International Journal of Pure and Applied Physics. ISSN 0973-1776 Volume 13, Number 1 (2017), pp. 84-88

The systematic study of N/Z dependence on surface diffuseness parameter in the fusion of heavy neutron-rich colliding nuclei by using Skyrme energy density formalism

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***Abstract*— Surface diffuseness parameter used in Woods-Saxon form of potential have been extracted from a large number of experimentally studied neutron-rich fusion cross sections at near barrier energies. It is clear from the literature that the exact value of diffuseness parameter suitable for fusion process is still not clear. On the other hand, various theoretical model developed so far have also employed quite arbitrary value for this parameter. We systematically analyze the collisions of ^{12}C + ^{16,18}O, ^{12}C + ^{28,30}Si, ^{12}C + ^{46,48,50}Ti, ^{16}O + ^{28,30}Si, ^{16}O + ^{70,72,74,76}Ge, ^{16}O + ^{144,148,154}Sm, ^{28}Si + ^{28,30}Si and ^{32,34,36}S + ^{58}Ni within the framework of Skyrme energy density formalism using different Skyrme forces and observed that the value of diffuseness parameter is not fixed like 0.63 fm from elastic scattering data but shows significantly deviation from it for different colliding partners. Further, surface diffuseness shows linear dependence with N/Z content of the colliding nuclei.**

Keywords: Fusion, Fusion barrier height, Fusion cross-section, Potential, Surface Diffuseness

I. INTRODUCTION

During last few decades, huge experimental data has been accumulated on the fusion cross-sections and excitation function involving large number of stable (symmetric and asymmetric) as well as unstable colliding nuclei [1-8]. The enriched experimental data has led to the development of large number of theoretical model. The bench mark of these model is either proximity formalism or Woods-Saxon form of parametrization. More than dozen of such potentials are available in the literature with slight change in the radius, diffuseness, surface energy coefficient and universal function parameter. The outcome of these models are drastically affected by slight change in these parameters. In particular the surface diffuseness parameter significantly affect the shape of nuclear potential. Surface diffuseness parameter used in Woods-Saxon form of potential have been extracted from a large number of experimentally studied neutron-rich fusion cross sections at near barrier energies [3-4].

The beneficial of parametrizing ion-ion potential calculated from the microscopic concept and from first principle is that one does not compromise with the physics and at the same time one can also have the simple form of ion-ion potential. Without parameterization, one need to derive the theory and model that to calculates the ion-ion potential. If one has parametrized form, ion-ion potential can be calculated directly. If one looks into the literature and alternatively can find out that most of the attempts far ion-ion potential are within energy density formalism based on Skyrme force – a force and idea that introduced a great simplification over Hartee-Fock calculations [5]. Heavy ion collisions from a few MeV/nucleon to a few GeV/nucleon provide a variety of phenomena which occur at different bombarding energies. Due to the lack of free phase space of low energies, almost all nucleon-nucleon collisions are absent and thus the collisions of two heavy nuclei at low energy leads to phenomena like complete fusion or deep inelastic collision, depending upon the impact parameter. As one goes to higher energies, frequent nucleon-nucleon collisions govern the fate of heavy ion dynamics and therefore, processes, like incomplete fusion, particle production can be observed. A large amount of work has been done using the phenomenological forces which account for the mean field potential of nucleons [6-8]. In phenomenological force, one fits the parameter to known ground state properties of nuclei and thus can generate several sets of parameters which can reproduce the ground state properties of nuclei with similar accuracy. Among various phenomenological forces the Skyrme interaction is widely used to study the heavy ion collisions from low to intermediate energies. The Skyrme force has been fitted to generate the standard hard and soft parametrization depending upon the incompressibility. Very few attempts have been made using the different Skyrme forces to study the low energy phenomena like fusion of two nuclei, their excitation functions etc. [9-16].

