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Vibration Behaviour of Laminated Composite Flat Panel Under Hygrothermal Environment

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Abstract

In this article, vibration responses of laminated composite shear deformable plate under combined temperature and moisture environment is investigated. A mathematical model based on the higher order shear deformation theory (HSDT) is developed and discretised using an isoparametric finite element method. The sets of governing equations are obtained using Hamilton's principle and the responses are obtained through a computer code in MATLAB environment. The convergence and the validation study of the model have been performed and discussed. The efficacies of the present developed model have been shown by solving different numerical examples and the vibration responses under combined environmental effects are discussed in detail.

Keywords: Laminated plate; hygrothermal load; HSDT; FEM.

1. Introduction

Laminated structures made of composites are exposed to extreme temperature and/or moisture during their service and manufacturing. The mechanical properties of composites are significantly influenced as well as the original geometry gets distorted substancially. It is well known that most structural component made up of composites (aerospace, marine, automotive, agriculture, sports, and biomedical etc.) are exposed to vibration during working consequent upon these structures experiences fatigue loading. Especially these structures when subjected to high strain rate loading conditions, distorted mechanical properties like Young's modulus, rigidity modulus and volume fraction of different material content affect the final behaviour and distortion in geometry induces nonlinearity in the system behaviour. This attracted the attention of many researchers to analyze the structural responses of laminated structure by considering all different points discussed in earlier lines. In order to predict the responses correctly the studies have been completed using theoretical and/or experimental approaches. Some of the important contributions in the said area are discussed in the following line briefly to address the objective and necessity of the present study. Nonlinear free vibration analysis of laminated composite plates under dissimilar temperature load is investigated by Liu and Huang (1996) by considering the nonlinear kinematics in von-Karman sense in the framework of the first order shear deformation theory (FSDT). Huang and Zheng (2003) studied the nonlinear free and forced vibration of simply supported moderately thick laminated plates subjected to transient loading. They employed Ready's HSDT and general von-Karman type equations for formulation. Nonlinear FEM steps have been employed by Naidu and Sinha (2007) to investigate the nonlinear vibration response of laminated composite shells in hygrothermal environment. They have developed the model based on the FSDT and the Green-Lagrange type nonlinearity. Benkhedda *et al.* (2008) proposed an analytical approach to evaluate the hygrothermal stresses in laminated composite thick plates considering temperature and moisture dependent material properties. The hygrothermal effects on multilayered composite plates by taking the temperature dependent material properties through a global–local higher order theory is reported by Lo *et al.* (2010). Nanda and Pradyumna (2011) presented nonlinear transient response of cylindrical/spherical panels with imperfection under hygrothermal loading based on the FSDT mid-plane kinematics and von-Karman nonlinearity. The nonlinear vibration behaviour and the severity due to combined variation in temperature and moisture concentrations is reported by Ashraf (2012) in conjunction with moisture-dependent material properties and the sinusoidal shear deformation plate theory. Combined numerical and experimental investigation on the effects of temperature, moisture on the natural frequencies of woven fiber Glass/Epoxy delaminated composite plates is carried out by Panda *et al.* (2013). Their model is based on the FSDT and changes in thermo-mechanical properties due to hygrothermal variation are considered. Panda and Mahapatra (2013) studied the nonlinear thermal free vibration behavior of laminated composite spherical shell panel using the HSDT and taking the nonlinearity in Green-Lagrange sense.

In the present paper, an attempt is made to develop a general nonlinear FEM based model by taking the HSDT mid-plane kinematics and Green-Lagrange type nonlinearity to achieve a general case under combined hygro-thermal loading. In addition to this all the nonlinear higher order terms are taken in the mathematical model to obtain the exact structural flexure. The composite material properties are the functions of the temperature and the moisture. The system of equation is obtained using Hamilton's principle and solved using direct iterative method.

