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Effect of EWMA on the Power Function of the Mean Test

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Abstract

In this paper, an attempt has been made to study the effect of EWMA on the power function of the mean test. In the testing of the hypothesis, we derive the power function under the assumption that mean $\overline{w}$ follow the normal law with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}(\lambda/(2-\lambda))^{1/2}$ under EWMA model. However, if we find after testing, that there is EWMA model from where the observations are taken. Hence the parameters of the normal distribution would necessary be changed. This in turn would cause change in the power function. As a visual illustration, we have drawn power curve for few selected value of λ.

Key words: Power Function, EWMA model, Testing of hypothesis.

1. Introduction:

Neyman and Pearson, in their first paper (1928), defined these errors and investigated an intuitive principle for test construction that yields likelihood ratio tests. Thus, we may say that the errors made in testing a statistical hypothesis are of two types, (i) we may wrongly reject it, when it is true, (ii)we may wrongly accept it, when it is false. These are known as Type-I and Type-II errors respectively. The probability of a Type-I error is equal to the size of critical region used α. The probability of a Type-II error is, of course, a function of the alternative hypothesis H_1 considered, and usually denoted by β and $(1-\beta)$ is called the power function of the test hypothesis. The Kendall and Staurt (1961), Fraser (1957), Hogg and Craig (1965) have been studied about the testing of statistical hypothesis. Z-statistic provide a suitable test of significance for the mean when the sample comes from a normal population. The power of the normal theory test has been studied by Neyman (1935). Scheff'e (1949) studied the power characteristics under the alternative hypothesis but without regard to EWMA model.

A standard assumption when using a control chart to monitor a process is that the observations from the process output are independent. Chou-wen Lu and Renolds (1999) represented a paper entitled " EWMA control charts for monitoring the mean of Auto correlated Process " in which they suggested that when the level of autocorrelation is low or moderate, the two EWMA charts require about the same amount of time to detect various shifts, but for high levels of autocorrelation and large shifts, the EWMA chart of the residuals is a little faster. Barror, Montgomery and Runger (1999) have shown that an EWMA control chart can be designed so that it is robust to the normality assumption, that is, so that the in-control average run length (ARL) is reasonably close to the normal-theory value for both skewed and heavy tailed symmetric non-normal distributions.

2. Mathematical Model:

For testing $H_0 : \mu = \mu_0$ against $H_1 : \mu = \mu_1$ for the normal population $f(x;\mu,\sigma_p^2)$ where σ_p^2 is known. Let $x_1, x_2, ..., x_n$ be a random sample from the population under consideration. Adopting usual notations and using Neyman-Person lemma, the Best Critical Region (BCR) is obtained as

$$\frac{L(x|\mu_1)}{L(x|\mu_0)} = \frac{e^{-\frac{1}{2\sigma_p^2}\sum(x_i-\mu_1)^2}}{e^{-\frac{1}{2\sigma_p^2}\sum(x_i-\mu_0)^2}} > K \tag{1}$$

taking logarithm and solving, we obtain

$$\bar{x} > \frac{\sigma_p^2}{n(\mu_1-\mu_0)}\log K + \frac{1}{2}(\mu_1+\mu_0). \tag{2}$$

Case I: when $\mu_0 < \mu_1$, BCR is determined as $\bar{x} > \lambda_1$,

Case II: when $\mu_0 > \mu_1$, BCR is determined as $\bar{x} < \lambda_2$,

Where λ_1 and λ_2 are equal to the right hand of the inequality under the two situations.

We know that under H$_0$, $\bar{x} \sim N\left(\mu_0, \frac{\sigma_p^2}{n}\right)$ using Neyman-Person and under H$_1$, the value of λ_1 can be obtained by the relation.

$$P(\bar{x} > \lambda_1 \mid H_0) = \frac{\sqrt{n}}{\sigma_p\sqrt{2\pi}}\int_{\lambda_1}^{\infty} e^{-\frac{n}{2\sigma_p^2}(\bar{x}-\mu_0)^2}\, d\bar{x} = \alpha \tag{3}$$

and for the value of λ_2

$$P(\bar{x} < \lambda_2 \mid H_0) = \frac{\sqrt{n}}{\sigma_p\sqrt{2\pi}}\int_{-\infty}^{\lambda_2} e^{-\frac{n}{2\sigma_p^2}(\bar{x}-\mu_0)^2}\, d\bar{x} \tag{4}$$

substituting $\frac{\sqrt{n}(\bar{x}-\mu_0)}{\sigma_p} = y$ for the normal distribution function,

$$\alpha = 1 - \Phi\left(\sqrt{n}(\lambda_1 - \mu_0)\right)$$
$$= \Phi\left(-\sqrt{n}(\lambda_1 - \mu_0)\right)$$

since

$$\Phi(x) = \int_{-\infty}^{x} \sqrt{2\pi}\,\exp\left(-\frac{1}{2}y^2\right)dy \tag{5}$$

