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Controlling self-similar waves through two Riccati parameter

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Abstract— **We present bright similaritons and self-similar first-order and second-order rogue waves for generalized nonlinear Schrödinger equation (GNLSE) modeling wave propagation in tapered graded-index nonlinear waveguide. The exact solutions have been worked out by employing similarity transformations which involves the mapping of GNLSE to constant-coefficient standard nonlinear Schrödinger equation (NLSE) for certain constraints on parameters. The constraint equation for tapering resembles Schrödinger equation in Quantum mechanics enabling us to introduce Isospectral Hamiltonian approach of supersymmetric Quantum Mechanics in nonlinear optics employing which we can control the intensity and width of self-similar waves. We extend the Isospectral Hamiltonian approach for two Riccati parameters and study their combined control on the intensity and width of self-similar waves in tapered graded-index nonlinear waveguide. This further widens the parameter space to amplify the signals which can be useful for communication purposes.**

Index Terms— ***tapered graded-index nonlinear waveguide; similarity transformation technique; self-similar waves; isospectral Hamiltonian approach; two Riccati parameters***

I. Introduction

In recent years, there has been considerable interest experimentally as well as theoretically in the use of tapered graded-index nonlinear waveguide in optical communication systems [1]. Tapering effect tends to reduce reflection losses and mode mismatch [2] which results in improving the coupling efficiency between fibers and waveguides. Tapering also finds applications in highly efficient Raman amplification [3] and extended broadband supercontinuum generation [4]. We are considering tapered graded-index nonlinear waveguide with refractive index given as

$$n(X,Z) = n_0 + n_1 F(Z) X^2 + n_2 I(X,Z) \qquad (1)$$

where X and Z stand for the dimensionless spatial co-ordinate and the propagation distance respectively. The first two term represent the linear contribution towards the refractive index and the third term is intensity dependent $I(X,Z)$ which arises due to Kerr type nonlinearity. The shape of the taper $F(Z)$ can be modeled appropriately depending upon the practical requirements. There exist variety of techniques to obtain solutions of GNLSE and one of the important technique is to use self-similarity transformation [5]-[7]. The basic idea behind the self-similar transformation technique is to reduce the GNLSE to standard NLSE using gauge and similarity transformation and then inverse map the solutions of standard NLSE to that of GNLSE. Thus, the solutions are named as self-similar solutions. Self-similar waves maintain their overall shape but their amplitude and width change with the modulation of system parameters [8]. In this work, we analyze bright similaritons, self-similar first-order and second-order rogue wave solutions. Bright optical solitons exist due to delicate balance between nonlinearity and dispersion. Rogue waves or freak waves or monster waves have their early observation in oceans and seas [9]. They are defined as the waves having height approximately three times the height of average wave crest. They have the characteristics of appear from nowhere and disappear without a trace [10]. There are not so many explanations for formation of rogue waves, but a possible explanation can be modulation instability also known as Benjamin-Fier instability [11] which describes an instability of certain class of initial conditions which tend to grow exponentially. Peregrine, in 1983, first reported the rational solutions of NLSE [12] which are prototype of rogue waves. Rogue wave solutions can be obtained as limiting solutions of breather (Akhmediev breather and Ma breather) solutions. Higher order rogue wave solutions can be obtained from the lower ones with the aid of Darboux transformation technique. Recently, Solli et al.[13] have found rogue wave solutions in nonlinear optics which opens the doors for a wide range of applications such as communications [14], supercontinuum generation [15] etc. The management and control of optical rogue waves as they propagate through nonlinear optical fiber has been discussed in [16], [17].

Recently, authors [18], [19] have presented a theoretical technique, to control the intensity and widths of self-similar waves, by invoking the Isospectral Hamiltonian approach introduced by Infeld and Hull [20] and Mielnik [21]. While reducing GNLSE to standard NLSE to obtain self-similar solutions, the constraint equation for tapering function resembles Schrödinger equation in quantum mechanics (QM), with tapering function as potential and width function as ground-state wavefunction. Thus, employing Isospectral Hamiltonian approach from supersymmetric quantum mechanics (SUSYQM) [22] in tapered graded-index waveguide, we can generate a class of tapering function $F(Z)$ and width function $W(Z)$ by introducing Riccati parameter. In this work, we extend the same approach for two Riccati

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parameter and study their effect on the intensity of self-similar waves.