2. Theoretical Development and Finite Element Formulation

The displacement field for laminated flat panel (Fig. 1) considered for deriving the mathematical model is based on the Ready's (2004) HSDT.

$$\begin{aligned}\bar{u}(x,y,z,t) &= u(x,y,t) + z\phi_1(x,y,t) + z^2\psi_1(x,y,t) + z^3\theta_1(x,y,t) \\ \bar{v}(x,y,z,t) &= v(x,y,t) + z\phi_2(x,y,t) + z^2\psi_2(x,y,t) + z^3\theta_2(x,y,t) \qquad \left[\frac{h}{2} \le z \le \frac{h}{2}\right] \\ \bar{w}(x,y,t) &= w(x,y,t)\end{aligned} \tag{1}$$

where, $(\bar{u},\bar{v},\bar{w})$ are the displacements at any point of the panel along the (X, Y, Z) coordinates, (u, v, w) are the displacements associated with a point on the mid plane of the panel and ϕ_1 and ϕ_2 are the rotations about the y and x-axes, respectively. ψ_1, ψ_2, θ_1, θ_2 are the higher order terms of the Taylor series expansion defined in the mid-plane.

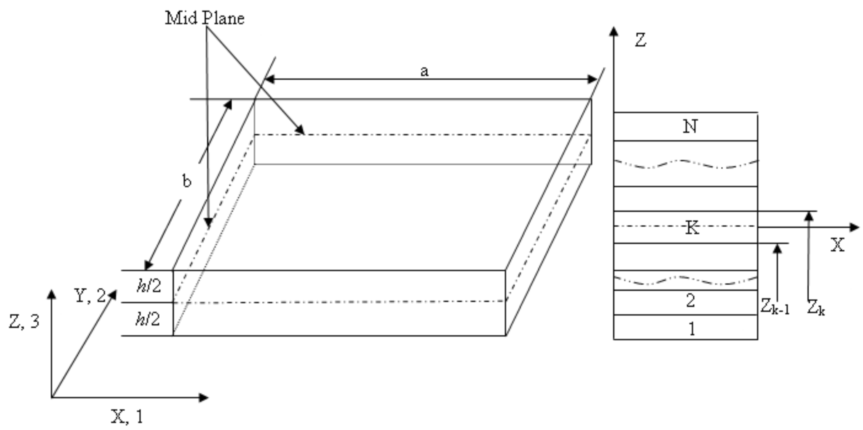

Fig. 1: Geometry and stacking sequence of laminated plate.

The nonlinear strain displacement relations are considered for the present analysis as in Panda and Mahapatra (2013)

$$\{\varepsilon\} = \{\varepsilon_L\} + \{\varepsilon_{NL}\} \tag{2}$$

Substituting Eq. (1) into Eq. (2), the linear and nonlinear strain terms are rearranged as:

$$\{\varepsilon\} = [\mathrm{H}_L]\{\bar{\varepsilon}_L\} + \frac{1}{2}[\mathrm{H}_{NL}]\{\bar{\varepsilon}_{NL}\} \tag{3}$$

where, $\{\bar{\varepsilon}_L\}$, $\{\bar{\varepsilon}_{NL}\}$ are the functions of x and y and $[\mathrm{H}]$ is the function of thickness coordinate. The detail terms of $\{\bar{\varepsilon}_L\}$ **Error! Bookmark not defined.**, $[\mathrm{H}_L]$ and $[\mathrm{H}_{NL}]$ can be seen in Panda and Mahapatra (2013).

The stress-strain relationship for a general k^{th} orthotropic composite lamina of any arbitrary fiber orientation angle with reference to co-ordinate axes (1, 2, and 3) under combined hygrothermal loading is given by

$$\left\{\sigma_{ij}\right\}^{k} = \left[\overline{Q_{ij}}\right]^{k} \left\{\varepsilon_{ij} - \alpha_{ij}\Delta T - \beta_{ij}\Delta C\right\}^{k} \tag{4}$$

