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Optimization Technique for Deteriorating Items under Advertisement Dependent Demand

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Abstract:
The current paper deals an inventory model in which demand is a function of time and advertisement. The model is developed for a finite time horizon. Shortages are fully backlogged. Advertisement policy is also incorporated. Numerical analysis is performed to demonstrate the application of the model. Mathematica 5.1 software is used to find the optimal solutions and ascertain the minimum cost in the business. Sensitivity analysis of the important parameters associated with the model has been carried out.

Key words: EOQ, time horizon, optimization technique, Advertisement dependent demand.

Introduction

Inventory management is the backbone of any product-based business. It ensures that the right products are available in the right quantities, at the right time and in the right place. It's a delicate balancing act, aiming to satisfy customer demand while minimizing the costs associated with holding stock. Maihami & Kamalabadi (2012) explained non-instantaneous deteriorating items with time and price dependent demand. Amutha (2017) introduced an inventory model for deteriorating items with quadratic demand, time dependent Holding cost and salvage value. Vora & Gothi (2023) analysed an inventory model where the demand rate is quadratic and deterioration follows two parameter weibull distribution. Mohan & Civil (2016) dealt in partial backlogging model with time dependent deterioration. Pandey & Pandey (2018) presented a production inventory model with variable holding cost. Behera & Tripathy (2017) dealt with an inventory model with power pattern demand under regulatory demand. Sahoo et al (2019) proposed an optimal decision support model with two parameter weibull demand. Tripathy & Mohanty (2018) explained an inventory model with the policy of combined criteria. Tripathy & Behera (2019) focused a mixture inventory model where demand is depends on time and stock dependent selling rate. Mohanty (2024) focused on green inventory management and carbon emissions. Amutha & Chandrasekaran (2013) proposed an inventory model where demand is polynomial function of time and holding cost is time dependent. The intensity of advertisement is mainly focused on sales and brand building. The connection lies in the information and insights that advertising provides, enabling businesses to make more informed decisions about what to stock, how much to stock and when to replenish. It also used to manage slow moving or excess inventory. When certain products are not selling as quickly, advertising campaigns can be launched to boost their visibility and appeal. Palanivel and Vetriselvi (2024) explained an inventory model with polynomial demand, advance payment and shortages. Bag and Patro (2023) introduced a partially backlogged inventory model with perishable goods with controllable deterioration and advance payment.

In this research paper attempts to investigate an inventory model featuring quadratic time dependent demand and an integrated advertising policy. This model provides a novel managerial strategy for retailers seeking to minimize total inventory costs. The optimization problem was formulated and solved with Mathematica 5.1.

Related Work

Table 1: Comparative Study of Review Literature

Literature	Deterioration	Rate of Demand	Shortages	Advertisement Policy
Maihamis & Kamalbadi (2012)	Non-instantaneous	Time & Price Dependent	Partially Backlogged	No
Mohan & Civil (2016)	Time Dependent	Quadratic	Partially Backlogged	No
Amutha (2017)	Time Dependent Exponential Distribution	Quadratic	No	No
Behera & Tripathy (2017)	Time Dependent	Power Demand	Partially Backlogged	No
Pandey & Pandey (2018)	Constant	Time Dependent	Completely Backlogged	No
Tripathy & Behera (2019)	Constant	Selling Price Dependent	Partially Backlogged	No
Bag & Patro (2023)	Reduced by Using	Selling Price Dependent	Partially Backlogged	No

	Preservation Technology			
Vora & Gothi (2023)	Two Parameter Weibull Distribution	Quadratic	No	No
Palanivel & Vetriselvi (2024)	Non-instantaneous	Polynomial	Completely Backlogged	No
Present Paper	Constant	Advertisement & Time Dependent	Completely Backlogged	Yes

Assumptions:
The inventory deals with single items only.
The rate of deterioration is unchanged.
The demand rate is depending on time and advertisement.
The rate of replenishment is instantaneous with lead time is zero.
The planning horizon is infinite.
The holding cost is unchanged.
Shortages are considered which is completely backlogged.

Notations:
$Z(t)$ = inventory level at time t.
$Q(t)$ = order quantity at time t.
O_C = cost of ordering.
H_C =cost of holding
D_C =cost of deterioration
S_C =cost of shortage
θ =rate of deterioration
B =cost of advertisement
λ =frequency of advertisement
t_1 = length of the positive inventory.
$TC(t_1, T)$ = total cost of the system.

Mathematical Model:

The on-hand inventory $Z(t)$ decreases due to the merged effect of customer requirement and depreciation during the time interval $(0, t_1)$. The level of inventory $Z(t)$ at any time t is given by

$$\frac{dZ(t)}{dt} + \theta Z(t) = -(a + bt + ct^2)B^{\lambda}, \quad 0 < t < t_1 \tag{1}$$

Using the boundary restrictions $Z(t_1) = 0$ and $Z(0) = Q$, the solution of equation (1) yields

$$Z(t) = a(t_1 - t) + \frac{b + a\theta}{2}(t_1^2 - t^2) + \frac{c + b\theta}{3}(t_1^3 - t^3) + \frac{c\theta}{4}(t_1^4 - t^4) \tag{2}$$

The order quantity is given by

$$Q(t) = at_1 + (\frac{b + a\theta}{2})t_1^2 + (\frac{c + b\theta}{3})t_1^3 + \frac{c\theta}{4}t_1^4 \tag{3}$$

Then shortages occur during the time interval (t_1, T).The level of inventory at any time t is given by

$$\frac{dZ(t)}{dt} = -(a + bt + ct^2)B^{\lambda} \qquad t_1 \le t \le T \tag{4}$$