II. MODEL EQUATION AND SELF-SIMILAR SOLUTIONS

The propagation of pulses through tapered graded-index nonlinear waveguide amplifier with refractive index given by Eq.(1) is governed by following GNLSE

$$i\frac{\partial U}{\partial Z}+\frac{1}{2}\frac{\partial^2 U}{\partial X^2}+F(Z)\frac{X^2}{2}U-\frac{i}{2}G(Z)U+|U|^2U=0, \qquad (2)$$

where X and Z are dimensionless variables and $F(Z)$, $G(Z)$ are the dimensionless tapering function and gain profile respectively. Eq.(2) has already been solved for non-autonomous 1- and 2-soliton solutions using inverse scattering technique [23]. Bright and dark similariton solutions [24], [25] and self-similar optical rogue wave solutions for Eq.(2) has also been obtained by reducing it to standard NLSE using self-similar transformation [16]. The gauge and similarity transformation can be given as

$$U(X,Z)=\frac{1}{W(Z)}\psi[\chi(X,Z),\zeta(Z)]e^{i\Phi(X,Z)}, \qquad (3)$$

where $\chi(X,Z)=\frac{X-X_c(Z)}{W(Z)}$ .On substituting Eq.(3) in Eq.(2), it reduces to standard NLSE

$$i\frac{\partial\psi}{\partial\zeta}+\frac{1}{2}\frac{\partial^2\psi}{\partial\chi^2}+|\psi|^2\psi=0, \qquad (4)$$

where

$$\zeta(Z)=\zeta_0+\int_0^Z\frac{1}{W^2(S)}ds,$$

$$X_c(Z)=W(Z)\left(X_0+C_{02}\int_0^Z\frac{1}{W^2(S)}dS\right),$$

$$\Phi(X,Z)=\frac{X^2}{2W}\frac{dW}{dZ}+\frac{C_{02}X}{W}-\frac{C_{02}^2}{2}\int_0^Z\frac{1}{W^2(S)}dS, \qquad (5)$$

with X_0 and C_{02} are constants. The constraints for tapering, gain and width function are given as,

$$\frac{d^2W}{dZ^2}-F(Z)W=0, \qquad (6)$$

$$G(Z)=-\frac{d}{dZ}[\ln W(Z)], \qquad (7)$$

We can obtain analytical solutions of Eq.(2) by writing the well known solutions, solitons and rogue waves, of standard NLSE. The general expression of the intensity for bright similaritons I_B, self-similar first order rouge waves I_{R_1} and self-similar second-order rouge waves I_{R_2} for Eq.(2) is given as

$$I_B(X,Z)=\frac{a^2}{W^2}\operatorname{sech}^2[a(\chi-v\zeta)], \qquad (8)$$

$$I_{R_1}(X,Z)=\frac{1}{W^2}\left[1+8\frac{1+4\zeta^2-4\chi^2}{(1+4\zeta^2+4\chi^2)^2}\right], \qquad (9)$$

$$I_{R_2}(X,Z)=\frac{1}{W^2}\left[\left(\frac{d-g}{d}\right)^2+\left(\frac{h}{d}\right)^2\right], \qquad (10)$$

where

$$d=\frac{1}{3}(\chi^2+\zeta^2)^3+\frac{1}{4}(\chi^2-3\zeta^2)^2+\frac{3}{64}(12\chi^2+44\zeta^2+1),$$

$$g=\left(\chi^2+\zeta^2+\frac{3}{4}\right)\left(\chi^2+5\zeta^2+\frac{3}{4}\right)-\frac{3}{4},$$

$$h=\zeta\left(\zeta^2-3\chi^2+2(\chi^2+\zeta^2)^2-\frac{15}{8}\right).$$

Here, parameters a and v are the amplitude and velocity of bright soliton solution of NLSE. Eq.(6) resembles with Schrödinger equation of quantum mechanics where width $W(Z)$ acts as wave function and tapering $F(Z)$ as potential. Making this identification, authors have used Isospectral Hamiltonian approach [22] to generate a class of tapering and width function and thus obtain Riccati parameterized highly intense pulses, by introducing a free parameter, known as Riccati parameter [18], [19].