where, $\left\{\sigma_{ij}\right\}^{k} = \left\{\sigma_1 \quad \sigma_2 \quad \sigma_6 \quad \sigma_5 \quad \sigma_4\right\}^{T}$ and $\left\{\varepsilon_{ij}\right\}^{k} = \left\{\varepsilon_1 \quad \varepsilon_2 \quad \varepsilon_6 \quad \varepsilon_5 \quad \varepsilon_4\right\}^{T}$ are the stress and strain vectors respectively for the k^{th} layer. In addition to that, $\left[\overline{Q_{ij}}\right]^{k}$ is the transferred reduced elastic constant for the k^{th} layer. $\left\{\alpha_{ij}\right\}^{k} = \left\{\alpha_1 \quad \alpha_2 \quad 2\alpha_{12}\right\}^{T}$ and $\left\{\beta_{ij}\right\}^{k} = \left\{\beta_1 \quad \beta_2 \quad 2\beta_{12}\right\}^{T}$ are the thermal and hygral expansion/contraction coefficient vectors. ΔT and ΔC are the uniform change in temperature and moisture concentration respectively.

The in-plane hygro thermal forces are obtained using the steps in Cook et. al. (2000) and the forces

$$\begin{Bmatrix} \{N\} \\ \{M\} \\ \{P\} \end{Bmatrix} = \left[\sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \left\{\left[\bar{Q}\right]^{k} \left\{\{\alpha\}^{k}\left(1, z, z^3\right)\Delta T + \{\beta\}^{k}\left(1, z, z^3\right)\Delta C\right\}\right\} dz\right] \tag{5}$$

where, $\{N\}$, $\{M\}$ and $\{P\}$, are the resultant vectors of compressive in-plane hygrothermal forces, moments and the higher order terms due to combined temperature and moisture variation.

The work done (W) due to in-plane hygral and thermal load is given by

$$W = \frac{1}{2}\int_{A} \left\{\varepsilon_G\right\}^{T} \left[\mathrm{D}_G\right] \left\{\varepsilon_G\right\} dA \tag{6}$$

The kinetic energy of a vibrating laminated plate is expressed in following step

$$\mathrm{T} = \frac{1}{2}\int_{A} \left\{ \sum_{k=1}^{NL} \int_{z_{k-1}}^{z_k} \rho^{(k)} \left\{\dot{\delta}\right\}^{T} [f]^{T} [f] \left\{\dot{\delta}\right\} dz \right\} dA = \frac{1}{2}\int_{\mathrm{A}} \left\{ \left\{\dot{\delta}\right\}^{\mathrm{T}} [\mathrm{m}] \left\{\dot{\delta}\right\} \right\} \mathrm{dA} \tag{7}$$

where, $[\rho]$, $\{\delta\}$ and $\{\dot{\delta}\}$ are the density, global displacement vector and first order derivative of the global displacement vector with respect to time.

$[m]$ is the inertia matrix written as $[\mathrm{m}] = \sum_{k=1}^{NL} \int_{z_{k-1}}^{z_k} \rho^{(k)} [f]^{\mathrm{T}} [f] dz$ and $\{\delta\} = \{\bar{u} \quad \bar{v} \quad \bar{w}\}^{\mathrm{T}} = \{f\}\{\delta\}$ where, $\{f\}$ is the function of thickness coordinate.

The total strains energy of the plate expressed as the sum of linear and nonlinear strain energies as

$$U = \frac{1}{2}\int_{A}\left(\{\varepsilon_{\mathrm{L}}\}_i^{\mathrm{T}}[\mathrm{D}_1]\{\varepsilon_{\mathrm{L}}\}_i\right) dA + \frac{1}{4}\int_{A}\left(\{\varepsilon_{\mathrm{L}}\}_i^{\mathrm{T}}[\mathrm{D}_2]\{\varepsilon_{\mathrm{NL}}\}_i + \{\varepsilon_{\mathrm{NL}}\}_i^{\mathrm{T}}[\mathrm{D}_3]\{\varepsilon_{\mathrm{L}}\}_i + \frac{1}{2}\{\varepsilon_{\mathrm{NL}}\}_i^{\mathrm{T}}[\mathrm{D}_4]\{\varepsilon_{\mathrm{NL}}\}_i\right) dA$$

$$= U_L + U_{NL} \tag{8}$$

A nine nodded Lagrangian isoparametric quadrilateral element with nine degrees of freedom per node is considered for present discretisation process. By employing FEM the displacement vector can be expressed as $\{\delta^*\} = [N_i]\{\delta_i^*\}$ (9)

where, $[N_i]$ and $\{\delta_i\}$ is the nodal interpolation function and displacement vector for i^{th} node, respectively.

