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Numerical Analysis of Soil-Water Flow in Fixed Horizontal Pipe

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Abstract

The flow through unsaturated porous media is governed by extension of Darcy's law and Richard differential equation. These equations generally presented in standard forms as "h-based", "θ-based" and "mixed" form. A finite difference techniques using forward Euler time-marching coupling simulation approach for soil-water content-pressure head curve, is presented in this paper. Fokker-Planck equation backed by effective saturation, Θ and relative hydraulic conductivity, Kr is solved. Spherical monochromatic porous media packing is considered for simulation. Result obtained from simulation are further presented for simulation period and compared with observed hydraulic conductivity data. The simulation is found to be reasonable accurate for predicting unsaturated flow properties in soil water retention curve like variation characteristic of matric potential, moisture content, hydraulic conductivity with respect to rig linear dimension variation.

Keywords: Unsaturated porous media flow, Mathematical modelling, Porous media, Soil-water retention curve, Water Content

1. INTRODUCTION

Unsaturated (Multiphase) flow through porous medium play a significant role in number of industrial processes like enhance oil recovery, environmental problem, catalyst design, filtration process and ground water modelling. After recognition of the engineering importance of the porous media, numerous numerical and analytical studies are conducted for better understanding. Mathematical model for flow through porous media have been utilized since the late 1800s (Darcy, 1856, **[1]**). Rigors mathematical equations are solved using computational tools leading wide range of industrial application. Variables (direct/indirect), their range, dependencies on other independent variables i.e. coupling behaviour, fundamental and constitutive equations, assumption lead results to new parameter waiting for explore.

2. BACKGROUND

Numerous analytical, numerical and experimental studies had conducted in past to understand fluid propagation in porous media having complex internal structure pattern. Extension to Darcy's law focusing on capillary pressure-saturation-relative permeability relationships where proposed by Richards **[2]** and Gardner **[3]**. Many empirical relationships were used to identify the relationship of parameters like Brooks-Corey model, Van Genuchten model **[4]**. Edlefsen et al. **[5]** extended Darcy law by adding mechanical potential and capillary pressure. Several researchers presented the different form of a pore-scale model like Nieber et al. **[6]** (a bundle of tubes model) Dahle et al. **[7]** and Yang et al. **[8]** (a bed of having multiple diameter spheres). These simple models were able to predict the major features of a porous media.

Numerical approach based on finite difference method and finite element method were adopted by several researcher for the solution of Richards' equation like **[9]**, **[10]** etc. Computer programing based models are used today, to predict the water movement through unsaturated porous media.

These approaches are based on pore size distribution model like Darcy Law. In last several decades several researcher suggested several model in their literature like **[11]**, **[12]**, **[13]**, **[14]** and **[15]** etc.This paper present a versatile approach based on numerical simulaiton model to solve one dimensional Extended Richard equation. Finite difference based algorithm is solved using Matlab software and simulated results are presented. The simulated result

3. GOVERNING EQUATION

Governing equation for unsaturated flow in porous media generally presented in three form i.e. 'h-based', 'θ-based' and 'mixed' form, based on dependent variable such as moisture content, θ (x,t)$\{L^3/L^3\}$ and pressure head, h(x,t) $\{L\}$ or conversion of one form to another based on constitutive relationship between moisture content, θ and pressure head, h.

θ-based form

$$\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial x}\left[D(\theta)\frac{\partial\theta}{\partial x}\right]-\frac{\partial K(\theta)}{\partial x} \qquad \textbf{0-1}$$

h-based form

$$C(h)\frac{\partial h}{\partial t}=\frac{\partial}{\partial x}\left[K(h)\frac{\partial h}{\partial x}\right]-\frac{\partial K(h)}{\partial x} \qquad \textbf{0-2}$$

And Mixed form

$$\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial x}\left[K(h)\frac{\partial h}{\partial x}\right]-\frac{\partial K(\theta)}{\partial x} \qquad \textbf{0-3}$$

Where, Moisture Capacity C(h)=dθ/dh {1/L}, Unsaturated Hydraulic Conductivity K(θ) or K(h) {L/T}, Unsaturated Diffusivity D(θ)= K(θ)/C(θ) $\{L^2/T\}$ and x {L} represent the vertical coordinates.

θ–Based Fokker-Planck equation, extension of Richard equation for unsaturated flow in porous media [16] can be written as

$$\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial x}\left[D\frac{\partial\theta}{\partial x}+K\right] \qquad \textbf{0-4}$$

Above equation is solved for flow through one dimensional horizontal column having length L in horizontal direction for x position and t time interval. Unsaturated Diffusivity, D(θ) and Unsaturated Hydraulic Conductivity, K is assumed to be constant throughout the rig.