The enriched experimental data has also led to the development of large number of theoretical model [17]. The nuclear dynamics at low, intermediate and high energies depend on the interplay between the real and imaginary parts of the interaction potential [9]. At low incident energies, where fusion is dominant channel the contribution of the imaginary part is negligible, therefore, the real part of the interaction potential plays vital role. At relativistic energies ($\geq$ GeV/ nucleons), frequent nucleon-nucleon collision make the imaginary part of the potential are equally comparable. The dynamics at low energies carries vital information about the nuclear interaction. Most of the theoretical models are based on the either phenomenological idea or one based on microscopic/macroscopic picture. In this picture, Energy density model have been a great success. Among energy density model one due to Skyrme enjoy special status because each of the terms of Hamiltonian can be analyze separately

[12]. In this paper our interest is to study the fusion of colliding nuclei (surface phenomenon), therefore, we neglect the imaginary part of the interaction potential. Several phenomenon at high density phase are found to be very sensitive towards the nature of Skyrme force or equation of state. Since fusion is a low density phenomenon occurs at the outer surface of the nuclei, the sensitivity of different forces can reproduce the experimental data on fusion and also represented a parameterization on that [3,5,8,9].

In the following, we first discuss briefly the Skyrme energy density formalism which also includes the spin-orbit density contribution. Our results are presented in section 3.

II Skyrme Energy Density Formalism

In the Skyrme Energy Density Formalism (SEDF) [3,5,8,9,12,13,18], the nucleus-nucleus interaction potential V_N (R) is defined as the difference between the energy expectation of colliding system at a separation distance R and at complete isolation (i.e. at ∞)

$$V_N(R) = E(R) - E(\infty) \quad (1)$$

The energy expectation distance value E for the energy density functional H (r) of Vautherin and brink [17] is given by

$$E=\int H(\vec{r})d\vec{r} \ , \quad (2)$$

Where the Hamiltonian for an even-even spherical nucleus (N=Z) is given by

$$H(\rho,\tau,\vec{J})=$$

$$\frac{\hbar^2}{2m}\tau + \frac{1}{2}t_0\left[(1+\frac{1}{2}x_0)\rho^2 - (x_0+\frac{1}{2})(\rho_n^2+\rho_p^2)\right] + \frac{1}{4}(t_1+t_2)\rho\tau$$
$$+\frac{1}{8}(t_2-t_1)(\rho_n\tau_n+\rho_p\tau_p) + \frac{1}{16}(t_2-3t_1)\rho\nabla^2\rho$$
$$+\frac{1}{32}(3t_1+t_2)(\rho_n\nabla^2\rho_n+\rho_p\nabla^2\rho_p) + \frac{1}{4}t_3\rho_n\rho_p\rho$$
$$-\frac{1}{2}W_0(\rho\vec{\nabla}.\vec{J}+\rho_n\vec{\nabla}.\vec{J}_N+\rho_P\vec{\nabla}.\vec{J}_P). \quad (3)$$

Using Eqs. (1-3), the nucleus-nucleus interaction potential V_N (R) is given by

$$V_N(R)=\int[H(\rho,\tau,J)-H_1(\rho_1,\tau_1,J_1)-H_2(\rho_2,\tau_2,J_2)], \quad (4)$$

Here the Skyrme Hamiltonian density H is Eq. (3), comprising of nucleonic density *(ρ),* kinetic energy density (τ) and spin density ($\vec{J}$), and n, p, refers, respectively, to the neutron and proton numbers. One also has $\rho=\rho_n+\rho_p$, $\tau=\tau_n+\tau_p$ and $\vec{J}=\vec{J}_N+\vec{J}_P$, representing the total nucleonic density, kinetic energy density and spin density. The Skyrme force parameters t_0, t_1, t_2, t_3, W_0 and x_0 have been fitted by different authors to describe the ground state properties of large number of nuclei across the periodic table. The first set of parameterization was given by Skyrme himself (labeled as S) [19]. This set of parameters, however, gave too small radii for heavy nuclei. Later on, two different forces SI and SII were fitted by Vautherin and Brink [17] succeeded in reproducing the radii and binding energies of whole of the periodic table using microscopic self-consistent calculations within Skyrme force.