The governing equation for the vibrated laminated composite plate under hygrothermal loading is obtained using Hamilton's principle as follows.

$$\delta \int_{t_1}^{t_2} L\mathrm{dt} = 0 \tag{10}$$

where, $L = [T\text{-}(U\text{+}W)]$

Now employing the Eq. (9) in Eq. (6) and Eq. (8) the elemental form is obtained as in Panda and Mahapatra (2013) and subsequently Eq. (10) can be expressed as in Sundaramoorthy *et. al.* (1973)

$$[\mathrm{M}]\{\ddot{\delta}\} + [\mathrm{K_L}] + \frac{1}{2}[\mathrm{K_{NL}}]_1 + \frac{1}{3}[\mathrm{K_{NL}}]_2 = \{\mathrm{F}_{\Delta T} + \mathrm{F}_{\Delta C}\} \tag{13}$$

where, [M] is the global mass matrix, $[K_L]$ is the global linear stiffness matrix, $[K_{NL}]_1$ and $[K_{NL}]_2$ are the nonlinear mixed stiffness matrices which depend on the displacement vector linearly and quadratically respectively.

In order to obtain an eigen value solution of the nonlinear responses the Eq. (13) is modified as follows:

$$\left\{\left([\mathrm{K}]_\mathrm{L} - \lambda_{cr}[\mathrm{K_G}] + \frac{1}{2}[\mathrm{K_{NL}}]_1 + \frac{1}{3}[\mathrm{K_{NL}}]_2\right) - \omega^2[\mathrm{M}]\right\}\{\delta\} = 0 \qquad 14)$$

where, $[K_G]$ is the global geometry stiffness matrix. The inclusion of geometry matrix in the governing equation is possible by dropping and the hygrothermal force terms from Eq. (13). The above equation is solved using a direct iterative method and the detail steps are same as in Panda and Mahapatra (2013).

3. Results and Discussion

The model is validated for temperature independent properties for specific temperature load with those available published literature. Fig 2 shows the convergence and validation behaviour of the present model. Based on the convergence, a (5×5) mesh is found adequate for analysis. The nondimensional fundamental frequencies ($\varpi = \omega^2 h(\rho/E_2)^{0.5}/h$) are obtained using the present model is presented in Fig. 3. It is clearly observed that the present model gives lower nonlinear frequency values than the references for room temperature whereas the frequency increases as the temperature load increases. This is because the fact that the small strain and large deformation case the responses may not follow a monotonous trend as in the references. In addition to that the reference models are developed based on the FSDT and von-Karman type nonlinearity which is unrealistic nature when the nonlinear is severe.

The following material and support conditions are used to illustrate new parametric studies.

Graphite/epoxy: E_{11} = 130 GPa, E_{22} =7 GPa, G_{12} = G_{13}= 6 GPa, G_{23}/G_{12} = 0.5, ν_{12} = 0.3, α_1= -0.3×10^{-6}/^{0} C α_2 = 28.1×10^{-6}/^{0} C, β_1= 0 and β_2 = 0.44.

(a) Simply support (SSSS):
v = w = Φ2 = Ψ2 = θ2 = 0 at x = 0, a and u= w = Φ1 = Ψ1 = θ1= 0 at y = 0, b

(b) Clamped condition (CCCC):
u = v = w = Φ1 = Φ2 = Ψ1= Ψ2 = θ1 = θ2 = 0 for both x = 0, a and y = 0, b.

Illustration 1:- Simply supported antisymmetric cross ply $(0^0/90^0)_2$ laminated square plates (*a/b*=1) of different thickness ratios are subjected to varying hygrothermal load at amplitude ratio 0.2 and presented in Fig. 4. It is evident that, the nonlinear frequency parameter increases with increase in temperature and moisture concentration. However, higher rate of increase is observed for thinner plates than thicker plates. The effect of variation of moisture concentration on nonlinear frequency parameter is slight superior at lower temperatures.

