

## **STAP Capability of Sea Based MIMO Radar Using Virtual Array**

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### **Abstract**

MIMO(Multiple Input and Multiple Output) radar is different from conventional radar mainly due to using the non-coherent(orthogonal) waveforms in the transmitter system followed by a matched filter bank in the receiver system. A better spatial density and spatial resolution from the clutter can be obtained mainly for the use of non- coherent signal. In this paper it is explored how the “Space Time Adaptive Processing” for MIMO radar which is useful for increasing spatial resolution can be implemented in sea based MIMO radar. It involves adaptive array processing algorithms to aid in target detection. Radar signal processing benefits from STAP in areas where interference is a problem (i.e. ground clutter, jamming). In sea environment the clutter characteristics is rapidly changing for the dynamic nature of sea and dependence of the physical condition of the sea respect to weather. The degree of freedom can be increased by using the virtual array concept in STAP. Here the of virtual array is implemented in seaborne radar with respect to the signal model design of MIMO STAP. The improved value of SINR is also given as a function of Doppler shifted signal which indicates the performance of the radar in sea environment.

### **Introduction**

The MIMO (multiple-input multiple-output) radar system allows transmitting orthogonal (or incoherent) waveforms in each of the transmitting antennas. These waveforms can be extracted by a set of matched filters in the receiver. Each of the extracted components contains the information of an individual transmitting path. There are two different kinds of approaches for using this information. First, the

spatial diversity can be increased. Second, a better spatial resolution can be obtained.[1] The Doppler shifted signal on which the SINR is dependent can be derived in the form of an analog response.

The calculation for the STAP will be mostly same for both seaborne and airborne case. The main difference in these two environment will be the clutter characteristics. The clutter sensed by a radar which surveys a section of ocean probably encounters the most complex of all possible scenarios. Sea clutter is extremely complex to suppress because of its dynamic nature, and large dependence on weather conditions. The sea waves cause complex correlation patterns that are not observed in the presence of land clutter. The wind can pick up water droplets causing sea spray, or causes ripples on the surface of the waves which resonate electromagnetically. The white-caps of broken waves and scattering from the crest of a wave also influences the electro-magnetic properties of the sea surface. These effects are detected by a radar as large spikes in the magnitude of the returned radar signal. The velocity component of these effects also cause a Doppler shift relative to the main body of the clutter. Since the battle is moving away from blue water scenarios toward to the sea shore, these effects are even more profound. These effects trigger false detections, which causes the radar designer to increase the detection level to minimize the probability of false alarms. This approach inevitably increases the probability that small targets such as boats used in asymmetric warfare go undetected. This trade-off has to be optimized by the radar designer. Testing for all these conditions in real time in a lab environment is a game changer that aids the radar designer in building the best system in the shortest amount of time. The K-Distribution has become the approximation of choice for sea clutter modeling. The ocean swell (long wavelength) is responsible for a modulation effect known as the texture component of sea clutter. The shorter wavelengths are responsible for the speckle component of sea clutter.[2] All of these have to be considered at the time of calculation of MIMO STAP for sea based radars.

### **Introduction to Sea clutter**

The sea can have different states. Mostly depending on the wind, the sea can be perfectly calm or very rough. Table 1 shows the different sea-states related to the wind speed [3].

Sea clutter can be calculated using various model which are useful to design the radar respect to the proper sea clutter calculation. Some of the sea clutter that has to be considered are-

1. Pierson-Moskowitz model
2. Flat see range attenuation
3. Shadowing 3a. Earth curvature shadowing  
3b. Sea-waves shadowing
4. Earth curvature consideration
5. Electromagnetic sea response 5a.K- distribution  
5b. Tsallis distribution

**K-distribution**

This distribution has been created in order to model electromagnetic radar responses. It was considered as the most fit distribution for that kind of model because it has been created for that purpose. [6] uses the K-distribution to model the sea-clutter response for example. K-distribution probability density function is given by

$$p_K(x) = \frac{2}{x} \left( \frac{L\nu x}{\mu} \right)^{\frac{L+\nu}{2}} \frac{1}{\Gamma(L)\Gamma(\nu)} K_{\nu-L} \left( 2\sqrt{\frac{L\nu x}{\mu}} \right), \quad x > 0 \tag{4}$$

Where:

- $K_x$  Modified Bessel function of the second kind of  $x^{th}$  order
- $\mu$  Mean value for  $x$ .
- $\Gamma(z)$  Gamma function with  $z$  parameter defined by
- $\nu$  Shape parameter to be estimated.
- $L$  Shape parameter to be estimated.

