International Journal of Pure and Applied Physics. ISSN 0973-1776 Volume 13, Number 1 (2017), pp. 195-200
© Research India Publications http://www.ripublication.com

Implications of Weak Basis Transformations for the Texture Specific Quark Mass Matrices

Samandeep Sharma

***Abstract*— In view of some recent attempts to construct viable quark mass matrices using the facility of *weak basis transformations*, we carry out a detailed investigation of the texture specific fermion mass matrices in the quark sector. Interestingly, our analysis reveals that the Fritzsch like texture four zero quark mass matrices are very well in agreement with the present quark mixing data whereas both the Fritzsch like as well as non Fritzsch like scenarios in the texture five zero case fail to do so. These results, being in contradiction with some of the recent analyses, can have important implications for the model builders in flavor physics.**

***Index Terms*—Fermion mass matrices, weak basis transformations, texture zeros.**

I. Introduction

Understanding fermion masses and mixings constitutes one of the most important issues in the present day high energy physics. In this context, it may be noted that the mixing angles and CP violating phases are very much related to the corresponding mass matrices. In view of this relationship, one has to essentially formulate the fermion mass matrices to unravel some of the deeper aspects of flavor physics. In this context, it may be noted that the texture specific mass matrices [1, 2] seem to be very helpful in understanding the pattern of quark mixing as well as the CP violating phenomena. In particular, hermitian texture four zero mass matrices are found to be compatible with quark and lepton mixing data [3] for both Dirac as well as Majorana neutrino case. However, in the absence of any deep compulsion for assuming a particular texture structure, attempts have been made to connect the texture specific mass matrices with the weak basis (WB) transformations [4, 5]. WB transformations are essentially unitary transformations on the quark fields, i.e.

$$q_L \to W q_L \ , q_R \to W q_R$$
$$q'_L \to W q'_L \ , q'_R \to W q'_R \tag{1}$$

under which the gauge currents remain real and diagonal but the quark mass matrices transform as,

$$M_U \to M'_U = W^\dagger M_U W, \ M_D \to M'_D = W^\dagger M_D W, \tag{2}$$

where W is an arbitrary unitary matrix while M_U and M_D correspond to the up and down sector quark mass matrices respectively. It can easily be checked that the hermiticity and the physical content of the mass matrices remain unchanged under these transformations.

To this end, recently Giraldo [6] has attempted to construct explicitly experimentally viable texture four zero and texture five zero quark mass matrices using the facility of WB transformations. Texture four zero quark mass matrices are known [7] to be compatible with the quark mixing data, however the texture five zero quark mass matrices have been shown to ruled out in literature [8]. It, therefore, becomes interesting to re-investigate the texture five zero quark mass matrices considered in [6] in detail.

In another recent analysis, Emanuel Costa et al. [9] have attempted to reconstruct the quark mass matrices in the basis where the matrices are hermitian and have a maximum of three vanishing elements. For carrying out the reconstruction at the M_Z scale, they have randomly scanned the parameter space taking the experimental ranges of the input values, viz., the quark masses and the parameters of the quark mixing matrix, also referred to as the Cabibbo Kobayashi Maskawa (CKM) matrix. Further, they consider these electroweak scale mass matrices as the initial conditions to run the Yukawa couplings from M_Z scale to M_{GUT} scale considering the 1-loop renormalization group (RG) equations corresponding to various low energy models, viz., Standard Model (SM), Minimal Supersymmetric Standard Model (MSSM) and two higgs doublet model. Following this procedure, they find that the hermitian quark mass matrices with zeros located at (1,1) and (1,3) positions in the up and down sector mass matrices simultaneously, viz. the Fritzsch like texture four zero matrices, are not able to accommodate the present quark mixing data unless one accepts large rotations in the diagonalizing transformations of these mass matrices so that the smallness of the CKM mixing angles is obtained by huge cancellations. This observation, however, is in contradiction with the earlier analyses [3] wherein the hermitian texture four zero quark mass matrices are found to provide a satisfactory description of the quark mixing phenomena. This motivates one to have a closer look at the analysis by Emanuel-Costa et al. as well as to carry out a careful reinvestigation of the Fritzsch like texture four zero quark mass matrices to examine the validity of the above conclusion.