Illustration 2: Square plates (*a/b*=1) of thickness ratio (*a/h*=60) with clamped support conditions are considered under hygrothermal load at amplitude ratio 0.3. The values of nonlinear frequency parameters for different lamination schemes are presented in Fig. 5. It is noted that, nonlinear vibration responses of symmetric laminated plates are least affected under hygrothermal conditions. It is also seen that for equal numbers of layers, antisymmetric cross ply laminates exhibit higher nonlinear frequencies than the hybrid laminates. However, at very high temperature and moisture concentration both behaves equally. Antisymmetric angle ply laminates, as compared to other lamination schemes are least liable to variation so far as their response under hygrothermal loading is concerned.

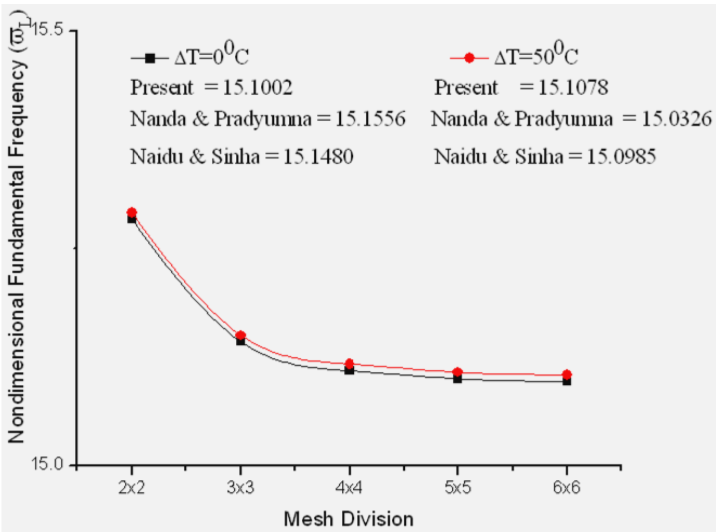

Fig. 2: Convergence and comparison of linear frequency parameter at ΔT=00C and ΔT=500C.

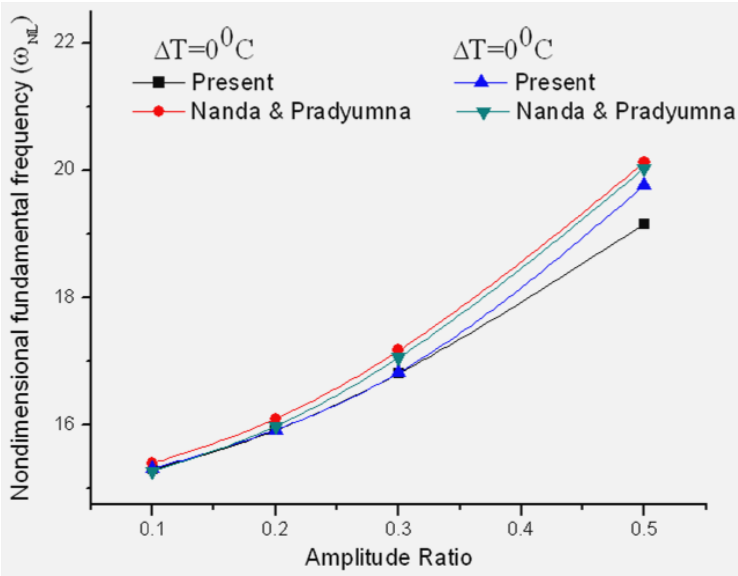

Fig. 3: Comparison of nondimensional nonlinear fundamental frequency responses at ΔT=00C and ΔT=500C.

4. Conclusion

The nonlinear free vibration behavior of laminated composite plates under hygrothermal loading is analyzed in the framework of the HSDT and the geometrical nonlinearity is taken in Green-Lagrange sense. All the nonlinear higher order terms are included in the mathematical formulation to obtain more realistic responses.

