

# Study of neck length parameter for complete fusion of ${}^9\text{Be}$ induced reactions

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**Abstract**—The reaction dynamics of heavy ion induced reactions is strongly influenced by the structure of interacting nuclei. The neck between two interacting nuclei plays a significant role in the reaction dynamics. Gupta and collaborators have successfully presented the interaction barrier modification characteristics within Dynamical Cluster Decay model (DCM) through the neck length parameter ( $\Delta R$ ). Recently the total fusion cross section ( $\sigma_{\text{fus}}$ ) in the reactions induced by loosely bound projectiles  ${}^7\text{Li}$ ,  ${}^9\text{Be}$ , on different targets were studied by employing unique neck length parameter  $\Delta R$  within DCM approach. However, the study was confined to only total fusion process. In the present work has been employed to study complete fusion (CF). CF has been observed experimentally in  ${}^9\text{Be} + {}^{144}\text{Sm}$  and  ${}^9\text{Be} + {}^{208}\text{Pb}$  reactions. Here we calculate the CF cross section for  ${}^9\text{Be} + {}^{144}\text{Sm}$  reaction by using particular value of empirically fixed  $\Delta R^{\text{CF}}$  at given incident energy  $E_{\text{lab}}=37$  MeV. The same value of  $\Delta R^{\text{CF}}$  is used to calculate the CF cross section of  ${}^9\text{Be} + {}^{208}\text{Pb}$  at same incident energy, which are in good comparison with experimental data.

## I. INTRODUCTION

The heavy ion induced reactions are strongly influenced by the structure of interacting/decaying fragments and the associated reaction dynamics. The neck of two interacting/decaying nuclei plays a significant role in the reaction. In low energy reactions when two nuclei come close to each other, there is interplay between, electrostatic force of repulsion and attractive nuclear force. Once this strong nuclear interaction overcomes the Coulombic repulsion, there is a formation of neck between the nuclei which held them together and is referred as neck length parameter  $\Delta R$ . Similar kind of dynamics comes into picture for the binary decay process, where the neck formation takes place between outgoing fragments. Gupta and collaborators successfully presented the Dynamical Cluster Decay Model (DCM) which based on two step process [1].

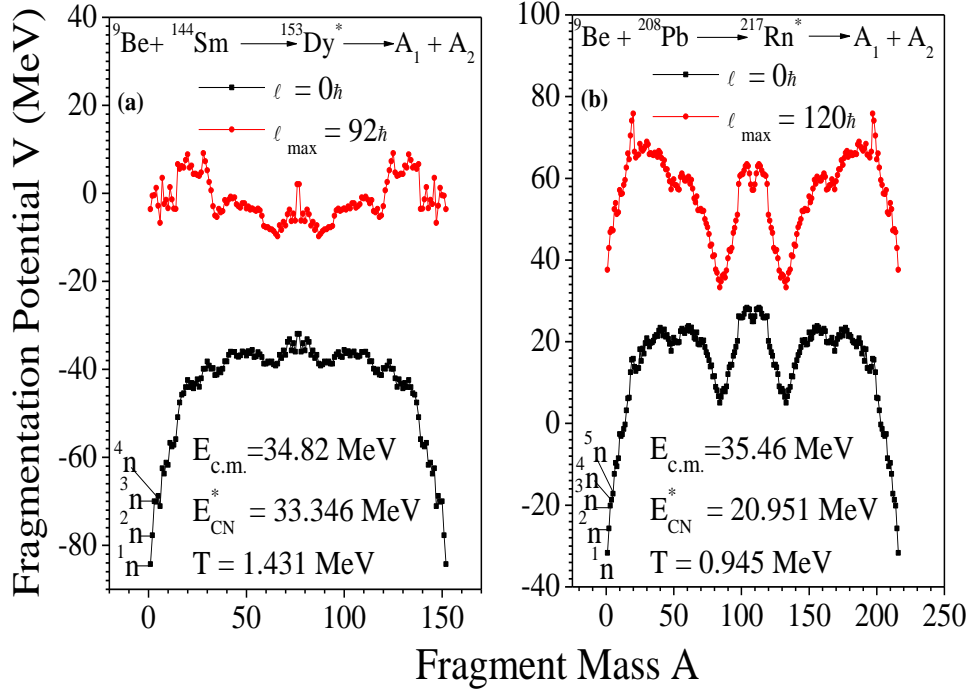
In DCM, collective mass motion of clusters (with certain preformation probability ( $P_0$ )) which lead to neck formation between the outgoing fragments, and their further penetration through the potential barrier (evaluated as penetration probability ( $P$ )), are treated quantum mechanically.