The approach adopted by Emanuel Costa et al. is quite interesting, however the correlation of the outcome of this random scanning with the physical quantities becomes little difficult, e.g. it is not possible to find the physical significance of the mass matrix elements obtained by this approach. Therefore, for a deeper analysis in this direction, probably one has to formulate a different strategy i.e. to assume a particular texture structure and analyze it to find its physical consequences. In this context, it is interesting to note [10] that while carrying out the RG evolution the hermiticity of the mass matrices is a affected to a very minor extent and the

Samandeep Sharma is with the GGDSD College, Sector 32-C, Chandigarh. (e-mail: samandeep.sharma@ggdsd.ac.in).

likely mass matrix element candidates for “texture zeros" at M_{GUT} are the same ones which are present at M_Z scale. Therefore, one essentially has to first formulate viable quark mass matrices at the M_Z scale even if one intends to use these mass matrices within the framework of grand unified theories.

The purpose of the present paper is to carry out a careful reinvestigation of texture four zero and texture five zero quark mass matrices, in order to explore their compatibility with the present quark mixing data as well as to compare the outcomes with those from some recent analyses in this direction. The paper is organized as follows. In section II, we detail the essentials of the formalism connecting the quark mass matrices to the quark mixing matrix. Inputs used in the present analysis have been given in section III. Section IV examines, on the one hand the Fritzsch like structure as well as the non Fritzsch like structure of texture five zero quark mass matrices considered in [6] while on the other hand the Fritzsch like texture four zero mass matrices considered in [9], for their compatibility with the present quark mixing data. Finally, section V summarizes our conclusions.

II. FORMALISM

The Fritzsch-like texture four zero quark mass matrices can be given as,

$$M_U = \begin{pmatrix} 0 & A_U & 0 \\ A_U^* & D_U & B_U \\ 0 & B_U^* & C_U \end{pmatrix}, \qquad M_D = \begin{pmatrix} 0 & A_D & 0 \\ A_D^* & D_D & B_D \\ 0 & B_D^* & C_D \end{pmatrix}, \tag{3}$$

where A_i and B_i (i=U, D) are complex elements and are defined as $A_i = |A_i| e^{i\alpha_i}$ and $B_i = |B_i| e^{i\beta_i}$. Fritzsch like texture five zero mass matrices can be obtained by taking either $D_U = 0$ and $D_D \neq 0$ or $D_U \neq 0$ and $D_D = 0$, thereby giving rise to two possible cases of Fritzsch like texture five zero mass matrices referred to as texture five zero $D_U = 0$ case pertaining to M_U being texture three zero type and M_D being texture two zero type and texture five zero $D_D = 0$ case pertaining to M_U being texture two zero type and M_D being texture three zero type. The non Fritzsch like mass matrices differ from the above mentioned Fritzsch like matrices in regard to the position of zeros in the structure of the mass matrices. It has to be kept in mind that one has to diagonalize these matrices in order to yield quark masses as eigenvalues as well as to obtain the quark mixing matrix. The details of the diagonalization procedure of the mass matrices can be looked up in our earlier works [3, 7, 8], however in order to make the manuscript self contained we discuss the essentials of the methodology followed. The hermitian matrix M_i (i=U, D) can be expressed as

$$M_i = Q_i M_i^r P_i$$

Or

$$M_i^r = Q_i^\dagger M_i P_i^\dagger, \tag{4}$$

where M_i^r is a real symmetric matrix with real eigenvalues and Q_i and P_i are diagonal phase matrices. The matrix M_i^r can be diagonalized by an orthogonal transformation, e.g.