By using the boundary condition $Z(t_1) = 0$, the Solution of equation (4) is found as

$$Z(t) = (t_1 - t)(a - a\lambda T) + (t_1^2 - t^2)(\frac{a\lambda}{2} + \frac{b}{2} - b\lambda T) + (t_1^3 - t^3)(\frac{b\lambda}{3} + \frac{c}{3} - \frac{c\lambda}{3}) + \frac{c\lambda}{4}(t_1^4 - t^4) \tag{5}$$

The ordering cost (OC) is O_C . (6)

The deterioration cost (DC) is expressed as

$$\text{DC} = D_C\left[Q - \int_0^{t_1}(a + bt + ct^2)dt\right] = D_C\left[\frac{a\theta}{2}t_1^2 + \frac{b\theta}{3}t_1^3 + \frac{c\theta}{4}t_1^4\right] \tag{7}$$

The holding cost (HC) is defined as

$$\text{HC} = H_C\int_0^{t_1} Z(t)dt = H_C\left[\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \frac{ct_1^4}{4} + \frac{at_1^3\theta}{3} + \frac{bt_1^4\theta}{4} + \frac{ct_1^5\theta}{5}\right] \tag{8}$$

The shortage cost (SC) is defined as

$$SC = S_C\int_{t_1}^{T} Z(t)dt = S_C\left[\frac{1}{60}\left(\begin{array}{l}\left(-12ct_1^5\lambda + 15ct_1^4T\lambda - 3cT^5\lambda + \right.\\ 5(3t_1^4 - 4t_1^2T + T^4)(c(-1+\lambda) - b\lambda) + \\ \left. 30a(t_1 - T)^2(-1 + T\lambda) + 10(t_1 - T)^2(2t_1 + T)(-b - a\lambda + 2bT\lambda)\right)\end{array}\right)\right] \tag{9}$$

The total cost of inventory system is

$$TC(t_1,T)=\frac{1}{T}\left[OC+HC+DC+SC\right] \tag{10}$$

The optimal points of t_1 and T which minimize the total cost of the system can be solved by using the following differential equations

$$\frac{dTC(t_1,T)}{dt_1}=0 \text{ and } \frac{dTC(t_1,T)}{dT}=0. \tag{11}$$

provided that,

$$\frac{d^2TC(t_1,T)}{dt_1^{\,2}}>0 \text{ and } \frac{d^2TC(t_1,T)}{dT^2}>0 \tag{12}$$

Numerical Example:

The below mentioned parameter values are reasonable and chosen at random. We were able to address the proposed problem effectively by utilizing established algorithms.

Example 1: Let us consider the values ordering cost O_c=400, Cost of Advertisement B=5, frequency of advertisement λ=1, holding cost H_c=30 per unit, deterioration cost D_c=10 per unit, shortage cost S_c=20, rate of deterioration θ=0.01. Then we get t_1=0.994016, T=1.73726, total cost $TC(t_1,T)$=315.808.

Example 2: Let us consider the values ordering cost O_c=500, Cost of Advertisement B=5, frequency of advertisement λ=2, holding cost H_c=50 per unit, deterioration cost D_c=10 per unit, shortage cost S_c=20, rate of deterioration θ=0.01. Then we get t_1=0.470784, T=1.15337, total cost $TC(t_1,T)$=837.183.

Discussion

The following significant perspectives can be noticed from the above numerical examples:

From Table 2, we get the optimum advertisement frequency and total Cost.

Table 2: Comparison of Frequency of Advertisement and Total Cost

λ	t_1	T	$TC(t_1,T)$
1	0.99401	1.7372	315.808
2	0.70915	1.1244	479.239
3	0.47522	1.7372	2659.780
4	0.26869	1.2393	6174.721
5	0.34664	1.45993	40942.25

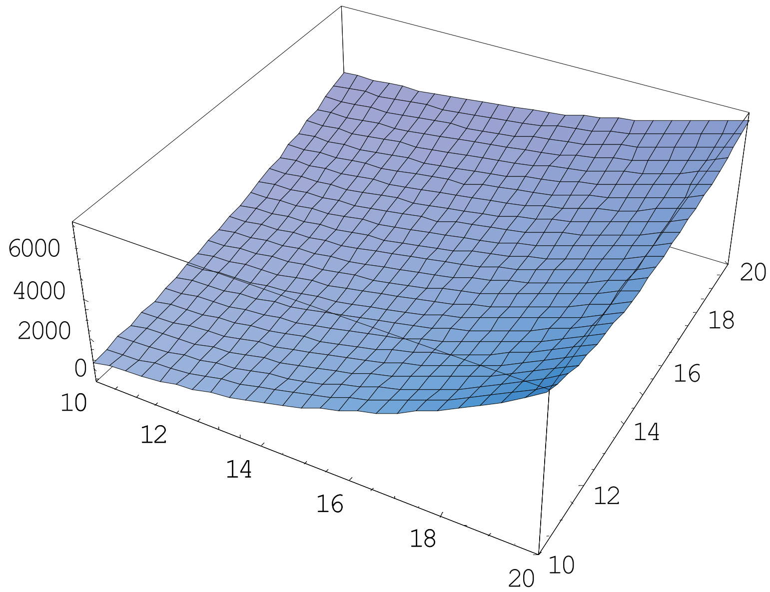

Figure 1: Optimal Cost

Conclusion

This research paper develops an inventory model that accounts for a quadratic time dependent demand and integrates an advertisement policy. The model introduces a new management strategy designed to help retailers reduce their overall inventory costs. We employed the proposed model using a numerical example to identify the most economically advantageous possibility. This analysis reveals that advertising significantly influences customer purchasing decisions, directly affecting product demand.

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