Interestingly, it has been worked out that the empirically fitted  $\Delta R^{\text{emp}}$  simply results in the corresponding barrier lowering  $\Delta V_B$  for a given reaction, which is inbuilt property of DCM [2]. The study of nuclear reactions induced by weakly bound projectiles around the coulomb barrier is of great interest since last decade, experimentally as well theoretically. Such studies play an important role for the formation of exotic nuclei. The DCM has been applied to number of nuclear reactions induced by stable as well as weakly bound projectiles. Recently, the major role of neck length parameter is found in the predictability of DCM [3-5]. The number of reaction induced through loosely bound projectiles  ${}^7\text{Li}$  and  ${}^9\text{Be}$  has been investigated to study the fusion cross section ( $\sigma_{\text{fus}}$ ). The  $\sigma_{\text{fus}}$  induced by the same loosely bound projectile, with same incident energy has been studied, while fixing  $\Delta R^{\text{emp}}$  empirically. The  $\sigma_{\text{fus}}$  for all other reactions, having same projectile and same incident energy on different targets, has been calculated or predicted by using the same value of  $\Delta R^{\text{emp}}$  [3]. We extended this study further for stable projectile  ${}^{32}\text{S}$  also at fixed  $E_{\text{lab}}=142$  MeV on different targets and have successfully fitted the experimental data of  $\sigma_{\text{fus}}$  for all the reactions at same value of  $\Delta R^{\text{emp}}$  [4]. We have also studied the isospin ( $N/Z$ ) effects for compound nuclei (CN) having  $A=60$ , within the DCM. The experimental data for CN  ${}^{60}\text{Fe}^*$ ,  ${}^{60}\text{Ni}^*$  and  ${}^{60}\text{Zn}^*$  is not available. However, following the above mentioned characteristic of DCM we have fixed  $\Delta R^{\text{emp}}$  for the reactions for which experimental data is available, having same projectile and the incident energy. The  $\sigma_{\text{fus}}$  for CN  ${}^{60}\text{Fe}^*$ ,  ${}^{60}\text{Ni}^*$  and  ${}^{60}\text{Zn}^*$  is then calculated with the fitted  $\Delta R^{\text{emp}}$ , which enabled us to investigate the decay of CN  ${}^{60}\text{Fe}^*$ ,  ${}^{60}\text{Ni}^*$  and  ${}^{60}\text{Zn}^*$  [5].

In present work, we further attempt to study CF process in  ${}^9\text{Be} + {}^{144}\text{Sm}$  and  ${}^9\text{Be} + {}^{208}\text{Pb}$  reactions. Here we calculate the CF cross section for  ${}^9\text{Be} + {}^{144}\text{Sm}$  reaction by using particular value of empirically fixed  $\Delta R^{\text{CF}}$  at given incident energy  $E_{\text{lab}}=37$  MeV. The same value of  $\Delta R^{\text{CF}}$  is used to calculate

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**Fig 1:** The variation of fragmentation potential  $V$  (MeV) with fragment mass  $A$  for  $^{153}\text{Dy}^*$  and  $^{217}\text{Rn}^*$  formed through  $^9\text{Be}$  induced reactions on different targets at same incident energy  $E_{\text{lab}}=37$  MeV for two extreme  $\ell$ -values at best fitted  $\Delta R=1.655$  fm.

the CF cross section of  $^9\text{Be} + ^{208}\text{Pb}$  at same incident energy and in good comparison with the experimental data [6,7].

