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Sequence of Exponential Ratio-type and Exponential Product-type estimators in Post-Stratification with their Efficiency

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Abstract

Following Panda, K.B. and Sahoo, N. [7] &[8] and Sahoo, N. and Dash, S. [9], we have advanced a sequence of exponential ratio-type and exponential product-type estimators in post-stratification. The proposed estimators of order k, are found to be more efficient than the usual ratio-type estimator and exponential ratio-type estimator in post-stratification and usual product-type estimator and exponential product-type estimator in post-stratification, respectively, under some practical condition. With a view to establishing the supremacy of the proposed estimators over the existing estimators, numerical illustrations in respect to real population have been considered.

Keywords: Post-Stratified Sampling, bias, mean square error, Predictive estimation.

I. INTRODUCTION

Stratification is a routinely techniques used in sample survey. Stratified sampling speculate the knowledge that the strata size and sampling frame for each stratum are known. But in many situations, it happens that the overall population size and also the percentage of unit that falls in the strata are known but the sampling frame for the stratum are may not be available or it quit time-consumable for preparing. In such situations we cannot supposed to use stratified random sampling. So, to overcome from this kind of problem we use post stratification technique where the sample size of 'n' is drawn by simple random sampling these sample is stratified in different strata and used as stratified samples.

Let us consider the population size as N that stratified into L strata of size $N_1, N_2, N_3 \dots \dots \dots N_L$ such that $\sum_{h=1}^{L} N_h = N$. Let n_h be the sample size falling in the h^{th} stratum such that $\sum_{h=1}^{L} n_h = n$.

Now by considering y as study variable and x as the auxiliary variable such that it is positively correlated with the study variable y. Let y_{hi} be the observation on i^{th} unit of h^{th} stratum for the study variable y and x_{h_i} be the observation on i^{th} unit of h^{th} stratum for the auxiliary variable x, then

$$\bar{Y}_h = \frac{1}{N_h}\sum_{h=1}^{N_n} y_{h_i} , \qquad \bar{Y} = \frac{1}{N}\sum_{h=1}^{L}\sum_{h=1}^{N_h} y_{h_i} = \sum_{h=1}^{L} W_h \bar{y}_h , \quad \bar{X}_h = \frac{1}{N_h}\sum_{h=1}^{N_n} x_{h_i} ,$$

$$\bar{X} = \frac{1}{N}\sum_{h=1}^{L}\sum_{h=1}^{N_h} x_{h_i} = \sum_{h=1}^{L} W_h \bar{x}_h$$

The usual unbiased estimator of population mean $\bar{Y}$ in post stratification is defined as

$$\bar{y}_{PS} = \sum_{h=1}^{L} W_h \bar{y}_h \tag{1.1}$$

Where $W_h = \frac{N_h}{N}$, and $\bar{y}_h$ is the mean of n_h sample units.

Then $$V(\bar{y}_{PS}) = \frac{N-n}{Nn}\sum_{h=1}^{L} W_h {S_{y_h}}^2 \tag{1.2}$$

Ige and Tripathy (1989) have proposed a product type estimator by using the knowledge about the population mean $\bar{X}$ of auxiliary variate x for population mean in post stratification as,

$$\bar{y}_{RPS} = \bar{y}_{PS}\left(\frac{\bar{X}}{\bar{x}_{PS}}\right) \tag{1.3}$$

$$\bar{y}_{PPS} = \bar{y}_{PS}\left(\frac{\bar{x}_{PS}}{\bar{X}}\right) \tag{1.4}$$

Where $\bar{y}_{PS} = \sum_{h=1}^{L} W_h \bar{y}_h$ and $\bar{x}_{PS} = \sum_{h=1}^{L} W_h \bar{x}_h$ are unbiased estimators of population mean $\bar{Y} = \sum_{h=1}^{L} W_h \bar{Y}_h$ and $\bar{X} = \sum_{h=1}^{L} W_h \bar{X}_h$ in post stratified random sampling, and $\bar{x}_h$ is the mean of the sample size n_h that falls in h^{th} stratum.