$$M_i^{\text{diag}} = O_i^T M_i^r O_i, \tag{5}$$

Where

$$M_i^{\text{diag}} = \text{diag}(m_1, -m_2, m_3),$$

the subscripts 1, 2 and 3 referring respectively to u, c and t for the U sector as well as d, s and b for the D sector. Using the invariants, tr(M_i^r), tr (M_i^{r2}) and det (M_i^r), the values of the elements of the mass matrices A_i, B_i and C_i, in terms of the free parameter D_i and the quark masses are given as

$$C_i = (m_1 - m_2 + m_3 - D_i)$$
$$|A_i| = (m_1 m_2 m_3 / C_i)^{1/2}$$
$$|B_i| = [(m_3 - m_2 - D_i)(m_3 + m_1 - D_i)(m_2 - m_1 + D_i) / C_i]^{1/2}. \tag{6}$$

The quark mixing matrix, also referred to as the Cabibbo Kobayashi Maskawa (CKM) matrix, which measures the non-trivial mismatch between diagonalizations of M_U and M_D can then be obtained using the relationship

$$V_{\text{CKM}} = O_U^T P_U P_D^\dagger O_D. \tag{7}$$

III. INPUTS USED FOR THE ANALYSIS

Before discussing the results of our analysis, we would first like to briefly mention the inputs used for carrying out the calculations. We have adopted the following ranges of quark masses [11] at the M_Z energy scale, e.g.

$$m_u = 1.38^{+0.42}_{-0.41}\,\text{MeV},$$
$$m_d = 2.82^{\pm 0.48}\,\text{MeV},$$
$$m_s = 57^{+18}_{-12}\,\text{MeV},$$
$$m_c = 0.638^{+0.043}_{-0.084}\,\text{GeV},$$
$$m_b = 2.86^{+0.16}_{-0.06}\,\text{GeV},$$
$$m_t = 172.1 \pm 1.2\,\text{GeV}. \tag{8}$$

The light quark masses m_u, m_d and m_s have been further constrained using the following mass ratios given by Leutwyler [12]

$$m_u / m_d = 0.553 \pm 0.043, \quad m_s / m_d = 18.9 \pm 0.8. \tag{9}$$

For the purpose of our calculations, we have considered the phases $\phi_1 = \alpha_U - \alpha_D$ and $\phi_2 = \beta_U - \beta_D$ as free parameters and given them full variation from 0 to 2π. The free parameters D_U and D_D have also been given full variation. While carrying out the analyses, we first attempt to reproduce the CKM matrix elements V_{us}, V_{ub} and V_{cb} corresponding respectively to the mixing angles s_{12}, s_{23} and s_{13} as well as the CP asymmetry parameter sin2β. The CKM parameters so produced are to be compared with the PDG 2016 data [13], e.g.,

$$\begin{gathered} |V_{us}| = 0.22506 \pm 0.00050 \\ |V_{ub}| = 0.00357 \pm 0.00015 \\ |V_{cb}| = 0.0411 \pm 0.0018 \\ \text{Sin}2\beta = 0.691 \pm 0.017 \end{gathered} \tag{10}$$

IV. RESULTS AND DISCUSSIONS

In order to carry out a careful study of the implications of WB transformations for the texture specific mass matrices in quark sector, we have essentially attempted to follow a three-prong approach. As a first step, we critically examine the non Fritzsch like texture five zero structure considered in [6], viz.,

$$M_U = P^{\dagger}\begin{pmatrix} 0 & 0 & C_U \\ 0 & A_U & B_U \\ C_U & B_U^* & D_U \end{pmatrix} P,$$

$$M_D = \begin{pmatrix} 0 & C_D & 0 \\ C_D^* & 0 & B_D \\ 0 & B_D^* & A_D \end{pmatrix}, \tag{11}$$