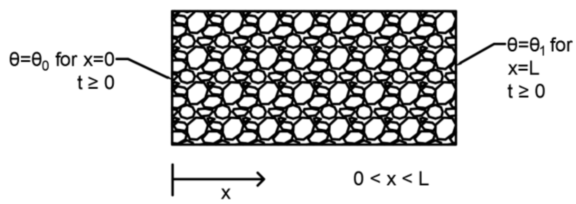

Figure 1 One Dimensional Horizontal Column

The infiltration depth along with its rate about the rig column can be evaluated based on equation . Simplifying the case for horizontal flow, neglecting gravitation effect the equation can be reduce as

$$\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial x}\left[D\frac{\partial\theta}{\partial x}\right] \qquad 0\text{-}5$$

By assuming diffusivity, D for test rig constant, we can simplify the equation as

$$\frac{\partial \theta}{\partial t} = D\frac{\partial^2 \theta}{\partial x^2} \quad 0\text{-}6$$

Further simplified solution of above equation based on the boundary condition (Dirichlet type (constant h or θ) and Neumann type (constant flux)) as assumed for simulation purpose is stated in Figure 1.

The volume averaging method relates the volume average of a spatial derivation to the spatial derivative of the volume average, and makes the transformation from microscopic equations to macroscopic equations. **[17]**, **[18]** simplified the volume averaging method for making it widely useful for engineering application.

The difference between the higher one and the lower one of the two pressure reading is known as Capillary Pressure (Bear, 1972 **[19]**). Often the symbol ψ is used for the capillary pressure head, that is

$$\psi \equiv h_c = \frac{P_c}{\rho_w g} \quad 0\text{-}7$$

The relationship for pressure head and water content is available in literature, Van Genuchten **[4]**.The commonly used relationship for water content and pressure head is

$$\text{EffectiveSaturation}\Theta = \frac{\theta - \theta_{res}}{\theta_{sat} - \theta_{res}} = \left[\frac{1}{1 + (\alpha\Psi)^n}\right]^m \quad 0\text{-}8$$

Where

θ = Volumetric water content, ($m^3\,m^{-3}$) at pressure head (m), $\theta_{sat} = \varepsilon =$ Porosity or saturated water content, θ_{res} = Residual water content, α = Constant - related to the inverse of the air-entry pressure (generally > 0, in m^{-1}) = y_b-1, y_b = bubbling pressure (m), n= Constant = λ+1, is a measure of the pore size distribution (generally >1), m = Constant = 1-(1/n), λ = pore size distribution parameter affecting the slope of the retention function, Ψ = Matrix Potential or pressure head= h-z (m), Θ = Effective saturation , z = Elevation head (m) and h = Total hydraulic head (m).

The relationship between the relative hydraulic conductivity (K_r) and pressure head (h) as a function of the dimensionless water content (Θ) as derived by Mualem **[15]** is given as

$$K_r = \Theta^{1/2}\left[\int_0^{\Theta} \frac{1}{h(x)}dx \Big/ \int_0^1 \frac{1}{h(x)}dx\right]^2 \quad 0\text{-}9$$

In simple word

$$\theta(h) = \begin{cases} \theta_r + \dfrac{\theta_s - \theta_r}{[1 + |\alpha h|^n]^m} h \le 0 \\ \theta_s h > 0 \end{cases} \quad 0\text{-}10$$

Among several empirically function proposed to describe soil water retention curve, Brook and Corey **[20]** widely used.

$$\theta = \begin{cases} \theta_r + (\theta_s - \theta_r)(\alpha h)^{-\lambda} & (ah > 1) \\ \theta_s & (ah \leq 1) \end{cases} \quad 0\text{-}11$$

Residual water content, θ_r is the maximum amount of water in a soil that will not contribute to liquid flow due to strong adsorption or blockage in flow paths **[21]**. θ_r may be defined as the water content at which both dθ/ dh and K got zero when h becomes large. Different functions can then be fitted directly to the measured curves using a nonlinear least squares regression procedure.

4. NUMERICAL SOLUTION

The finite difference discretization is used to solve governing equation for numerical solution. The fluid velocity in the medium is not constant but varies with space and time. The numerical solution of pressure in the unsaturated bed is obtained from the finite difference discretization of Equation 0-5 . The equation is solved using implicit differencing for time derivative and backward differencing for spatial derivatives as

$$\frac{h_i^{n+1} - h_i^n}{\Delta t} = \frac{K}{\Delta x^2}[k_r^e * (h_{i+1}^{n+1} - h_i^{n+1}) - k_r^w * (h_i^{n+1} - h_{i-1}^{n+1})] * \left(\frac{dc}{dh}\right)^{-1} \quad 0\text{-}1$$

And kr are obtained by harmonic mean of values at downstream and upstream grid points of point i. These are respectively called as east and west values. In other words

$$\frac{1}{k_r^e} = \frac{1}{k_{r\ i+1}} + \frac{1}{k_{r\ i}} \quad 0\text{-}2$$

$$\frac{1}{k_r^w} = \frac{1}{k_{r\ i-1}} + \frac{1}{k_{r\ i}} \quad 0\text{-}3$$

5. SOLUTION OF UNSATURATED FLOW CONSIDERING CONSTANT DENSITY:

Van Genuchten [4] soil water retention and hydraulic conductivity functions are used to estimate partially saturated hydraulic conductivity and the effective saturation for the initial head and capillary pressure distribution.