The bias and mean-square error of the Ige and Tripathy (1989) estimator is,

$$B(y_{RPS}) = \left(\frac{1}{n} - \frac{1}{N}\right)\frac{1}{\bar{X}}\sum_{h=1}^{L} W_h \left(R_h {S_{x_h}}^2 - S_{yxh}\right) \tag{1.5}$$

$$MSE(y_{RPS}) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h \left({S_{y_h}}^2 + {R_h}^2 {S_{xh}}^2 - 2R_h S_{yxh}\right) \tag{1.6}$$

$$B(y_{PPS}) = \left(\frac{1}{n} - \frac{1}{N}\right)\frac{1}{\bar{X}}\sum_{h=1}^{L} W_h S_{yxh} \tag{1.7}$$

$$MSE(y_{PPS}) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h\left({S_{yh}}^2 + {R_h}^2{S_{xh}}^2 - 2R_h S_{yxh}\right) \tag{1.8}$$

where

$R_h = \frac{\bar{Y}_h}{\bar{X}_h}$, ${S_{xh}}^2 = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(x_{hi} - \bar{X}_h)^2$ and $S_{yxh} = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(y_{hi} - \bar{Y}_h)(x_{hi} - \bar{X}_h)$.

Tailor & Tailor and Chouhan (2017) proposed a new product type exponential estimator for population mean in post stratification as,

$${\bar{y}_{Re}}^{PS} = \bar{y}_{PS} exp\left(\frac{\bar{X} - \bar{x}_{PS}}{\bar{X} + \bar{x}_{PS}}\right) \tag{1.9}$$

$${\bar{y}_{Pe}}^{PS} = \bar{y}_{PS} exp\left(\frac{\bar{x}_{PS} - \bar{X}_h}{\bar{x}_{PS} + \bar{X}_h}\right) \tag{1.10}$$

Bias and mean-square error of Tailor & Tailor and Chouhan (2017) estimator is expressed as,

$$B\left({\bar{y}_{Re}}^{PS}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\frac{1}{\bar{X}}\left[\sum_{h=1}^{L} W_h\left(\frac{3}{8}{R_h}^2{S_{xh}}^2 - \frac{1}{2}R_h S_{yxh}\right)\right] \tag{1.11}$$

$$MSE\left({\bar{y}_{Re}}^{PS}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\left[\sum_{h=1}^{L} W_h {S_{yh}}^2 + \frac{1}{4}(1 - 4K){R_h}^2\sum_{h=1}^{L} W_h {S_{xh}}^2\right] \tag{1.12}$$

$$B\left({\bar{y}_{Pe}}^{PS}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\frac{R_h(4K-1)}{8\bar{X}}\sum_{h=1}^{L} W_h {S_{xh}}^2 \tag{1.13}$$

$$MSE\left({\bar{y}_{Pe}}^{PS}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\left[\sum_{h=1}^{L} W_h {S_{yh}}^2 + \frac{1}{4}(1 + 4K){R_h}^2\sum_{h=1}^{L} W_h {S_{xh}}^2\right] \tag{1.14}$$

where $K = \frac{\sum_{h=1}^{L} W_h S_{yxh}}{R\sum_{h=1}^{L} W_h {S_{xh}}^2}$.

II. A SEQUENCE OF EXPONENTIAL RATIO-TYPE ESTIMATOR IN POST STRATIFICATION WITH PERFORMANCE

With the predictive set-up, we have the population total Y as

$$Y = \sum_{i\in s} y_i + \sum_{i\in \bar{s}} y_i \tag{2.1}$$

Where s denoting the selected sample units and the $\bar{s}$ denoting the complement of s. To estimate the Y we supposed to predict $y_i(i \in \bar{s})$. In other way we have to use the predictive format for the estimation of Y i.e.

$$\hat{Y} = \sum_{i\in s} y_i + \sum_{i\in \bar{s}} \hat{y}_i \tag{2.2}$$

Where $\hat{y}_i$ known as the implied predictor of $y_i(i \in \bar{s})$.

