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On Statement Analysis and Logical Anomaly

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Abstract

The paper focuses on certain aspects of set theory which do not hold its consistency when applied to some ambiguous arguments, few of the examples are described further. It aims at the shedding light on the few of the key concepts which were first introduced by Bertrand Russell (Russells Antimony and the supporting argument, we shall use these principles as a supporting arguments for our proof. These uncertainties although proved priorly, it is necessary to explore many such possibilities to come close the axiomatic concept of set theory. The result includes showing the contradictions explained by the means of paradox, namely Time travellers Paradox and explained through simple logic about the equivalence of both parts of the paradox. We will try to develop a relationship between linguistic syntax and propositional logic. Utmost care has been taken to describe the result by means of available much information both mathematically, logically and theoretically.

INTRODUCTION

In the middle of the nineteenth century, due to immense contributions primarily by George Boole, Augustus De Morgan for logic and of course the father of set theory Georg Cantor, a well defined field of logic had emerged which found it’s application in a number of places. This led to a new system based on logic which gave rise to its application to newer fields such as set theory, model theory, etc. This has been important since the time of its in- ception and has been used in numerous fields from philosophy to electronics ad most recently in computer science.

In 1874 the famous paper" On a Property of the Collection of All Real Algebraic Numbers" was published by Georg Cantor. This single paper bough all the bits and pieces together and led to an entirely new field of set theory. His proofs that there are more real numbers than natural numbers, infinities of infinity were the direct results fo the power set operation. Many mathe- matical concepts can be explained using the set theory such as equivalence and ordered relations which are essential in mathematics. Soon set theory had become a foundational system for the field of mathematics. We know begin to explore a tiny corner of this vast universe by means of an abstract concept. This will be possible since the relation of set theory and certain other principles have led to a better understanding of philosophical syntax and language semantics. Thus, we shall try to derive a relation between the mathematical set theory and philosophical grammar.

Set Theory and Paradoxes

The fields of set theory and logic are closely related, although the later points out some specific uncertainties in set theories principles' over the time. The Russell's Paradox has been an important concept developed by the famous mathematician, logician and philosopher Bertrand Russell with its development being cited in the beginning of the 20th century while he was in the process of working on the Principia Mathematica along with Alfred North Russell. The same principle will be used here in order to prove the result. The Russell's paradox itself will be further discussed in detailed as it closely relates to the idea of coinciding laws of set theory and their contradictions.

Formal Definitions and Terminology

The symbol U , as in the general convention denotes the Universal set or a system .

The following symbols are used in U - The notation ¬ refers to NOT(negation).

"∧" refers to AND(conjunction).

"−→" is to implication

"←→" is to double implication or equality.

U includes the all the elements and sets. In our case it also includes all the dimensions.

R is the set which includes the set of all the elements in its present time as the observer.

¬R is the set of elements in the different time dimension, in our case the future.

”∃” for there exists.

”∀” for all.

”⊆” for subset

Therefore, as per the convention all the elements in the set R will be in its original form and the elements in R will be the negations of the ele- ments in the set R.(Detailed explanation about this will be in the Chapter

6. Representation in its logical form).

Understanding a Paradox

A paradox is a statement despite sounding reasonable leads to a rather self-contradictory or a logically unacceptable conclusion. Thats all that it is to it? Or maybe there is more. Out of all the things it leads to, it leads to critical thinking. Set theory and paradoxes have been closely related. It finds it;s application in various fields like discrete mathematics, philosophy,medicine,etc. A paradox, to a layman may just sound as a mind twisting fancy statement, but its true sense lies beneath its very surface which a lot of people dont notice. A mathematician, dissects it, breaks it down, comes up with it logical abstract to truly understand it. Paradoxes have been widely used by logicians and philosophers in some of their ground breaking works. One of the example that one can think of is the liar paradox used by Kurt Godel in his magnum opus ”Uber formal unentscheidbare Stze der Principia Mathematica und verwandter Systeme I”(”On Formally Undecidable Propositions of Principia Mathematica and Related Systems I”).

In the following paper Godel begins the proof of his first incompleteness the- orem by using a modified version of the liar paradox. The traditional liars paradox is the following- This statement is false. Godel uses the modified version of this paradox i.e This statement is unprovable. He calls this sen- tence Godel sentence G. Thus for a theory ”T”, ”G” is true, but not provable in ”T”. What this paper went on to achieve can be called as one of the best results achieved in the field of mathematics, specifically logic in 20th century. But the most important of them all, atleast in our context is the Russells paradox which was thought out by him in the beginning of the 20th century and is the first instance which showed that some attempted formalizations of naive set theory created by Georg Cantor led to a contradiction.

Russell’s Paradox

A new wave of excitement began around 1900 when mathematician Bertrand Russell came up with the famous Russells Paroadox. This was the first instance since the emergence of Canotrian set theory that its interpretations led to a few points of contradictions which are known as antimonies or paradoxes.Russell and Ernesto Zermelo independently found the simplest paradox to describe the contradiction. This is now known as the Russels paradox. Let us now define the Russells Paradox formally:

The principle states that-

$$(NC)\ \exists A\ \forall x\ (x \in A \equiv \psi)$$

where A is not free in the formula ψ.