where $P = diag(e^{-i\phi_{cu}}, e^{-i\phi_{bu}}, 1)$ corresponds to the phase matrix with $\phi_{bu} \equiv arg(B_U)$ and $\phi_{cu} \equiv arg(C_U)$. We have first carried out a detailed analysis regarding the compatibility of these mass matrices with the latest quark mixing data. To this end, we first present the magnitudes of the elements of CKM matrix obtained from this structure using the methodology discussed in section II,

$$\begin{pmatrix} 0.974-0.974 & 0.225-0.226 & 0.005-0.012 \\ 0.016-0.083 & 0.002-0.146 & 0.096-0.989 \\ 0.0015-0.229 & 0.001-0.976 & 0.002-0.999 \end{pmatrix}. \tag{12}$$

A look at this matrix immediately reveals that the ranges of some of the CKM elements, in particular $|V_{ub}|$, $|V_{cd}|$, $|V_{cs}|$ and $|V_{cb}|$, show no overlap with those obtained by the recent global analyses [13]. This, therefore, leads one to conclude that the non-Fritzsch like texture 5 zero quark mass matrices considered in [6] are not in agreement with the recent quark mixing data. To re-emphasize our conclusion, we have examined the dependence of one of the CKM matrix element $|V_{ub}|$, obtained from the mass matrices considered here, on the parameters A_U and B_D. This variation has been plotted in Fig. (1) from which one can easily find that for all possible values of A_U and B_D, the values of $|V_{ub}|$ are much larger than the allowed range (0.00357±0.00015) [13], thereby clearly ruling out these mass matrices.

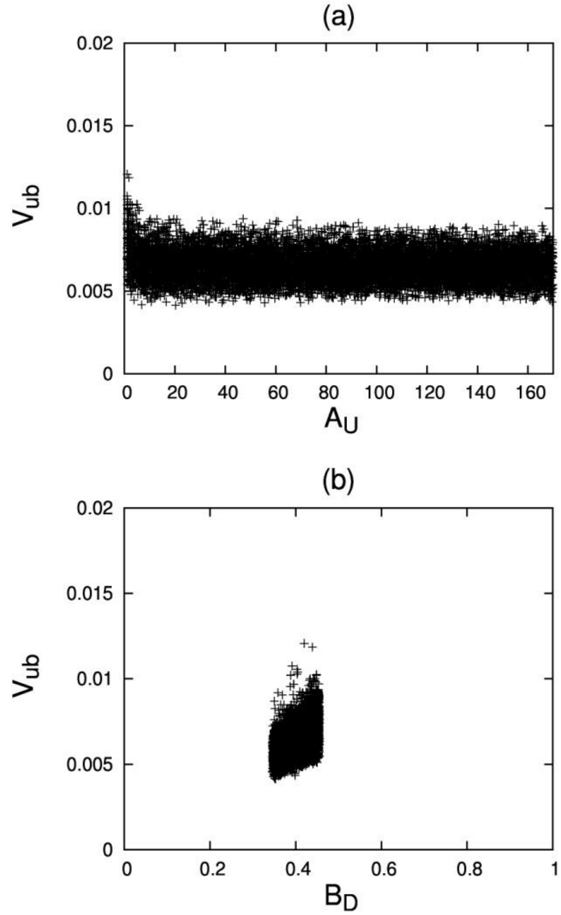


Fig. 1. Plots showing the dependence of the CKM matrix element V_{ub} on (a) the parameter A_U of the mass matrix M_U (b) the parameter B_D of the mass matrix M_D. The values of A_U and B_D are in GeV units.

It is interesting to note that this conclusion is in contradiction with that obtained in [6]. This can be understood from the fact that in [6] fit for the quark mixing parameters has been obtained by adding several additional phases in the unitary matrices used for diagonalizing the matrices M_U and M_D. It may be noted that the introduction of additional phases is not justified as it is against the basic spirit of the texture specific mass matrices which were introduced to reduce the number of free parameters.