Now if we use $\bar{y}_{Re}{}^{Ps}$ as the intuitive predictor of $y_i (i \in \bar{s})$ in (2.2) then we obtain,

$$\hat{Y} = \sum\nolimits_{i \in s} y_i + (N - n)\bar{y}_{Re}{}^{Ps}$$

Equivalently we have $\hat{\hat{Y}} = \bar{y}_{Re}{}^{PS(1)}$

where $\bar{y}_{Re}{}^{PS(1)} = \emptyset_1 \bar{z}_{Re}{}^{PS} + \bar{y}_{Re}{}^{PS}$ (2.3)

with $\emptyset_1 = 1 + \lambda\emptyset_0, \emptyset_0 = 0$, $\lambda = 1 - \frac{n}{N}$ and $\bar{z}_{Re}{}^{PS} = \frac{n}{N}\bar{y}_{PS}\left[1 - exp\left(\frac{\bar{X} - \bar{x}_{PS}}{\bar{X} + \bar{x}_{PS}}\right)\right]$

Second iteration with $\bar{y}_{Re}{}^{PS(1)}$ as an intuitive predictor of $y_i (i \in \bar{s})$ in (2.2) will give the result in $\bar{y}_{Re}{}^{PS(2)}$i.e.

$$\bar{y}_{Re}{}^{PS(1)} = \emptyset_2 \bar{z}_{Re}{}^{PS} + \bar{y}_{Re}{}^{PS}$$

where $\emptyset_2 = 1 + \lambda\emptyset_1$.

Continuing in this way we will reach to the k^{th} iteration i.e.

$$\bar{y}_{Re}{}^{PS(k)} = \emptyset_k \bar{z}_{Re}{}^{PS} + \bar{y}_{Re}{}^{PS}$$

where $\emptyset_k = 1 + \lambda\emptyset_{k-1} = \frac{1-\lambda^k}{1-\lambda}$.

Now, $\bar{y}_{Re}{}^{PS(k)}$ will expressed as,

$$\bar{y}_{Re}{}^{PS(k)} = (1 - \lambda^k)\bar{y}_{PS} + \lambda^k \bar{y}_{Re}{}^{PS} \qquad (2.4)$$

We will consider $\bar{y}_{Re}{}^{PS(k)}$ as the k^{th} order exponential ratio-type estimator in post-stratification. Here we may note that for $k = 0$, $\bar{y}_{Re}{}^{PS(k)}$ becomes the usual exponential ratio-type estimator in post-stratification in (1.9) and as $k \to \infty$, we have $\lambda^k \to 0$ so $\bar{y}_{Re}{}^{PS(k)} \to \bar{y}_{PS}$.

We will assume that N is finite because, if we select samples of fixed size for infinite population then $\bar{y}_{Re}{}^{PS(k)}$ will be same as $\bar{y}_{Re}{}^{Ps}$.

The bias of $\bar{y}_{Re}{}^{PS(k)}$ up to $O(n^{-1})$can be obtained as,

$$B\left(\bar{y}_{Re}{}^{PS(k)}\right) = \lambda^k \bar{Y}\left(\frac{1}{n} - \frac{1}{N}\right)\left[\frac{3}{8}\frac{1}{\bar{X}^2}\sum\nolimits_{h=1}^{L} W_h {S_{xh}}^2 - \frac{1}{2\bar{X}\bar{Y}}\sum\nolimits_{h=1}^{L} W_h S_{yxh}\right] \qquad (2.5)$$

The mean square error or variance of $\bar{y}_{Re}{}^{PS(k)}$ up to $O(n^{-1})$ can be obtained as,

$$MSE\left(\bar{y}_{Re}{}^{PS(k)}\right) =$$

$$\left(\frac{1}{n} - \frac{1}{N}\right)\left[\sum\nolimits_{h=1}^{L} W_h {S_{yh}}^2 + \frac{\lambda^{2k}}{4}\frac{\bar{Y}^2}{\bar{X}^2}\sum\nolimits_{h=1}^{L} W_h {S_{xh}}^2 - \lambda^k \frac{\bar{Y}}{\bar{X}}\sum\nolimits_{h=1}^{L} W_h S_{yxh}\right] \qquad (2.6)$$

To determine k optimally we will minimize (2.6) by taking derivative with respect to λ^k and equating with zero we will get,

$$\lambda^k = 2\frac{\bar{X}}{\bar{Y}}\frac{\sum_{h=1}^{L} W_h S_{yxh}}{\sum_{h=1}^{L} W_h {S_{xh}}^2} \quad (2.7)$$

Its shows that $MSE\left({\bar{y}_{Re}}^{PS(k)}\right)$ under condition (2.7) will be,

$$MSE\left({\bar{y}_{Re}}^{PS(k)}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\left[\sum_{h=1}^{L} W_h {S_{yh}}^2 - \frac{\sum_{h=1}^{L} W_h S_{yxh}}{\sum_{h=1}^{L} W_h {S_{xh}}^2}\right] \quad (2.8)$$

It will be a natural problem when the optimal value of k is a non-integer and in such case it may be replaced by the nearest integer.