The kinetic energy density τ In Eq. (4) can further be written in terms of Thomas-Fermi kinetic energy density τ_{TF} corrected for additional surface effects as [20-21]:

$$\tau = \tau_{TF} + \tau_\lambda = \frac{3}{5}\left(\frac{3}{2}\pi^2\right)^{2/3}\rho^{5/3} + \lambda\frac{(\vec{\nabla}\rho)^2}{\rho}, \quad (5)$$

τ_λ gives the surface correction due to von-Weizsäcker, with λ having values between 1/9 and 9/36. The Skyrme Hamiltonian density H now becomes a functional of ρ and $\vec{J}$ only. We apply the standard Fermi mass density distribution for nucleonic density:

$$\rho_i(R) = \frac{\rho_{0i}}{1+\exp\left\{\frac{R-R_{0i}}{a_i}\right\}}, \qquad -\infty \le R \le \infty \quad (6)$$

The average central density ρ_{0i} given by

$$\rho_{0i} = \frac{3A_I}{4\pi R_{0i}^3}\frac{1}{1+\frac{\pi^2 a_i^2}{R_{0i}^2}}, \quad (7)$$

$R_{0i}(=C_i$, the Sussman central radii are related to the effective sharp radii as C_i = (R_i -1/ R_i, R_i = 1.28A$^{1/3}$ - 0.76 + 0.8A$^{1/3}$) and a_i are, respectively, the half-density radii and surface diffuseness parameters.

Earlier, the contribution of spin-orbit density ($\vec{J}$) was neglected either by studying the spin saturated ($\vec{J}$ = 0) nuclei or neglecting its contribution. Vautherin and Brink calculated the spin-orbit density $\vec{J}$ for nuclei with closed J-shell. Later on, Puri et al. extended the calculations of Vautherin and Brink [17] to spin unsaturated ($\vec{J}\neq 0$) nuclei. the spin-orbit density reads as:

$$\vec{J}_q(\vec{r}) = \frac{\vec{r}}{4\pi r^4}\sum\left(2j_\alpha+1\right)\left[j_\alpha(j_\alpha+1)-l_\alpha(l_\alpha+1)-\frac{3}{4}\right]R_\alpha^2(\vec{r}), \quad (8)$$

Here summation over α refers to all the occupied single particle orbitals, with α = q, n, *l*. The above Eq. (8) is valid if nucleus is a closed J-shell nucleus i.e. either j = $l+\frac{1}{2}$ or j = $l-\frac{1}{2}$ is filled. Note that if both j = $l\pm\frac{1}{2}$ are occupied, the contribution of J ($\vec{r}$) vanishes. For an even-even nucleus with valence particles (or holes) outside (or inside) the closed core, Puri et al. [5], divided the $\vec{J}_q$ ($\vec{r}$) into two parts (for q = n or p) as [12]

International Journal of Pure and Applied Physics. ISSN 0973-1776 Volume 13, Number 1 (2017), pp. 84-88
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$$\vec{J}_q(\vec{r}) = \vec{J}_q^{\,c}(\vec{r}) \pm \vec{J}_q^{\,n_v}(\vec{r}), \quad (9)$$

Where $\vec{J}_q^{\,c}(\vec{r})$ and $\vec{J}_q^{\,n_v}(\vec{r})$, respectively, gives the contribution due to the core consisting of closed shells and due to the valence n_v particles or holes outside the closed core, with (+) for particles and (-) for holes. Puri et al. [12], calculated $\vec{J}_q^{\,n_v}(\vec{r})$ for $n_v = 4$ particles and generalized it to even number of valence particles (or holes) to the following form:

$$\vec{J}_q^{\,n_v}(\vec{r}) = \frac{n_v\vec{r}}{4\pi r^4}\left\{ j(j+1) - l(l+1) - \frac{3}{4}\right\} R_l^2(\vec{r}). \quad (10)$$

Using Eq. (5), the Hamiltonian $H(\rho, \tau, \vec{J})$ reduces to a function of nucleonic density ρ and spin-orbit density $\vec{J}$ only. The nuclear interaction potential $V_N(R)$ is solved by.