There is a set A such that for any object x, x is an element of A if and only if the condition expressed by ψ holds. Russells paradox arises by taking ψ to be the formula: $x \in / x$.

To understand it closely we need to break it further down and simplify it, since only at its simplest level that its inner beauty is revealed.

In a simple form the paradox means that most sets are not not elements of themselves. At instance it does not make sense.

Consider S={Horse | Horse is a set such that Horse ∈/ Horse}

How can a set of all horses cannot be horses?

Let us dig in further by unformalizing it and present it in a simple language form.

In a certain town there is a male barber who shaves all those men, and only those men, who do not shave themselves. Question: Does the barber shave himself ?

Well, neither yea nor no. If the barber in the particular town shaves himself then he belongs to the set of men who shave themselves. But the discrepancy occurs when the we understand that no men in this set shave themselves. We can say that the barber does not shave himself. But the barber shaves every man in this set therefore, it wont be wrong if we say that the barber does shave himself.

Therein lies our contradiction the since the answer is neither nor no. The barber shaves himself is neither true nor false. Therefore, we can conclude that the paradox does not have a definite solution to it.

Formally presenting,

Let U be a universal set and consider that all sets under discussion are sub- sets of U.

Let $S = \{ A \mid A \subseteq U \text{ and } A / \in A \}$.

In Russells paradox, both implications S ∈ S → S /∈ S and S / ∈ S−→S

∈ S are proved, and the contradictory conclusion neither S ∈ S nor S / ∈ S

is therefore deduced. In the situation in which all sets under discussion are subsets of U, the implication S ∈ S −→ S / ∈ S is proved in almost the same way as it is for Russells paradox: (Suppose S ∈ S. Then by definition of S, S ⊆ U and S / ∈ S. In particular, S / ∈ S.) On the other hand, from the supposition that S / ∈ S we can only deduce that the statement S ⊂ U and S / ∈ S is false. By one of De Morgans laws, this means that S U or S ∈ S. Since S ∈ S would contradict the supposition that S / ∈ S, we eliminate it and conclude that S U. In other words, the only conclusion we can draw is that the seeming definition of S is faultythat is, that S is not a set in U.[3]

Russells paradox was just the beginning of contradiction. What was followed in the 1931 ground breaking paper by Kurt Godel was exponentially much more astonishing.He showed that we cannot prove that mathematics is free from contradictions. Ideally this results shouldve caused the mathematicians to surrender their weapons and give up. But the activity in the field of mathematics increased exponentially since 1931 than in any other time in history.

Statement Analysis and Logical Anomaly

Note:Reader is advised to pay attention to the following statements lin- guistic syntax rather than digging deep into it’s physical truth to have a clearer understanding of the exposition

Let is consider the following statement-

" If a time traveller travels to the future, does he meet himself in the future? If does meet himself in the future, did he travel time? "

Well, just by glancing at the statement it plays with our mind. Let us try to break down the statement so that we understand it better.

Consider a time traveller ” p ” who travels to the future, let’s say about a month ahead.(We assume time to be cyclical in our case).

Now, when he arrives in the future there are two possibilities. Will he meet his own self in the future or not. If he does not then has he really time travelled? If he did then he would’ve been able to meet himself in the future. We’ll take this case a bit ahead and define the logical representation of it.

We have ” U ” which is our Universal set.

Let the person who travels time be represented by ”p” This set consists of two sample sets: ” R ”: we will assume this to be the sample set of all the elements of the present time.

Let R={R | R ⊆ R∈ U}

(This would be a natural assumption, but refer chapter.5, we see that, it may not always be true.)

Now,{ ∃ R ∀ p | p ∈ R}

also, {∃ R ∀ r | r ∈ R}

Therefore, here we have defined a set ”R” which contains all the elements and conditions of the present time.

Now, let us represent a set for the future time. We’ll call that ”¬R”.

Let ¬R={¬R | R ⊆ R∈ U}

Now, { ∃ R∀ ¬p | ¬p ∃ R}

and, {∃ R∀ ¬r | ¬p ∃ R}.

Now, that we have defined both the time frames, we will try to conceptualize the statement

There reason why different variable names aren’t assigned to the sets of two different time frames is because time in itself is one whole dimension. Therefore, it is only appropriate if we use the complementary terms instead of the assigning new values. That wouldn’t make sense sine we would be dealing with two completing different objects,

We can think time as a straight line with two ends. Since there are two opposite ends one end can be called ”r” and second one it’s complement, ”¬r” as it will display its opposite characteristics.

The statement poses a question that if a time travel has really travelled time based on the assumption of meeting himself.

Now, if he does not meet himself, that wouldn’t make sense too. Then the question would arise where has his future self vanished?(since we are perceiving time cyclically).

Conversely consider the below diagram.