Since $\Phi(x) = 1 - \Phi(-x)$ by symmetry. If we now write

$$d_\alpha = \frac{\sqrt{n}(\lambda_1 - \mu_0)}{\sigma_p} \tag{6}$$

We have

$$\Phi(-d_\alpha) = \alpha \tag{7}$$

In the normal case, the power of the test is

$$1-\beta = P(x \in W \mid H_1) = P(\bar{x} > \lambda_1 \mid H_1)$$

$$1-\beta = \frac{\sqrt{n}}{\sigma_p\sqrt{2\pi}} \int_{\lambda_1}^{\infty} e^{-\frac{n}{2\sigma_p^2}(\bar{x}-\mu_1)^2}\, d\bar{x} \tag{8}$$

substituting $y = \frac{\sqrt{n}(\bar{x}-\mu_1)}{\sigma_p}$, equation (8) reduces as

$$1-\beta = 1 - \Phi\left(\sqrt{n}(\mu_0 - \mu_1) + d_\alpha\right)$$
$$= \Phi\left(\sqrt{n}(\mu_1 - \mu_0) - d_\alpha\right) \tag{9}$$

3.The Power Function of the Mean Test Under EWMA:

We suppose that a process is on target μ initially and successive measurements $\bar{X}_t, (t = 1,2,.....)$ are taken (it could even be average of several measurements taken at time t) to check whether there is a shift from the target. Then to use a control chart based on the statistic

$$W_t = \lambda\bar{X}_t + (1-\lambda)W_{t-1} \quad , \qquad t \geq 0 \tag{10}$$

where $W_0 = \mu$, which is a geometric mean of all the observations with $\bar{X}_t$ the most recent observation getting the greatest weight ($=\lambda$) and all previous observations weight decreasing in geometric progression.

Here the mean and variance of W_t will be

$$E(W_t) = \mu$$

and

$$Var(W_t) = \frac{\sigma^2}{n}T^2 \text{,where } T^2 = \left(\frac{\lambda}{2-\lambda}\right) \tag{11}$$

From equation(9), the power under EWMA may be calculated by

$$1-\beta = 1-\Phi\left(\frac{\sqrt{n}}{T}(\mu_0-\mu_1)+d_\alpha\right)$$

$$= \Phi\left(\frac{\sqrt{n}}{T}(\mu_1-\mu_0)-d_\alpha\right) \qquad (12)$$

where

$$d_\alpha = \frac{\sqrt{n}(\lambda_1-\mu_0)}{\sigma T} \qquad (13)$$

4. Numerical Illustration and Result:

Suppose we have to test the hypothesis $H_0 : \mu = \mu_0 = 0.8$ against the set of alternative $H_1 : \mu \neq \mu_0$.

Table 1: Values of the Power Function for Different Values of λ and n=7 For α =0.05