separating the terms containing spin-independent $V_P(R)$ and spin-dependent $V_J(R)$ parts as:

$$V_N(R) = V_P(R) + V_J(R),$$

Where

$$V_P(R) = \int \left\{ H(\rho,\tau) - \left[H_1(\rho_1,\tau_1) + H_2(\rho_2,\tau_2)\right]\right\} d\vec{r}$$

And,

$$V_J(R) = \int \left\{ H(\rho,\vec{J}) - \left[H_1(\rho_1,\vec{J}_1) + H_2(\rho_2,\vec{J}_2)\right]\right\} d\vec{r} \quad (11)$$

Following Puri et al. [5], $V_P(R)$ in Eq. (11) can be calculated in the spirit of proximity theorem. Since the spin-orbit density term involves the structural effects of the colliding nuclei, one cannot use the above proximity concept here. One has to solve this part of the potential directly. The total interaction potential is calculated by adding Coulomb potential to Eq. (11) as:

$$V_T(R) = V_N(R) + \frac{Z_1 Z_2 e^2}{r}. \quad (12)$$

The barrier height V_B and position R_B are then determined from the condition:

$$\frac{dV_T(r)}{dr}\Big|_{r=R_B} = 0; \text{and} \frac{d^2V_T(r)}{dr^2}\Big|_{r=R_B} \le 0; \quad (13)$$

In other words, the fusion cross barrier is defined by its position at the distance $R = R_B$, and the height $V(R = R_B) = V_B$. From the knowledge of the height and the position of the barrier, the fusion cross-section can be calculated by using the so called sharp-cutoff model [3],

$$\sigma_{\text{fus}}(mb) = \pi R_B^2 (1 - \frac{V_B}{E_{cm}}). \quad (14)$$

III RESULT AND DISCUSSION

The present study is conducted within the framework of SEDF by using the above set of Skyrme forces and even-even colliding nuclei. The diffuseness parameter is firstly extracted using SEDF by comparing the outcome with experimental fusion cross section values for the collision of ^{12}C + ^{16,18}O, ^{12}C + ^{28,30}Si, ^{12}C + ^{46,48,50}Ti, ^{16}O + ^{28,30}Si, ^{16}O + $^{7(0,2,4,6)}$Ge, ^{16}O + ^{144,148,154}Sm, ^{28}Si + ^{28,30}Si and ^{32,34,36}S + ^{58}Ni [22-30]. As a next step the diffuseness parameter is taken here to be a free parameter. We systematically vary the value of '*a*' to best fit the available experimental data on fusion cross-sections. Total interaction potential is calculated by using Eq. (12). Once the total interaction potential is calculated, the fusion cross-section is calculated using Wong model [31] with parabolic approximation. The best suited value we obtained using Skyrme force – SIII is plotted with N/Z ratio in figure 1.