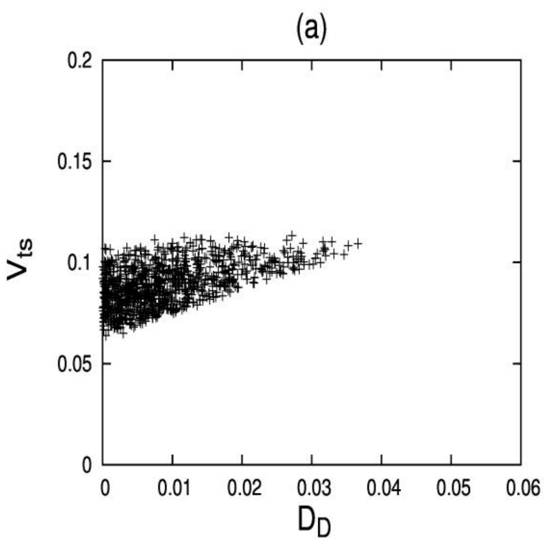
As a next step, we carry out a detailed analysis for both the cases for Fritzsch-like texture five zero quark mass matrices, viz. the $D_U = 0$ case, which can be given as

$$M_U = \begin{pmatrix} 0 & A_U & 0 \\ A_U^* & 0 & B_U \\ 0 & B_U^* & C_U \end{pmatrix},$$
$$M_D = \begin{pmatrix} 0 & A_D & 0 \\ A_D^* & D_D & B_D \\ 0 & B_D^* & C_D \end{pmatrix}, \quad (13)$$

and the $D_D = 0$ case, which can be given as

$$M_U = \begin{pmatrix} 0 & A_U & 0 \\ A_U^* & D_U & B_U \\ 0 & B_U^* & C_U \end{pmatrix},$$
$$M_D = \begin{pmatrix} 0 & A_D & 0 \\ A_D^* & 0 & B_D \\ 0 & B_D^* & C_D \end{pmatrix}. \quad (14)$$

These matrices are again analyzed using the methodology outlined in section II. On comparing the CKM matrix so obtained with the latest PDG data [13], we find that ranges of some of the CKM elements, e.g. $|V_{cb}|$, $|V_{td}|$ and $|V_{ts}|$ show no overlap with the PDG data. To this end, in figure (2)(a) and 2(b), we have plotted the magnitude of the CKM matrix element V_{ts} with the free parameters D_D and D_U for the $D_U = 0$ case and the $D_D = 0$ case of the Fritzsch like texture five zero quark mass matrices respectively. It is evident from the Fig. (2) that the values of V_{ts} show no overlap with the experimentally allowed region (0.0403 ± 0.0013) which clearly rules out both the cases of Fritzsch like texture five zero quark mass matrices.



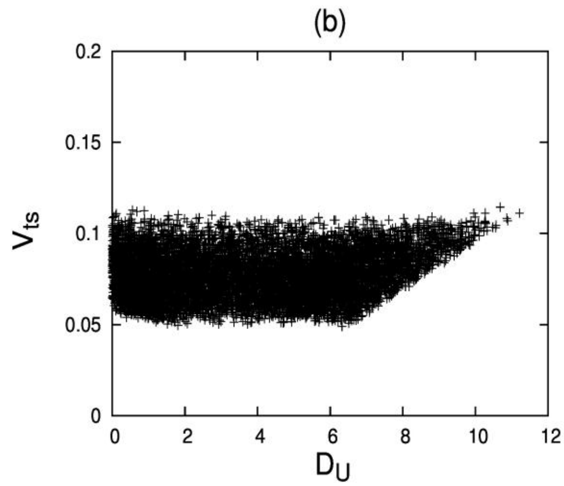


Fig. 2. Plots showing the dependence of the CKM matrix element V_{ts} on (a) the parameter D_D(b) the parameter D_U for the D_U=0 case and the D_D=0 case of the Fritzsch like texture five zero quark mass matrices respectively.

