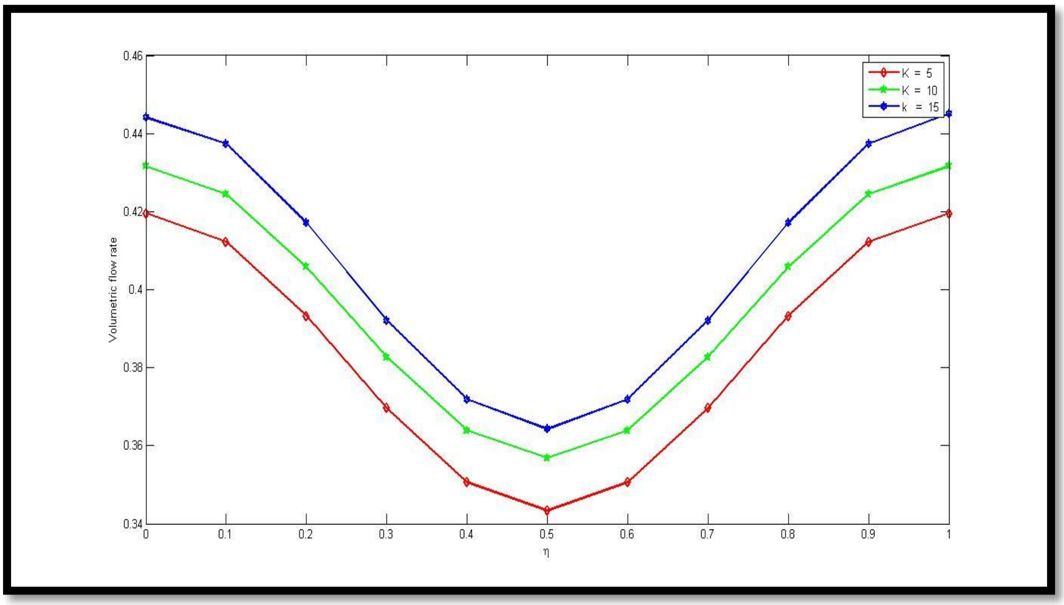


Fig 2: Variation of volumetric flow rate (Q) with longitudinal constricted region H(x) for different values of permeability K for fixed Hartman number (M=2) and $\delta = 0.2$.

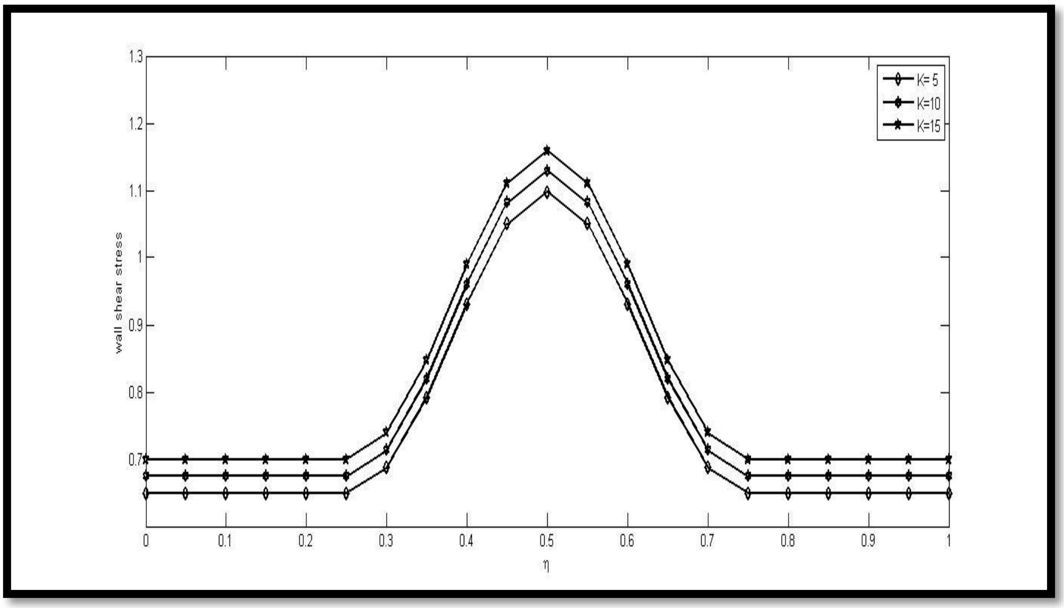


Fig 3: Variation of wall shear stress ($\bar{\tau}_w$) with longitudinal constricted region H(x) for different values of permeability K for fixed Hartman number (M=2) and $\delta = 0.2$

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