If the value of $\frac{\sum_{h=1}^{L} W_h S_{yxh}}{\sum_{h=1}^{L} W_h {S_{xh}}^2}$ exceed 1, then we can apparently have that a suitable value of k cannot be found from (2.7).

In general, by comparing (1.2) and (2.6) we have, ${\bar{y}_{Re}}^{PS(k)}$ is more efficient than $\bar{y}_{PS}$ if,

$$\frac{\lambda^k}{4}\frac{\bar{Y}}{\bar{X}} < \frac{\sum_{h=1}^{L} W_h S_{yxh}}{\sum_{h=1}^{L} W_h {S_{xh}}^2} \quad (2.9)$$

Also, by comparing (1.6) and (2.6) we have, ${\bar{y}_{Re}}^{PS(k)}$is more efficient than $\bar{y}_{RPS}$ if,

$$\frac{(\lambda^{2k}-4)}{4(\lambda^k-2)}\frac{\bar{Y}}{\bar{X}} > \frac{\sum_{h=1}^{L} W_h S_{yxh}}{\sum_{h=1}^{L} W_h {S_{xh}}^2} \quad (2.10)$$

Thus, by combining (2.9) and (2.10) we found that ${\bar{y}_{Re}}^{PS(k)}$ will be better than $\bar{y}_{PS}$ and $\bar{y}_{RPS}$ if,

$$\frac{\lambda^k}{4} < \frac{\sum_{h=1}^{L} W_h S_{yxh}}{\sum_{h=1}^{L} W_h {S_{xh}}^2} < \frac{(\lambda^{2k}-4)}{4(\lambda^k-2)} \quad (2.11)$$

The efficiency bounds of $\frac{\sum_{h=1}^{L} W_h S_{yxh}}{\sum_{h=1}^{L} W_h {S_{xh}}^2}$ in (2.11) is calculated with the sampling fraction $f = \frac{n}{N}$ and $\lambda = (1-f)$. We have given Table-1 that represent the bounds of $\frac{\sum_{h=1}^{L} W_h S_{yxh}}{\sum_{h=1}^{L} W_h {S_{xh}}^2}$ for various values of k for which ${\bar{y}_{Re}}^{PS(k)}$ will be more efficient then $\bar{y}_{PS}$ and $\bar{y}_{RPS}$.

Table-1

f	k					
	1	**2**	**5**	**8**	**10**	**50**
0.05	**(0.237,0.737)**	**(0.226,0.725)**	**(0.193,0.693)**	**(0.166,0.665)**	**(0.150,0.649)**	**(0.019,0.519)**
0.10	**(0.225,0.725)**	**(0.202,0.702)**	**(0.148,0.647)**	**(0.108,0.607)**	**(0.087,0.587)**	**(0.001,0.501)**
0.20	**(0.200,0.700)**	**(0.160,0.600)**	**(0.082,0.581)**	**(0.042,0.541)**	**(0.027,0.526)**	**(0,0.499)**
0.50	**(0.125,0.625)**	**(0.062,0.562)**	**(0.008,0.507)**	**(0,0.500)**	**(0,0.500)**	**(0,0.500)**
0.80	**(0.050,0.550)**	**(0.010,0.510)**	**(0,0.500)**	**(0,0.500)**	**(0,0.500)**	**(0,0.500)**
0.90	**(0.025,0.525)**	**(0.002,0.502)**	**(0,0.500)**	**(0,0.500)**	**(0,0.500)**	**(0,0.500)**

Generally, Table-1 is helpful in choosing the suitable value of k and f to ensure that the performance of $\bar{y}_{Re}{}^{PS(k)}$ will be more efficient then $\bar{y}_{PS}$ and $\bar{y}_{RPS}$. However, the optimal value of k can be obtained from equation (2.7) with the condition that $\frac{\sum_{h=1}^{L} W_h S_{yxh}}{\sum_{h=1}^{L} W_h S_{xh}{}^2} < 1$. Even the exact value of k is not available rather than that a suitable value can be taken that renders $\bar{y}_{Re}{}^{PS(k)}$ superior to $\bar{y}_{PS}$ and $\bar{y}_{RPS}$ may be still found as evidence by Table-1.