## II. Dynamical Cluster Decay Model (DCM)

The DCM, QMFT based cluster model is different from another statistical model as it treats the evaporation residues (ERs), intermediate mass fragments (IMFs) and fusion fission (ff) on equal footings. The missing nuclear structure information of compound nucleus in statistical model enters in DCM via preformation probability  $P_0$  of the fragments and is calculated by solving Schrodinger equation in  $\eta$  co-ordinate. For  $\ell$ -partial waves, the compound nucleus decay cross-section is given by

$$\sigma = \frac{\pi}{k^2} \sum_{l=0}^{l_{\text{c,orl,max}}} (2l+1) P_0 P \quad k = \sqrt{\frac{2\mu E_{\text{c.m.}}}{\hbar^2}} \quad (1)$$

Where,  $\mu = [A_1 A_2 / (A_1 + A_2)]m$ , is the reduced mass, with  $m$  as the nucleon mass and  $l_{\text{max}}$  is the maximum angular momentum. The angular momentum  $l_{\text{max}}$  is fixed for vanishing the fusion barrier of incoming channel  $\eta_i$  or light particles cross-section  $\sigma_{\text{LP}}$ . The total fusion cross-section within this model is given by

$$\sigma_{\text{fus}} = \sigma_{\text{ER}} + \sigma_{\text{IMF}} + \sigma_{\text{ff}} + \sigma_{\text{nCN}} \quad (2)$$

Where  $\sigma_{\text{ER}}$ ,  $\sigma_{\text{ff}}$  and  $\sigma_{\text{nCN}}$  are respectively evaporation residue/LPs, IMFs, ff and non-compound nucleus cross sections which sum up to give fusion cross section  $\sigma_{\text{fus}}$ . mass motions of preformed clusters or fragments through the barrier.

In Eq. (1), the preformation probability  $P_0$  is obtained by solving the stationary Schrodinger equation in  $\eta$ , at a fixed  $R = R_a$ , and is given by

$$P_0 = \sqrt{B_{\eta\eta}} |\psi[\eta(A_i)]|^2 (2/A) \quad (3)$$

The structure information of the compound nucleus enters the preformation probability  $P_0$  through the fragmentation potential  $V(R, \eta, \beta_{\lambda i}, \theta_i, T)$ , defined as

$$V_R(\eta, T) = \sum_{i=1}^2 [V_{\text{LDM}}(A_i, Z_i, T)] + \sum_{i=1}^2 [\delta U_i] \exp(-T^2 / T_0^2) + V_c(R, Z_i, \beta_{\lambda i}, \theta_i, T) + V_p(R, A_i, \beta_{\lambda i}, \theta_i, T) + V_\ell(R, A_i, \beta_{\lambda i}, \theta_i, T) \quad (4)$$

Here  $V_{\text{LDM}}$  and  $\delta U$  are, respectively, the T-dependent liquid drop and shell correction energies,  $V_c$  is the coulomb potential,  $V_p$  indicates the proximity potential and  $V_\ell$  denotes the angular momentum dependent potential.

The penetrability calculated as the WKB tunneling probability

$$P = \exp\left[-\frac{2}{\hbar} \int_{R_a}^{R_b} \{2\mu[V(R) - V(R_a)]\}^{1/2} dR\right] \quad (5)$$

$R_a$ , defined above, is the first turning point of the penetration path used for calculating the WKB penetrability  $P$ . Apparently, in the

decay of hot CN, the first turning point of the penetration path, used for calculating the penetrability  $P$ , is postulated by Gupta *et al.* as

$$R_a(T, \eta, \alpha) = R_t(T, \eta, \alpha) + \Delta R(T) \quad (6)$$

Where  $R_t = R_1(T, \eta, \alpha_1) + R_2(T, \eta, \alpha_2)$  and  $\Delta R$  is the neck length parameter that assimilates the deformation and neck formation effects between two nuclei.  $\Delta R$  allows us to define equivalently, the barrier lowering parameter  $\Delta V_B$ , which simply relates  $V(R_a, \ell)$  and top of the barrier position  $V_B(\ell)$ , for each  $\ell$  shown in Fig.2:

$$\Delta V_B = V(R_a, \ell) - V_B(\ell) \quad (7)$$

Here  $V(R_a, \ell)$  represents the actual barrier used for the penetration. Note that  $\Delta V_B$  appears as a negative quantity,  $V(R_a, \ell)$  being always smaller than  $V_B(\ell)$ , implying that the barrier actually used is effectively lowered.