Finally, we carry out a detailed analysis of the Fritzsch like texture four zero quark mass matrices given in equation (3) for their compatibility with the latest quark mixing data [13] using the methodology discussed in section II. To this end, in eqn. (15) we present the magnitudes of the CKM matrix elements so obtained viz.,

$$\begin{pmatrix} 0.9741-0.9744 & 0.2246-0.2259 & 0.00337-0.00365 \\ 0.2245-0.2258 & 0.9732-0.9736 & 0.0407-0.0422 \\ 0.0071-0.0100 & 0.0396-0.0417 & 0.9990-0.9992 \end{pmatrix} \quad (15)$$

A general look at the above matrix reveals that the ranges of all the CKM elements are very well in agreement with the present quark mixing data [13]. Interestingly, the ranges of various CP violating parameters such as the Jarlskog's rephrasing invariant parameter J obtained here are also compatible with the latest PDG data. It is, therefore, evident that the Fritzsch-like texture 4 zero hermitian quark mass matrices present a viable case.

In view of the fact that one of the aims of the present paper is to examine the issue of rotations involved in the diagonalization procedure in order to construct experimentally viable Fritzsch like texture four zero quark mass matrices, we present the ranges of the elements of the mass matrices M_U and M_D reconstructed from the CKM matrix which is fully compatible with the latest experimental data, viz.

$$M_U = \begin{pmatrix} 0 & 0.027-0.039 & 0 \\ 0.027-0.039 & 13.0-64.0 & 46.57-83.58 \\ 0 & 46.57-83.58 & 107.46-158.46 \end{pmatrix}, \quad (16)$$

$$M_D = \begin{pmatrix} 0 & 0.011-0.015 & 0 \\ 0.011-0.015 & 0.15-0.95 & 0.748-1.37 \\ 0 & 0.748-1.37 & 1.86-2.65 \end{pmatrix}. \quad (17)$$

All the elements of M_U and M_D listed in the above matrices are in the units of GeV. The elements of orthogonal matrices O_U and O_D which diagonalize the above mass matrices have been given in equations (18) and (19) respectively.

$$\begin{pmatrix} 0.998-0.999 & 0.0307-0.0504 & -0.027-(-0.0135) \\ 0.038-0.0526 & -0.957-(-0.790) & 0.281-0.611 \\ 0.00005-0.0001 & 0.280-0.612 & 0.791-0.960 \end{pmatrix} \quad (18)$$

$$\begin{pmatrix} 0.972-0.976 & 0.217-0.233 & 0.00124-0.003 \\ 0.184-0.222 & -0.941-(-0.79) & 0.266-0.585 \\ -0.138-0.00 & 0.259-0.568 & 0.811-0.963 \end{pmatrix} \quad (19)$$

A careful look at the eqns. (18) and (19) reveals that one does not require large rotations in the diagonalization procedure for the Fritzsch like texture four zero quark mass matrices to be compatible with the present quark mixing data. This observation is in contradiction with that by Emanuel-Costa et al. [9], wherein the authors find that a parallel structure with zeros located at (1, 1) and (1, 3) elements for both quark mass matrices is not compatible with the electroweak data unless large rotations are incorporated in the diagonalizing transformations of these mass matrices.