III. A SEQUENCE OF EXPONENTIAL PRODUCT-TYPE ESTIMATORS IN POST-STRATIFICATION WITH PERFORMANCE

Now, $\bar{y}_{Pe}{}^{PS(k)}$ will expressed as,

$$\bar{y}_{Re}{}^{PS(k)} = (1-\lambda^k)\bar{y}_{PS} + \lambda^k \bar{y}_{Pe}{}^{PS} \tag{3.1}$$

Bias of $\bar{y}_{Pe}{}^{PS(k)}$ is found as,

$$B\left(\bar{y}_{Pe}{}^{PS(k)}\right) = \lambda^k \bar{Y}\left(\frac{1}{n}-\frac{1}{N}\right)\left[-\frac{1}{8}\frac{1}{\bar{X}^2}\sum_{h=1}^{L} W_h S_{xh}{}^2 + \frac{1}{2\bar{X}\bar{Y}}\sum_{h=1}^{L} W_h S_{yxh}\right] \tag{3.2}$$

and

$$MSE\left(\bar{y}_{Pe}{}^{PS(k)}\right) =$$

$$\left(\frac{1}{n}-\frac{1}{N}\right)\left[\sum_{h=1}^{L} W_h S_{yh}{}^2 + \frac{\lambda^{2k}}{4}\frac{\bar{Y}^2}{\bar{X}^2}\sum_{h=1}^{L} W_h S_{xh}{}^2 + \lambda^k \frac{\bar{Y}}{\bar{X}}\sum_{h=1}^{L} W_h S_{yxh}\right] \tag{3.3}$$

Then

$$\lambda^k = -2\frac{\bar{X}}{\bar{Y}}\frac{\sum_{h=1}^{L} W_h S_{yxh}}{\sum_{h=1}^{L} W_h S_{xh}{}^2} \tag{3.4}$$

After substituting the optimum value of λ^k in (3.3) we have,

$$MSE\left(\bar{y}_{Pe}{}^{PS(k)}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\left[\sum_{h=1}^{L} W_h S_{yh}{}^2 - \frac{\sum_{h=1}^{L} W_h S_{yxh}}{\sum_{h=1}^{L} W_h S_{xh}{}^2}\right] \tag{3.5}$$

It can be seen easily by comparing (1.8) and (3.4) we have, $\bar{y}_{Pe}{}^{PS(k)}$is more efficient than $\bar{y}_{RPS}$ if,

$$-\frac{(\lambda^{2k}-4)}{4(\lambda^k-2)}\frac{\bar{Y}}{\bar{X}} < \frac{\sum_{h=1}^{L} W_h S_{yxh}}{\sum_{h=1}^{L} W_h S_{xh}{}^2} \tag{3.7}$$

Also comparing (1.2) and (3.4) we have, $\bar{y}_{Pe}{}^{PS(k)}$is more efficient than $\bar{y}_{PS}$ if,

$$-\frac{\lambda^k}{4}\frac{\bar{Y}}{\bar{X}} > \frac{\sum_{h=1}^{L} W_h S_{yxh}}{\sum_{h=1}^{L} W_h S_{xh}{}^2} \tag{3.8}$$

Hence, by combining (3.7) and (3.8) we get $\bar{y}_{Pe}{}^{PS(k)}$is more efficient than $\bar{y}_{PS}$ and $\bar{y}_{RPS}$ if,

$$-\frac{(\lambda^{2k}-4)}{4(\lambda^k-2)} < \frac{\sum_{h=1}^{L} W_h S_{yxh}}{\sum_{h=1}^{L} W_h S_{xh}{}^2} < -\frac{\lambda^k}{4} \tag{3.9}$$

IV. Numerical Illustration

The following examples are showing the potential gains by the use of exponential ratio-type estimators in post-stratification of k^{th} order and exponential product-type estimator in post-stratification of k^{th} order in place of classical and existing estimators.

