

Satisfactory Roommates Problem with Incomplete List

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Abstract

In this paper, satisfactory roommate's problem with incomplete list is described and an algorithm SMAR is applied to find the satisfactory matching based on preference value, using assignment method.

Keywords: Satisfactory roommate's problem with incomplete list, Preference Value, Satisfactory value matrix, Satisfactory Level.

Introduction

The stable roommate's problem is the extension of stable marriage problem [1]. It is a well known problem of matching $2n$ people into n disjoint pairs to achieve a certain type of stability. The input to the problem is a set of $2n$ preference lists, one for each person i , where person i 's list is a preference of the $2n-1$ people other than i . Roommate assignment is a pairing of $2n$ people into n disjoint pairs and is said to be unstable if there is a pair in the assignment for which there exist a better matching in the preference list than the present mate, such a pair is said to be a block assignment. An Assignment which is not unstable is called stable. An instance of the stable roommate's problem is called solvable if there is at least one stable assignment [3]. It is known that there are unsolvable instances of the stable roommate's problem [1]. An efficient algorithm to determine stable matching if an instance is solvable was proposed by Knuth and only recently solved by R.Irving [2].

The classical satisfactory roommate's problem (SFRP) is closely related to the stable roommate's problem. In the satisfactory roommate's problem each person in

the set of even cardinality n ranks the $n-1$ others in order of preference. The objective is to find satisfactory matching of roommate's problem. This is the partition of the set into $n/2$ pairs of roommates based on the individual satisfactory level. It is known that some of the instances of the satisfactory roommate's problem are unsolvable.

As an extension of the above, the same SMAR algorithm is applied for finding satisfactory matching for roommate's problem with incomplete list.

Satisfactory Roommate's Problem with Incomplete list

A classical Satisfactory Roommate's problem (SFRP) in which complete preference lists will be given for each member of the group. There are Satisfactory Roommates problem described with incomplete preference list and denoted as SFRPI. That is for some member in the group of n members the preference list consists of less than $n-1$ members. The satisfactory matching for this type of SFRPI problem can be obtained by using SMAR algorithm described in [4]. In SFRPI the participant p is acceptable to participant q if p appears on the preference list of q , and unacceptable otherwise.

For the related terminologies such as satisfactory value matrix, satisfactory level, satisfactory matching, assignment model and SMAR algorithm refer [4].

Example 2.1: Consider the problem instance of size 4 based on order of preference.

1. 3 4 2
2. 4 1
3. 1 4
4. 2 3 1

The Satisfactory value Matrix is

$$\text{SVM} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \left(\begin{array}{cccc} - & \frac{3}{3} & 2 & \frac{3}{3} \\ \frac{3}{3} & - & - & 2 \\ 2 & - & - & \frac{4}{3} \\ \frac{3}{3} & 2 & \frac{4}{3} & - \end{array} \right) \end{matrix}$$

The resultant matching for the above instance is (1, 3) and (2, 4). The above result shows that the satisfactory level of matching (1, 3) is 100% and for (2, 4) is 100%.

Example 2.2: Consider the problem instance of size 4 based on order of preference

1. 2 3 4
2. 3 1
3. 4 2 1
4. 3 1

The Satisfactory value Matrix is

$$\text{SVM} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{pmatrix} - & \frac{5}{3} & \frac{3}{3} & \frac{3}{3} \\ \frac{5}{3} & - & \frac{5}{3} & - \\ \frac{3}{3} & \frac{5}{3} & - & 2 \\ \frac{3}{3} & - & 2 & - \end{pmatrix} \end{matrix}$$

The resultant matching for the above instance is (1, 2) and (3, 4). The above result shows that the satisfactory level of matching (1, 2) is 83.3% and for (3, 4) is 100%.

Example 2.3: Consider the problem instance of size 6 based on order of preference.

1. 2 6 4
2. 3 5 1 6 4
3. 2 5 4
4. 5 2 3 6 1
5. 6 3 4 2
6. 4 2 5 1

The above instance will not give any solution.

Conclusion

The Satisfactory Roommates problem with incomplete list is essentially a version of Satisfactory Roommates problem, involving just one set. An algorithm SMAR is applied on the instances of SFRPI and discussed satisfactory level of matching for each pair. This algorithm results a matching in which each pair attains maximum satisfactory level. So SMAR results satisfactory matching of roommates with incomplete lists.

References

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