The general form of tapering for sech^2–type waveguide is

$$F(Z)=n^2-n(n+1)\operatorname{sech}^2 Z, \qquad (11)$$

with width and gain profiles given by

$$W(Z)=\operatorname{sech}^n Z, \quad G(Z)=n\tanh Z. \qquad (12)$$

For n = 1, the Riccati parameterized intense pulses have been obtained for single Riccati parameter by generating a class of tapering function $F(Z)$, width function $W(Z)$ and gain function $G(Z)$.

III. EXTENSION TO TWO RICCATI PARAMETER FAMILY

We extend this analytical approach to introduce two Riccati parameters which further widens the parameter space to control the intensity and width of self-similar waves. We can generate a class of tapering, width function and gain, involving two Riccati parameters c_1 and c_2, given as [22],

$$\hat{F}(c_1,c_2)=F(Z)-2\frac{d^2}{dZ^2}\ln\left[W_1\left(c_2+\int_{-\infty}^Z W_2^2dZ\right)\phi(c_1,c_2)\right], \qquad (13)$$

$$\hat{W}(Z)=\frac{3\sqrt{c_1(c_1+1)}}{\phi(c_1,c_2)}, \qquad (14)$$

$$\hat{G}(Z)=-\frac{d}{dZ}[\ln\hat{W}(Z)], \qquad (15)$$

where

$$W_1=\sqrt{\frac{3}{4}}\operatorname{sech}^2 Z,$$

$$W_2=\sqrt{\frac{1}{4}}\operatorname{sech} Z,$$

$$\phi_i=\frac{\int_{-\infty}^Z W_i^2+c_i}{W_i},$$

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$$\phi_1(c_1,c_2) = -\frac{d^2\phi_1}{dZ^2} + \frac{d\phi_1}{dZ}\frac{d(\ln W_2)}{dZ} + \phi_1\frac{d^2(\ln W_2)}{dZ^2} + \frac{d\phi_1}{dZ}\frac{d(\ln \phi_2)}{dZ} - \phi_1\frac{d(\ln \phi_2)}{dZ}\frac{d(\ln W_2)}{dZ}.$$

The Riccati parameters c_1 and c_2 have significant effect on the intensity profiles of self-similar waves. We can control the intensity of self-similar waves by suitably choosing c_1 and c_2. The intensity profiles for bright similaritons, first-order and second-order self-similar rogue waves are plotted for different values of c_1 and c_2 in Fig.1, Fig.2 and Fig.3 respectively. For smaller values of c_1 and larger values of c_2 the intensity of self-similar waves increases significantly. Also, the relative compression of pulses occur with the modulation of Riccati parameters c_1 and c_2 which can be proved useful in the design and realization of ultrahigh repetition rate sources.

The tapering function $F(Z)$ and gain function $G(Z)$ given by Eq.(13) and Eq.(15) are plotted in Fig.4.

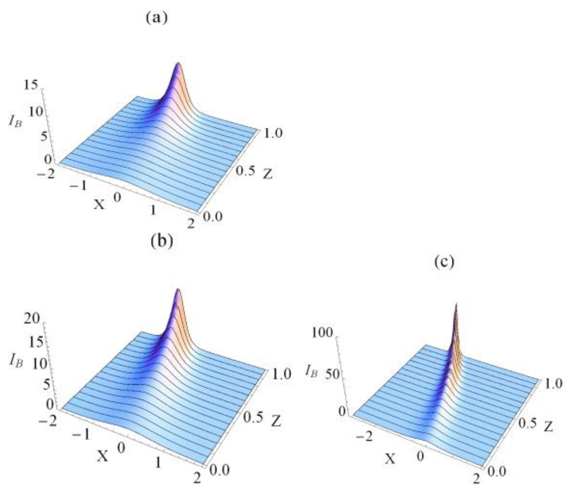

Fig.1. Intensity profiles for bright similaritons for (a) c_1= 1, c_2= 0.1, (b) c_1= 1, c_2= 10 and (c) c_1= 0.1, c_2= 10. The other parameters used in the plots are C_{02}= 0.3, v= 0.3 and a= 1.