Formula used for percentage gain in efficiencies are,

$$G_1 = \left[\frac{V(\bar{y}_{PS})}{V(\bar{y}_{RPS})} - 1\right] \times 100$$

$$G_2 = \left[\frac{V(\bar{y}_{PS})}{V(\bar{y}_{Re}{}^{PS(k)})} - 1\right] \times 100$$

Formula used for percentage gain in efficiencies or exponential ratio-type estimators in post-stratification are,

$$G_1^* = \left[\frac{V(\bar{y}_{PS})}{V(\bar{y}_{PPS})} - 1\right] \times 100$$

$$G_2^* = \left[\frac{V(\bar{y}_{PS})}{V(\bar{y}_{Pe}{}^{PS(k)})} - 1\right] \times 100$$

IV(A): Exponential ratio-type estimator in post-stratification

Illustration 1: we use the data in Onyeka (2012).

Population	N=96	$n = 20$	$\bar{X}$=68.13	$\bar{Y}$=2.44	S_x=7.03	S_y=0.57	${S_x}^2$=49.37	${S_y}^2$=0.33	S_{yx}=3.26
Stratum I	N_1=72	n_1=8	$\bar{X}_1$=68.11	$\bar{Y}_1$=2.44	S_{x1}=7.28	S_{y1}=0.60	${S_{x1}}^2$=52.97	${S_{y1}}^2$=0.35	S_{yx1}=0.80
Stratum II	N_2=24	n_2=12	$\bar{X}_2$=68.17	$\bar{Y}_2$=2.46	S_{x2}=6.36	S_{y2}=0.50	${S_{x2}}^2$=40.41	${S_{y2}}^2$=0.25	S_{yx2}=2.75

Based on the above information the gain in efficiencies are calculated as

$G_1 = 113.33$ and $G_2 = 197.67$

Illustration 1: we use the data in Onyeka (2012).

Stratum I	N_1=36	n_1=6	$\bar{X}_1$=366.667	$\bar{Y}_1$=135	S_{x1}=52.0256	S_{y1}=8.9443	S_{yx1}=440
Stratum II	N_2=72	n_2=12	$\bar{X}_2$=310.8333	$\bar{Y}_2$=99.1667	S_{x2}=43.3712	S_{y2}=15.0504	S_{yx2}=618.93
Stratum III	N_3=42	n_3=7	$\bar{X}_3$=317.1429	$\bar{Y}_3$=80.7142	S_{x3}=53.7631	S_{y3}=10.9653	S_{yx3}=444.04

Based on the above information the gain in efficiencies are calculated as

$G_1 = 155.2712$ and $G_2 = 267.3313$

IV(B): Exponential product-type estimator in post-stratification

Illustration 1: Data Source (National horticulture Board)

y: Productivity in MT/ Hectare

x: Area in '000 Hectare

Stratum I	N_1=10	n_1=4	$\bar{X}_1$=6.32	$\bar{Y}_1$=1.70	S_{x1}=1.19	S_{y1}=0.50	S_{yx1}= -0.05
Stratum II	N_2=10	n_2=4	$\bar{X}_2$=80.67	$\bar{Y}_2$=3.67	S_{x2}=10.82	S_{y2}=1.41	S_{yx2}= -7.04

Based on the above information the gain in efficiencies are calculated as

$G_1 = 16.8997$ and $G_2 = 49.0341$

Illustration 2: Data Source, Chouhan (2012)

y: Snowy days

x: Total annual sunshine hours

Stratum I	N_1=10	n_1=4	$\bar{X}_1$=1629.9	$\bar{Y}_1$=149.7	S_{x1}=102.17	S_{y1}=13.46	S_{yx1}= -1072.8
Stratum II	N_2=10	n_2=4	$\bar{X}_2$=2035.9	$\bar{Y}_2$=102.6	S_{x2}=103.26	S_{y2}=12.60	S_{yx2}= -655.25

Based on the above information the gain in efficiencies are calculated as $G_1 = 66.6862$ and $G_2 = 106.6269$

The above illustrations clearly establishing the fact that the proposed exponential ratio-type estimator of order k in case of post-stratification is better than $\bar{y}_{PS}$ and $\bar{y}_{RPS}$ and exponential product-type estimator of order k in case of post-stratification is better than $\bar{y}_{PS}$ and $\bar{y}_{PPS}$.

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