### III. Calculations and Discussions

In reference to our previous studies [3-5], we extend our present work to see the role of  $\Delta R$  in CF for the reactions induced by same projectile ( ${}^9\text{Be}$ ) at same incident energy ( $E_{\text{lab}} = 37 \text{ MeV}$ ) on two different targets  ${}^{144}\text{Sm}$  and  ${}^{208}\text{Pb}$ .  ${}^9\text{Be}$  fuses

with the target  ${}^{144}\text{Sm}$  and there is a formation of  ${}^{153}\text{Dy}^*$ , which enables us complete fusion cross section by fitting same  $\Delta R^{CF}$  for the decay of  ${}^{153}\text{Dy}^*$ . Secondly, similar kind of dynamics of CF comes into picture for another target  ${}^{208}\text{Pb}$ . When  ${}^9\text{Be}$  fuses with  ${}^{208}\text{Pb}$ , there is formation of  ${}^{217}\text{Rn}^*$ . The nuclear properties via fragmentation potential and barrier modification are also study for the decay of  ${}^{153}\text{Dy}^*$  and  ${}^{217}\text{Rn}^*$ .

Fig.1 gives the fragmentation potential  $V$  (MeV) with fragment mass ( $A$ ) for CN  ${}^{153}\text{Dy}^*$  and  ${}^{217}\text{Rn}^*$  formed through  ${}^9\text{Be}$  induced reactions at same incident energy  $E_{\text{lab}} = 37 \text{ MeV}$  for two extreme  $\ell$ -values. We observe that light particles (LPs) having  $A \leq 4$  are more favorable (lower fragmentation potential) at lower the  $\ell$ -values. In both cases  ${}^{153}\text{Dy}^*$  and  ${}^{217}\text{Rn}^*$ , LPs have lower fragmentation potential comparison to another competing fragments (IMFs, HMFs and FF fragments), but at higher  $\ell$ -values they are in strong competition with LPs. The fragments which have least fragmentation potential are highly favored. As we observe the observed LPs (1n, 2n, 3n, 4n) for  ${}^{153}\text{Dy}^*$  have least fragmentation potential than the neighboring fragments at  $\ell_{\text{min}}$ , but they are in competition with neighboring fragments at higher  $\ell$ -values. But 1n, 2n, 3n, 4n, 5n for  ${}^{217}\text{Rn}^*$  are minimized in the fragmentation potential are highly stable for both  $\ell$ -values. We may say that LPs are more prominent for both the reactions at given  $E_{\text{lab}}$ . One more observation is that