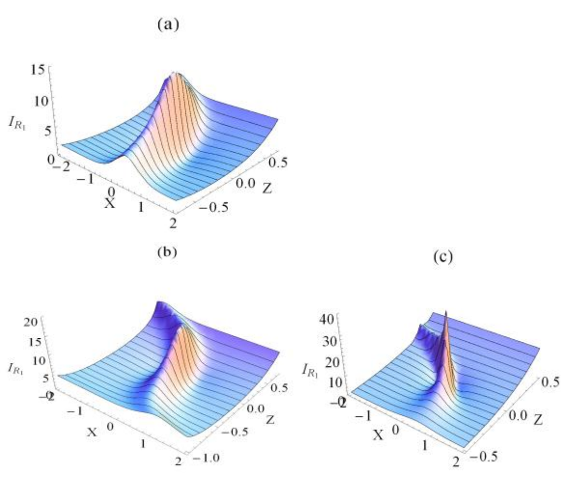

Fig.2. Intensity profiles for self-similar first-order rogue waves for (a) c_1= 1, c_2= 0.1, (b) c_1= 1, c_2= 10 and (c) c_1= 0.1, c_2= 10.

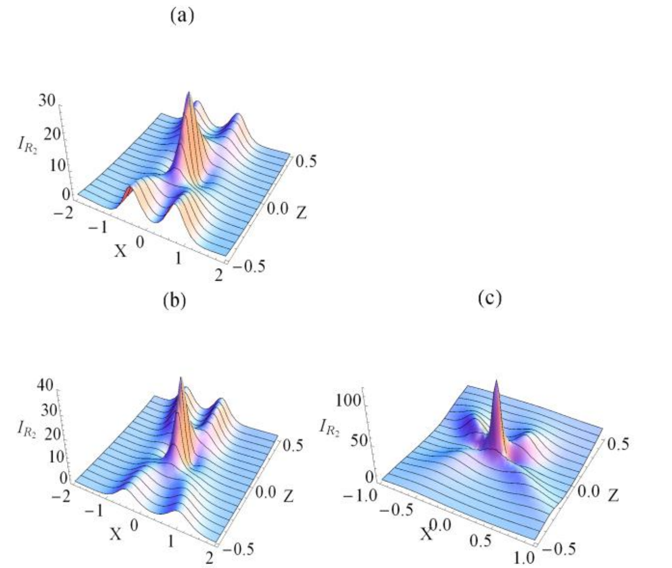

Fig.3. Intensity profiles for self-similar second-order rogue waves for (a) c_1= 1, c_2= 0.1, (b) c_1= 1, c_2= 10 and (c) c_1= 0.1, c_2= 10.

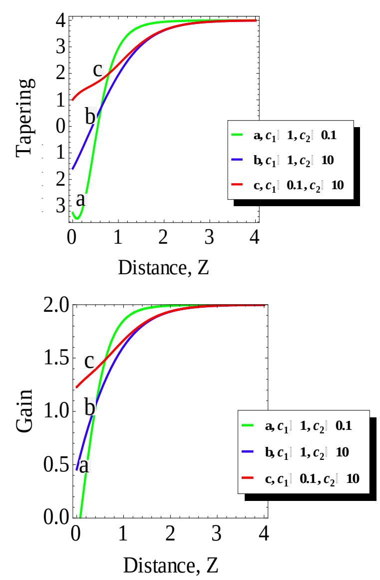

Fig.4 Tapering and Gain profile for different values of c_1 and c_2.

IV. CONCLUSIONS

While reducing GNLSE to standard NLSE using similarity transformation technique for tapered graded-index nonlinear waveguide, the constraint equation for tapering function resembles Schrödinger equation in quantum mechanics. This paves the way for introducing Isospectral Hamiltonian approach of SUSYQM in nonlinear optics. We generate a class of tapering, width function and gain extended to two Riccati parameter generalization. This widens the parametric space to control the intensity of self-similar waves propagating through tapered graded-index nonlinear waveguide. The two Riccati parameter generalization would be more useful for experimental realization of highly energetic pulses for practical applications.

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