V. Summary and Conclusions

In view of a recent attempt [6] to construct texture five zero quark mass matrices using the facility of WB transformations, we have attempted to carry out a critical examination of one category of the non Fritzsch like structure as well as both the possibilities pertaining to the Fritzsch like structure of texture five zero quark mass matrices considered therein. The viability of the non Fritzsch like mass matrices has been examined by comparing the CKM matrix obtained from these with the latest PDG data as well as by studying the dependence of the element V_{ub} on the elements A_U and B_D. Contradictory to the observation in [6], we find that these mass matrices are not in agreement with the present quark mixing data. We observe that in [6], the author has been able to obtain a fit to the quark mixing data by incorporating several additional phases, which cannot be justified as it leads to an increment in the number of free parameters of the mass matrices. Further, both the cases, viz. $D_U = 0$, $D_D \neq 0$ and $D_U \neq 0$ and $D_D = 0$, of Fritzsch like texture five zero quark mass matrices are found to be ruled out by the present quark mixing data. Furthermore, in view of a recent attempt by Emanuel-Costa et al. [9] to reconstruct the hermitian quark mass matrices at the electroweak as well as at GUT scale using the facility of WB transformations, we carry out a detailed analysis of Fritzsch like texture four zero quark mass matrices. Our analysis reveals that these mass matrices are compatible with the latest quark mixing data without the requirement of large rotations in the diagonalization procedure. This observation is in contradiction with that from Emanuel-Costa's work wherein they have observed that that the parallel Fritzsch like texture four zero quark mass matrices, with zeros at (1,1) and (1,3) positions in the up as well as down sector, are not in agreement with the present quark mixing data unless one accepts large rotations in the unitary matrices to diagonalize these mass matrices.

In conclusion, we would like to emphasize that both the Fritzsch like as well as non Fritzsch like texture five zero quark mass matrices seem to be ruled out by the present quark mixing data, which necessitates one to be cautious in analyzing the implications of WB transformations for the texture specific mass matrices. Also, it seems that instead of performing a random scanning of the entire parameter space, as carried out by Emanuel-Costa et al., a better strategy is to assume a particular texture structure, and anaylze it in detail to examine its physical consequences. This, in turn, would have important implications for model building of fermion mass matrices.

Acknowledgements

The author would like to thank the Principal, GGDSD College, Chandigarh for providing the necessary facilities to work.

References

[1] S. Weinberg , "The problem of mass", in Transactions of the New York Academy of Sciences 38, 185 (1977).

[2] H. Fritzsch , "Calculating the Cabibbo angle", Phys. Lett. B 70, 436 (1977).

[3] M. Gupta and G. Ahuja, "Flavor Mixings And Textures Of The Fermion Mass Matrices", Int. Jour. Mod. Phys. A 27, 1230033 (2012).

[4] G. C. Branco, D. Emmanuel-Costa and R. Gonzalez Felipe, "Textures zeros and weak basis transformations", Phys. Lett. B 477, 147(2000); G. C. Branco, D. Emmanuel-Costa, R. Gonzalez Felipe and H. Serodio, "Weak basis transformations and texture zeros in the leptonic sector", Phys. Lett. B 670, 340 (2009).

[5] H. Fritzsch and Z Z Xing, "Flavor symmetries and the description of flavor mixing", Phys. Lett. B, 413, 396 (1997).

[6] Y. Giraldo, "Texture zeros and WB transformations in the quark sector of the standard model", Phys. Rev. D. 86, 093021 (2012)

[7] R. Verma, G. Ahuja, N. Mahajan, M. Randhawa and M. Gupta, “Exploring the parameter space of texture four zero quark mass matrices”, Jour. of Phys. G 37, 075020 (2010).

[8] N. Mahajan, R. Verma and M. Gupta, “Investigating non-fritzsch-like texture specific quark mass matrices”, Int. Jour. Mod. Phys. A 25, 2037 (2010)

[9] D. Emmanuel Costa and C. Simoes, “Reconstruction of quark mass matrices with weak basis texture zeros from experimental input”. Phys. Rev. D 79, 073006 (2009).

[10] R. D. Peccei and K. Wang, “Natural Mass Matrices”, Phys. Rev. D 53, 5 (1996).

[11] Z. Z. Xing, H. Zhang and S. Zhou, “Impacts of High mass on Vacuum stability, running fermion masses, and two-body Higgs decays”, Phys. Rev. D 86, 013013 (2012).

[12] H. Leutwyler, “The raios of the light quark masses”, Phys. Lett. B 378, (1996) 313.

[13] C. Patrignani et al. (Particle Data Group), “The Review of Particle Physics(2016)”, Chin. Phys. C, 40, 100001 (2016).