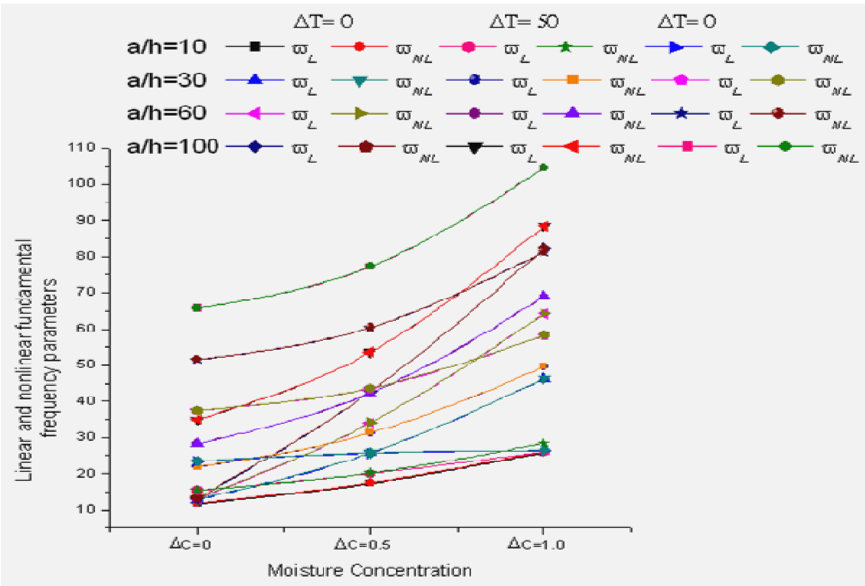

Fig. 4: Variation of nonlinear frequency parameter with thickness ratio under hygrothermal loading.

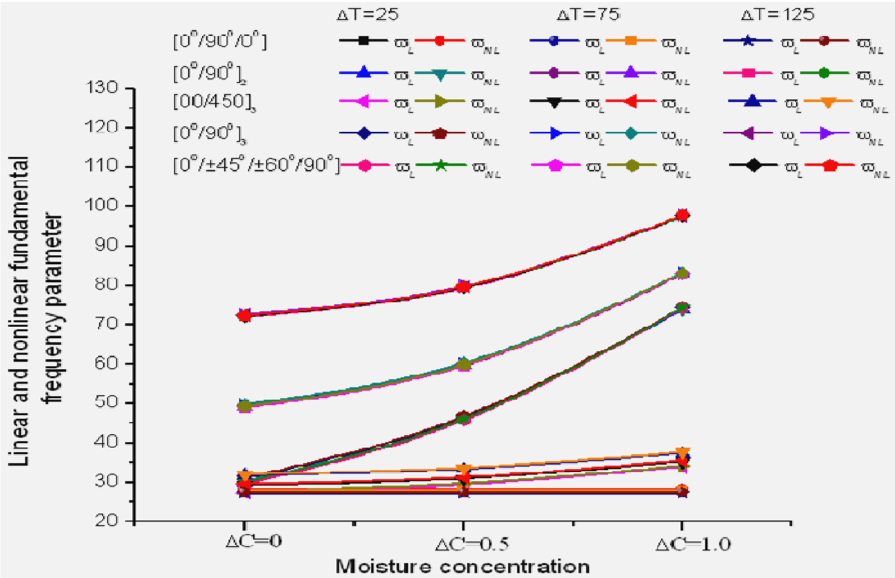

Fig. 5: Effect of lamination schemes of the plates on nonlinear frequency response under hygrothermal loading.

Proposed and developed model is discretised using nonlinear FEM and solved using a direct iterative method. Linear/nonlinear results are compared with those available results in open literatures. The convergence and comparison behaviour has been shown. The results demonstrated the efficacy of the developed model for evaluation of nonlinear responses. Based on the numerical experimentation it is observed that the nonlinear frequency parameter of laminated plates increases with increase in temperature and moisture concentration, irrespective of thickness ratios and lamination schemes. It is understood that the effect of hygrothermal conditions has to be considered for laminated structure, particularly when the structures are exposed to environmental severity.

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