μ_1	λ =0.2	λ =0.4	λ =0.6	λ =0.8	λ =1.0
-0.9	1.000	1.000	1.000	1.000	0.998
-0.8	1.000	1.000	1.000	0.999	0.994
-0.7	1.000	1.000	0.999	0.996	0.988
-0.6	1.000	1.000	0.997	0.991	0.977
-0.5	1.000	0.999	0.991	0.978	0.959
-0.4	1.000	0.996	0.976	0.955	0.929
-0.3	0.997	0.982	0.941	0.913	0.887
-0.2	0.974	0.942	0.877	0.851	0.829
-0.1	0.875	0.851	0.773	0.761	0.752
0.0	0.637	0.692	0.637	0.652	0.663
0.1	0.334	0.496	0.484	0.528	0.559
0.2	0.111	0.295	0.326	0.401	0.460
0.3	0.022	0.142	0.198	0.284	0.355
0.4	0.003	0.055	0.104	0.184	0.264
0.5	0.0002	0.017	0.048	0.111	0.184
0.6	0.00001	0.004	0.020	0.062	0.123
0.7	0.0000001	0.001	0.007	0.031	0.076
0.8	0.000000001	0.0001	0.002	0.014	0.045
0.9	0.0000001	0.001	0.007	0.031	0.076
1.0	0.00001	0.004	0.020	0.062	0.123
1.1	0.0002	0.017	0.048	0.111	0.184
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1.8	0.974	0.942	0.877	0.851	0.829
1.9	0.997	0.982	0.941	0.913	0.887
2.0	1.000	0.996	0.976	0.955	0.929
2.1	1.000	0.999	0.991	0.978	0.959
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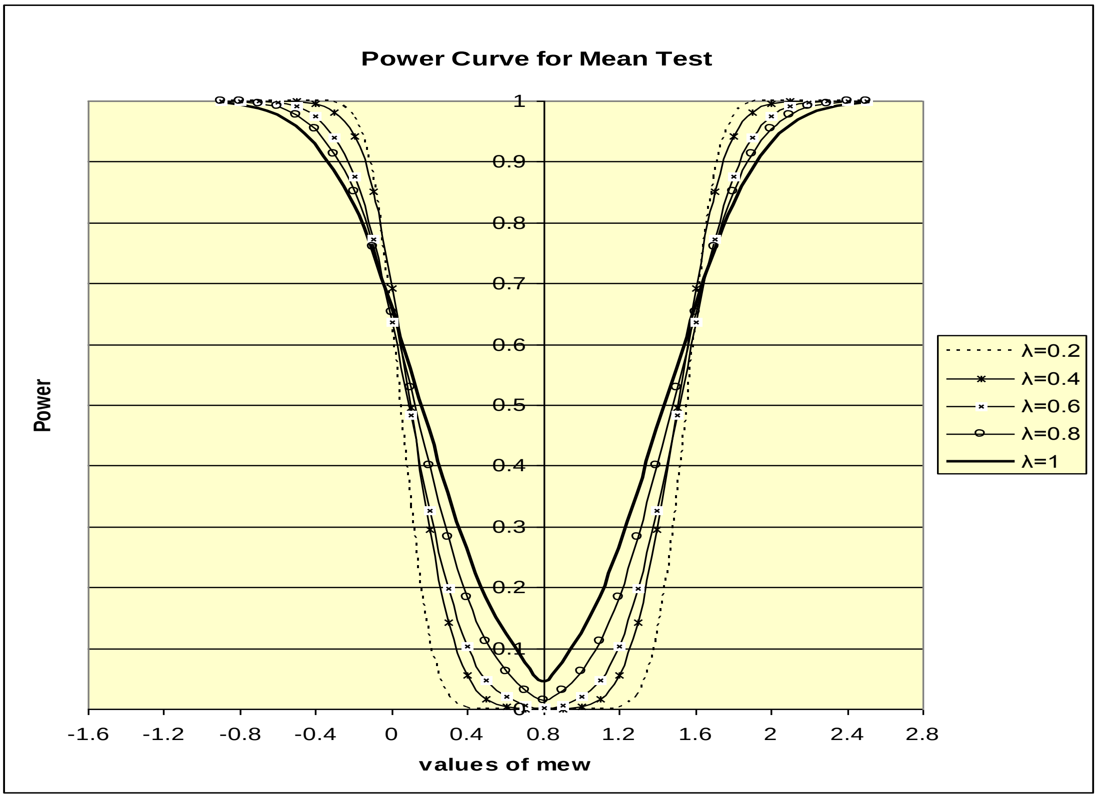

Figure-1

We have taken some illustrative example to show the effect of EWMA on the power functions corresponding to the significance level α =0.05.

Our tabulated value of power function in Table-1 for α =0.05 being calculated from a basically identical formula under EWMA model. The power curve Fig.-1, for sample size 7 from the EWMA population specified by different values of

$\lambda = 0.2, 0.4, 0.6, 0.8, 1.0$ and for known σ which is equal to one. The visual comparison shows that the value of α obtained on applying under EWMA model are increasing as the value of λ decreases. It may be seen from the Table -1 that the power increases, when the λ increases. So we derive the conclusion about the nature of the effect of EWMA model on the power function of the mean test. Critical regions based on the assumptions of normality and without EWMA of the parent population have been considered, which though erroneous have helped in bringing out how the power curve is distorted when the usual Z-test is applied to a sample from normal population.

Our results indicate that good choices for λ depends on the number of variables in the testing of hypothesis and the size of the shift. The recommendation for study-state analyses are not substantially different than those for zero-state. The worst state result show the amount of degration in power function that can occur.

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