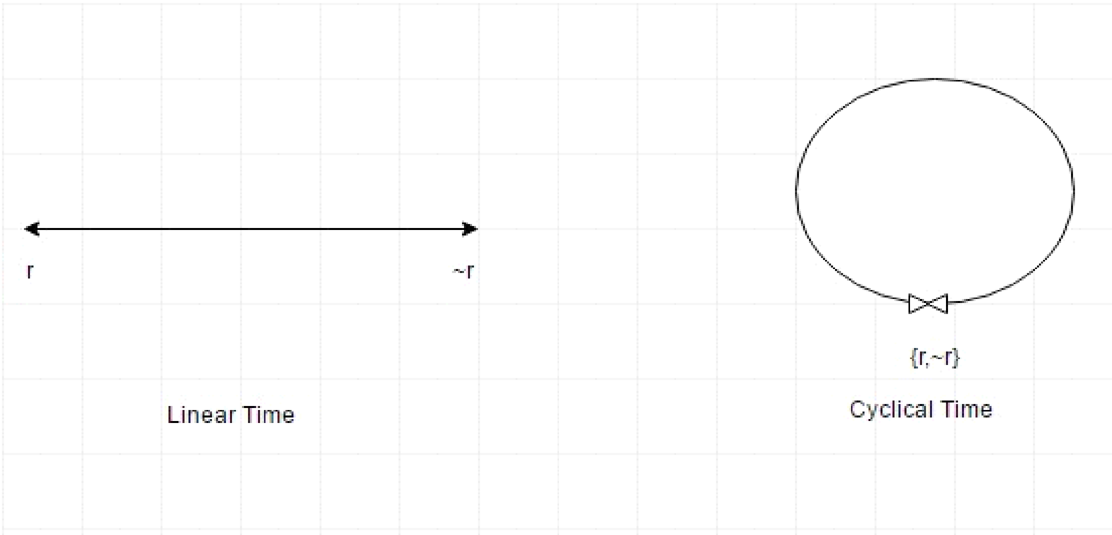

Fig 1.1

Representation in logical form

In the previous chapter we have tried to express the statement in the simplest language for a better understanding. But nothing enables a clearer view than expression in a mathematical form. It is important that the statement needs to be understood mathematically in order to reach its depth and analyse it correctly. Logic provides a successful bridge between linguis- tics and mathematics. Therefore, we shall thus try to include the statement in it's logical form.

We have already dissected the statement in two parts and defined two different sets for the same. However, both of them do belong to the common universal set " U ".

Amongst the two forms let us consider each form one by one.

Suppose the traveller does not meet himself then it is same as saying that he has travelled to the future.

Then we can say that. $p \wedge \neg p \longrightarrow \neg r$ ...(1) Where,

$p \in R$,

$\neg p \in \neg R$,

$\neg r \in \neg R$.

In the second form we posed an argument whether he has travelled to the future if he meets himself. We discussed this argument further that if he meets himself then there is a possibility that he hasn't travelled in time and is in fact in the present.

Therefore, representing the following:- $p \wedge \neg p \longrightarrow r$ ...(2)

Where, $p \in R$,

$\neg p \in \neg R$,

$\neg r \in R$.

Now that we have represented both of the sub-statements in logical form, we will test it's truth value using the truth table.

Table 1: Truth table for statement no.1

p	q	r
T	T	T
TFF	FTF	TTT

Table 2: Truth table for statement no.2

p	q	r
T	T	T
TFF	FTF	TTT

Therefore, by referring the truth table we see that both of the statements are "tautologies".

If we equate the following equation we get the following:- $((p \wedge \neg p) \longrightarrow r) \longleftrightarrow ((p \wedge \neg p) \longrightarrow r)$

Since both the statement are tautologies, we can say that above expression is also a tautology.

Now this has posed an even bigger problem. It literally means that both the possibilities are truth. Thus we have reached a point of the paradox since both the statements are true in its logical form.

Motivations and Interpretations

In this section we shall try to explain the significance of the concepts discussed in this paper. That is, instances of occurrence of the anomaly, linguistic syntax and its relation with the inconsistencies in representational or propositional logic.

By the means of the time traveller's paradox we can safely conclude that sometimes linguistic syntax when represented by propositional logic do not necessarily hold true in the literal sense but can be true in its logical representation. This maybe a rare occurrence but enough to acknowledge the existence of sentences with special phenomena such as Russell's sentence,(NC)

$\exists A\ \forall x\ (x \in A \equiv \psi)$. They do occur every now and then and cannot be ignored. Similarly, statements like time traveller's sentence do sound vague on the face of it. But after dissecting it and logically testing it, it proves it's equivalence, which in fact not hold true when we actually think about it. These anomalies are rare and even rarer that they occur in our everyday use. But after studying these facts we repeatedly arrive at the age old dilemma which was posed by Bertrand Russell, Kurt Godel. In it's true sense it does lead us to re-think the concepts about the systems closely. This shall lead to a rise of new branch of mathematical philosophy, although as Godel proved no system would truly be perfect in it's own sense but the question herein lies the need for a bypass route leading to the higher truth.

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