MATLAB program based on Rawls and Brakensiek regression equation (1989) used to calculate bubbling pressure, pore size distribution index, residual moisture content and soil conductivity. Equation used to evaluate the above values has limit range for clay percentage of 5% to 60% and sand percentage of 5% to 70%. Density of soil mineral are assumed to be 2650 Kg/m^3. Calculated bubbling pressure y_b (m), pore size distribution index λ, residual moisture content θ_r and soil conductivity K (cm/s) by assuming clay percentage to be 20% and sand percentage to be 15%, are shown in table 1 for varing porosity value.

Table 1 Calculated value of governing parameter based on porosity

Porosity, Ø	Bubbling pressure y_b(m)	Pore size distribution index λ	Residual moisture content θ_r	Soil conductivity K (cm/s)
0.39357	0.78522	0.35719	0.03084	0.0753

Soil Water Retention Curve graphically shows the relation between soil water content and soil moisture suction. For solving Van Genuchten [4] equations for soil water retention and hydraulic conductivity function to estimate partially saturated hydraulic conductivity and the effective saturation for the initial head and capillary pressure distribution input parameter like alpha, n, m etc. parameter are required. There are vast literature is available for solution of iteration differential equation.

Calculated value of alpha, n and m based on pedotransfer function through MATLAB program are shown in table 2 for different porosity.

Table 2 Calculated value of alpha, n and m based on porosity

Porosity Ø	N	m	Alpha α
0.39357	1.3572	0.26318	0.21478

The numerical solution of the finite difference discretize governing equation 0-1 is considered. The equation is solved using implicit differencing for time derivative and backward differencing for spatial derivatives. Two dimension matrix is used to storing moisture content. By inputting the boundary condition remaining unknown location are solved using the finite differencing scheme. The profiles are plotted for selected time intervals and plotted in figure 2 to figure 6.

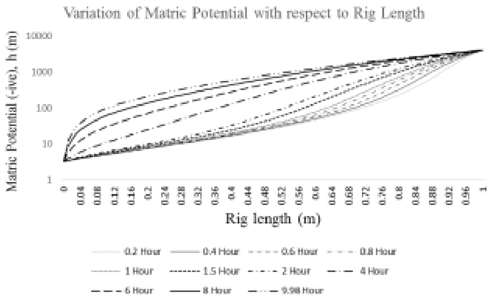

Figure 2 Variation of Matric Potential with respect to Rig Length

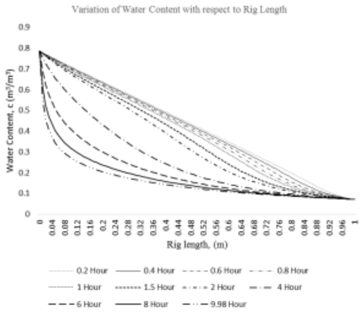

Figure 3 Variation of Water Content with respect to Rig Length

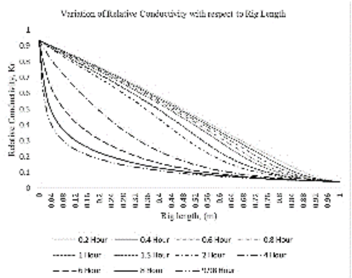

Figure 4 Variation of Relative Conductivity with respect to Rig Length

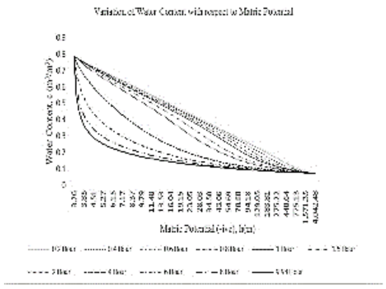

Figure 5 Variation of Water Content with respect to Matric Potential

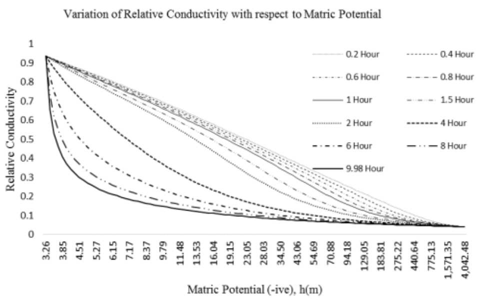

Figure 5 Variation of Relative conductivity with respect to Matric Potential

CONCLUSION

A comprehensive approach to predict the flow through horizontal porous media (unsaturated flow) are presented in the paper. Various relationship for predicting the special case of soil water retention curve are further presented. Simulation result behaviour from numerical simulation for unsaturated flow through porous media is identical to soil-water retention curve characteristic. Further work can be extended to comparing available soil-water analytical characteristic curve data with the various simulated results by varying the range of simulation.

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