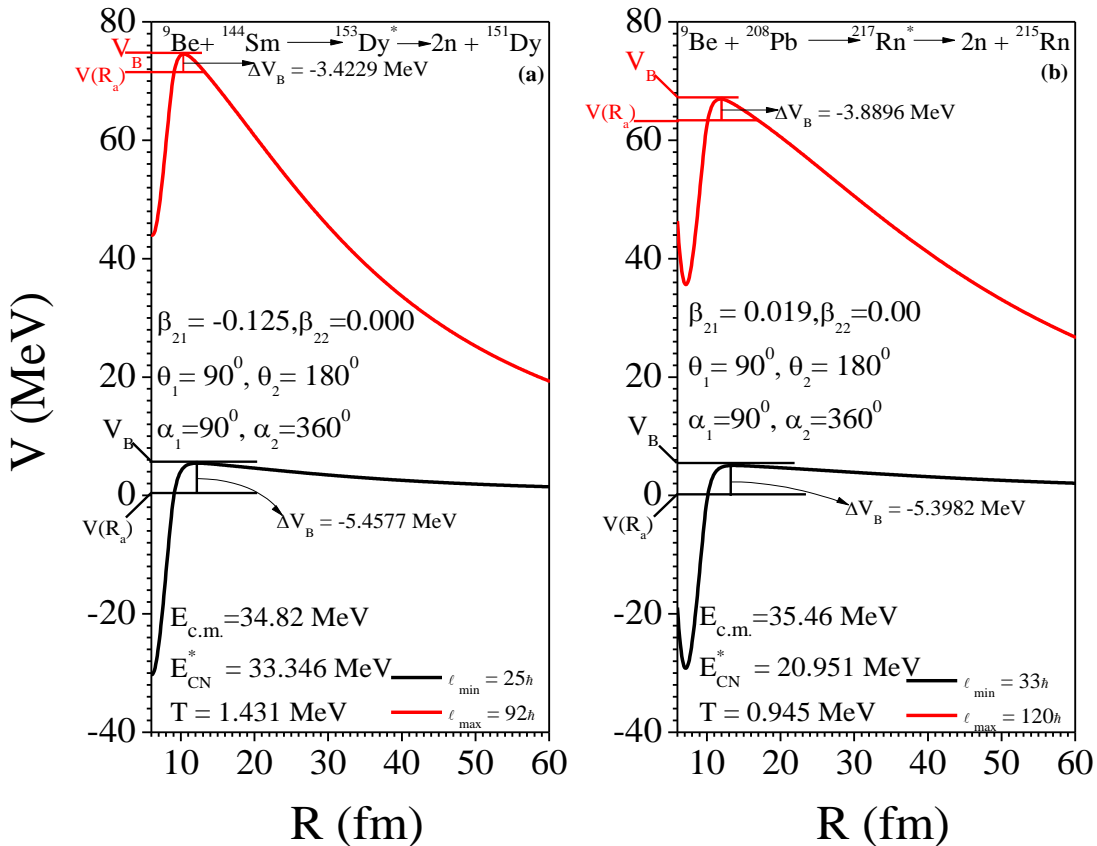


Fig 2: The scattering potentials  $V$  (MeV) for the compound systems (a)  ${}^{153}\text{Dy}^*$  (b)  ${}^{217}\text{Rn}^*$  decay through 2n exit channel at two extreme  $\ell$ -values. The barrier lowering parameter  $\Delta V_B$  is also shown at both  $\ell$ -values.

symmetric fragments are not highly favored for both the reactions, so asymmetric decay is possible for both the cases.

Fig.2 shows the Scattering potential for the decay of (a)  $^{153}\text{Dy}^*$  and (b)  $^{217}\text{Rn}^*$  through 2n exit channel is shown at two extreme  $\ell$ -values. The area under the curve decreases as the value of  $\ell$  increases, it means that penetrability increases with increase in the value of  $\ell$  due to the higher rotational energy. The barrier modification parameter  $\Delta V_B$ , which is an inbuilt property of DCM, is also shown in Fig 2. As we observed in our previous studies,  $\Delta V_B$  is almost similar for the reaction induced by same projectile at same incident energy. Here we observed that  $\Delta V_B$  is almost similar for the decay of both CN  $^{153}\text{Dy}^*$  and  $^{217}\text{Rn}^*$  through 2n exit channel at two extreme  $\ell$ -values. As discussed in methodology the  $\Delta R$  allows us to define the barrier lowering, which is inbuilt property of DCM.

So, if the barrier modification is similar for the decay of both CN, then  $\Delta R$  would be same for both the reaction to fit very The observation from Fig.1 emission of LPs for both the reactions is much similar to the given experimental data [6,7]. For  $^{153}\text{Dy}^*$  the observed fragments are 1n, 2n, 3n and 4n which contributes for  $\sigma_{CF}$ , but in the experimental data the contribution of  $\sigma_{CF}$  is from 1n, 2n, 3n, 4n, pn and p2n. To compare  $\sigma_{CF}$  with experimental data, within DCM approach we fit the data for 1n-4n by using the free parameter of DCM  $\Delta R^{CF}=1.655$  fm. The  $\sigma_{CF}$  for pn and p2n is also taken through replacement of 2n and 3n by pn and p2n in the minimization process by using same value of  $\Delta R^{CF}$ . Here we are able to compare calculated  $\sigma_{CF}$  within DCM with experimental data. By using this fixed value of  $\Delta R^{CF}$ , we have fitted  $\sigma_{CF}$  for  $^{217}\text{Rn}^*$  and nicely compared with the experimental data [6,7] as shown in Table I.

**Table I**

Reaction	$E_{c.m.}$ (MeV)	$E^*_{CN}$ (MeV)	T (MeV)	$\ell_{max}$ (CF) (h)	$\Delta R^{CF}$ (fm)	$\sigma_{CF}$ (DCM) (mb)	$\sigma_{CF}$ (Expt.) (mb)
$^9\text{Be} + ^{144}\text{Sm} \rightarrow ^{153}\text{Dy}^*$	34.62	33.35	1.431	92	1.655	293.3	295±21
$^9\text{Be} + ^{208}\text{Pb} \rightarrow ^{217}\text{Rn}^*$	35.46	20.95	0.953	120	1.655	2.25	2.79±0.43

**Table I:** The CF calculated cross section using DCM for  $^9\text{Be}$  induced reactions at  $E_{lab}=37$  MeV respectively.