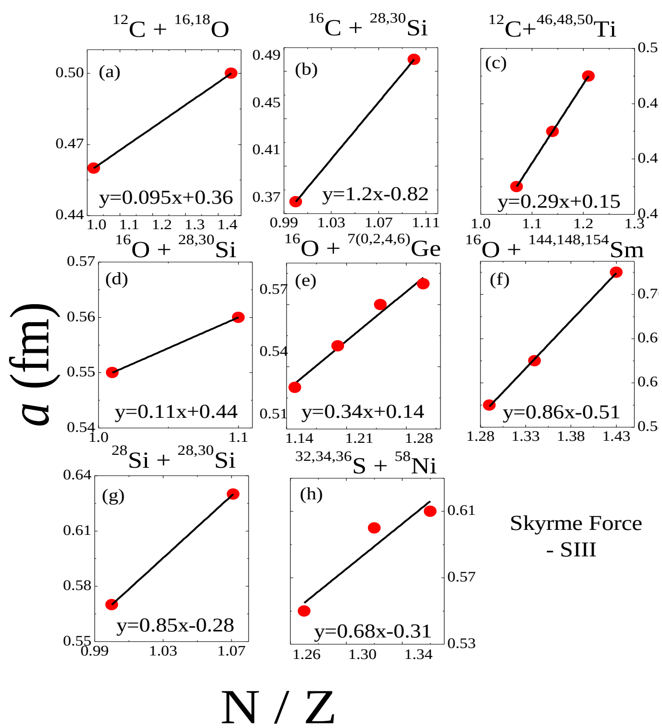

Fig 1: The variation of surface diffuseness parameter '*a*' with N/Z ratio of colliding nuclei namely ^{12}C + ^{16,18}O, ^{12}C + ^{28,30}Si, ^{12}C + ^{46,48,50}Ti, ^{16}O + ^{28,30}Si, ^{16}O + ^{70,72,74,76}Ge, ^{16}O + ^{144,148,154}Sm, ^{28}Si + ^{28,30}Si and ^{32,34,36}S + ^{58}Ni for Skyrme force - SIII. The solid line represents the straight line least square fit over the data point.

Also noticed that slight changes in the value of diffuseness parameter '*a*' drastically affect the fusion probabilities. Further, the colliding partners having larger N/Z ratio show strong dependence on '*a*'. In figure 2, the variation of surface diffuseness parameter with N/Z ratio of colliding nuclei namely ^{16}O + ^{70,72,74,76}Ge for different Skyrme forces S, SI, SII, SIII, SIV, SV, SVI, Ska, Skm and Skm*.

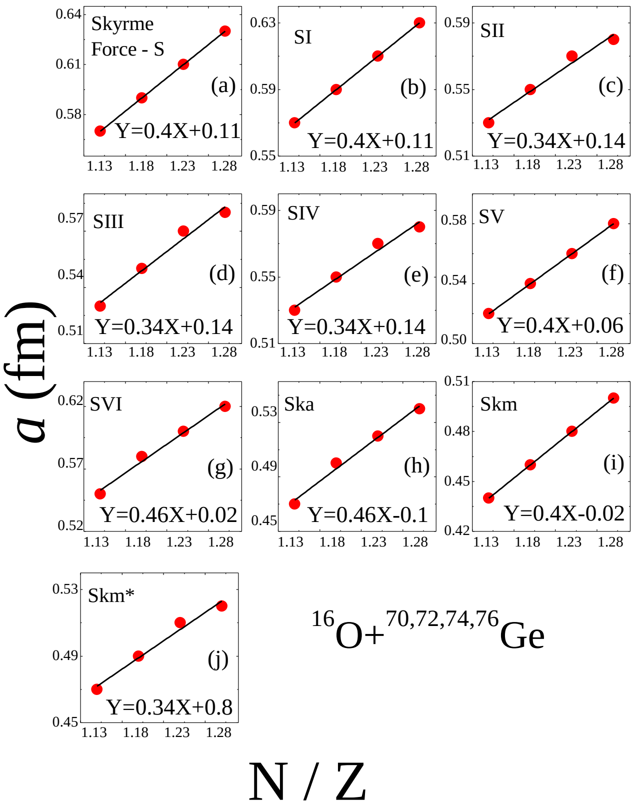

Fig 2: The variation of surface diffuseness parameter '*a*' with N/Z ratio of colliding nuclei namely ^{16}O + $^{70,72,74,76}Ge$ for different Skyrme forces S, SI, SII, SIII, SIV, SV, SVI, Ska, Skm and Skm*. The solid line represents the straight line least square fit over the data points.

Our systematic study over large number of colliding nuclei reveals that the surface diffuseness parameter extracted from measured fusion cross sections follow a linear trend with N/Z content of the colliding nuclei for different Skyrme forces. Further, the extracted value of diffuseness parameter is independent of the choice of Skyrme force parameter used [32].

ACKNOWLEDGEMENT

we are thankful to Prof. R. K. Puri, Department of Physics, Punjab University Chandigarh for many fruitful discussions. The authors are also thankful to SERB, New Delhi for financial support under Project No. SB/FTP/PS-025/2013 under fast track scheme for young scientists.

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International Journal of Pure and Applied Physics. ISSN 0973-1776 Volume 13, Number 1 (2017), pp. 84-88
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