#### IV. CONCLUSION

The dynamics of loosely bound projectile  $^9\text{Be}$  has been studied within dynamical cluster decay model (DCM). There is the formation of  $^{153}\text{Dy}^*$  and  $^{217}\text{Rn}^*$  are formed when  $^9\text{Be}$  fuses with  $^{144}\text{Sm}$  and  $^{208}\text{Pb}$  at same incident energy  $E_{lab}=37\text{MeV}$ . The comparative study of fragmentation potential and barrier modification parameter is also done. In fragmentation profile of both the CN  $^{153}\text{Dy}^*$  and  $^{217}\text{Rn}^*$  the LPs are more prominent and contribute to complete fusion cross section ( $\sigma_{CF}$ ) for both the cases. We have fitted  $\Delta R^{CF}$  for  $^{153}\text{Dy}^*$  to calculate  $\sigma_{CF}$  and compare with the experimental data. Same value of  $\Delta R^{CF}$  used to calculate the  $\sigma_{CF}$  for  $^{217}\text{Rn}^*$ . The barrier modification parameter  $\Delta V_B$  is also calculated for the decay of both  $^{153}\text{Dy}^*$  and  $^{217}\text{Rn}^*$  through 2n exit channel.  $\Delta V_B$  is almost similar for the decay of both CN at both  $\ell$ -values, which is an inbuilt property of DCM depend upon the value of  $\Delta R$ . The calculated  $\sigma_{CF}$  are in good comparison with the experimental data for both reactions.

#### REFERENCES

1. R. K. Gupta et al. (2008) Collective Clusterization in Hot and Rotating Nuclei: Preformed-cluster based Dynamical Cluster-decay. *Int. Rev. Phys (IREPHY)*, **2**, 369, [http://www.praiseworthyprize.org/latest\\_issues/IREPHY-latest/IREPHY\\_vol\\_2\\_n\\_6.html](http://www.praiseworthyprize.org/latest_issues/IREPHY-latest/IREPHY_vol_2_n_6.html); (2008). "Decay of  $^{246}\text{Bk}^*$  formed in similar entrance channel reactions of  $^{11}\text{B}+^{235}\text{U}$  and  $^{14}\text{N}+^{232}\text{Th}$  at low energies using dynamical cluster decay model". *Phys. Rev. C*, **77**, 054613, <http://journals.aps.org/prc/abstract/10.1103/PhysRevC.77.054613>.

2. Raj Kumar et al. (2009). "Angular momentum effects and barrier modification in sub barrier fusion reaction using different proximity potential in wong formula". *Phys. Rev. C* **80**, 034618. DOI: 10.1103/PhysRevC.80.034618.
3. M. Kaur et al. (2015). "Decay analysis of compound nuclei with mass A~30-200 formed in the reactions involving loosely bound projectiles". *Phys. Rev. C* **92**, 024623, <http://dx.doi.org/10.1103/PhysRevC.92.024623>.
4. M. Kaur et al. (2015) "Significance of neck length parameter in 32S induced fusion reactions". *DAE symposium on nuclear physics at Sri Satya Sai institute of higher learning*, on Nucl. Phys. **60**, 590.
5. M. Kaur et al. (2015). "Investigating N/Z effects in the decay of Compound nuclei with A=60". *DAE symposium on nuclear physics at Sri Satya Sai institute of higher learning*, on Nucl. Phys. **60**, 598.
6. P.R.S. Gomes et al. (2006). "Comprehensive study of reaction mechanisms for the  $^9\text{Be} + ^{144}\text{Sm}$  system at near and sub barrier energies". *Phys. Rev. C* **73**, 064606, 10.1103/PhysRevC.73.064606.
7. M. Dasgupta et al. (2004). "Effect of breakup on fusion of  $^6\text{Li}$ ,  $^7\text{Li}$  and  $^9\text{Be}$  with heavy nuclei". *Phys. Rev. C* **70**, 024606. 10.1103/PhysRevC.70.024606