We have to estimate  $\nu$  and  $L$  parameters. The distribution is composed by two independent distributions. One of those distributions represents the radar cross section and the other one the speckle component. The speckle component is the addition of randomly phased complex contributions [7].

Now considering all these clutter models we have to design the MIMO radar Space time adaptive processing for naval radars. Here we implement the virtual array concept in this field.

**Signal Model of MIMO-STAP**

Here we are discussing the MIMO-STAP with a signal model to make the concept more clear.

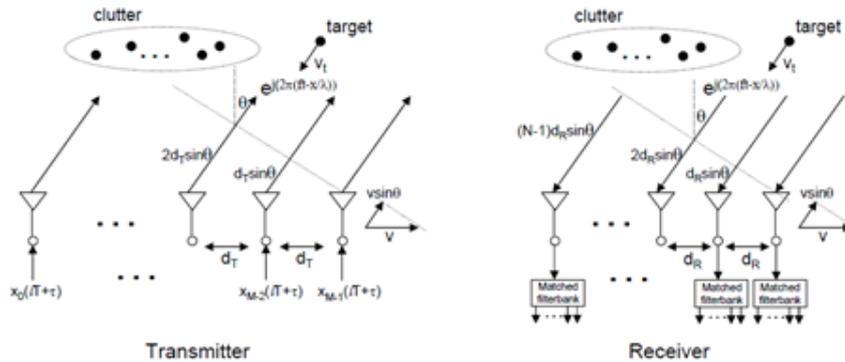


Figure 1: MIMO radar System with M transmitting and N receiving Antenna

At first mentioning the notations used here-

1.  $d_T$  is the spacing of the transmitting antennas
2.  $d_R$  is the spacing of the receiver antennas

3. M is the number of transmitting antennas,
4. N is the number of the receiving antennas,
5. T is the radar pulse period,
6. l indicates the index of radar pulse (slow time),
7.  $\tau$  represents the time within the pulse (fast time),
8.  $v_t$  is the target speed toward the radar station, and
9.  $v$  is the speed of the radar station

Notice that the model assumes the two antenna arrays are linear and parallel. The transmitter and the receiver are close enough so that they share the same angle variable  $\theta$ . The radar station movement is assumed to be parallel to the linear antenna array. This assumption has been made in most of the airborne ground moving target indicator (GMTI) systems. Each array is composed of

Omni directional elements. The transmitted signals of the mth antenna can be expressed as

$$x_m(lT + \tau) = \sqrt{E} \phi_m(\tau) e^{j2\pi f(lT + \tau)} \dots \dots \dots (5)$$

for  $m = 1; 2; \dots \dots \dots M - 1$ , where  $\phi_m(\tau)$  is the baseband pulse waveform, f is the carrier frequency, and E is the transmitted energy for the pulse. The demodulated received signal of the nth antenna can be expressed as,

$$y_n(lT + \tau + \frac{2r}{c}) \approx \sum_{m=0}^{M-1} \rho_t \phi_m(\tau) e^{j \frac{2\pi}{\lambda} (\sin \theta_t (2vTl + d_R n + d_T m) + 2v_t Tl)} + \sum_{i=0}^{N_c-1} \sum_{m=0}^{M-1} \rho_i \phi_m(\tau) e^{j \frac{2\pi}{\lambda} (\sin \theta_i (2vTl + d_R n + d_T m))} + y_n^{(J)}(lT + \tau + \frac{2r}{c}) + y_n^{(w)}(lT + \tau + \frac{2r}{c}), \dots \dots \dots (6)$$

Where,

1. r is the distance of the range bin of interest,
2. c is the speed of light,
3.  $\rho_t$  is the amplitude of the signal reflected by the target,
4.  $\rho_i$  is the amplitude of the signal reflected by the ith clutter
5.  $\theta_t$  is the looking direction of the target,
6.  $\theta_i$  is the looking direction of the ith clutter,
7.  $N_c$  is the number of clutter signals,
8.  $y_n^{(J)}$  is the jammer signal in the nth antenna output, and
9.  $y_n^{(w)}$  is the white noise in the nth antenna output.

For convenience, all of the parameters used in the signal model are summarized in table below

Table1 : List of the parameters used in the signal model

$d_T$	spacing of the transmitting antennas
$d_R$	spacing of the receiving antennas
$M$	number of the transmitting antennas
$N$	number of the receiving antennas
$T$	radar pulse period
$l$	index of radar pulse (slow time)
$\tau$	time within the pulse (fast time)
$v_t$	target speed toward the radar station
$x_m$	transmitted signal in the $m$ th antenna
$\phi_m$	baseband pulse waveforms
$y_n$	demodulated received signal in the $n$ th antenna
$v_t$	target speed toward the radar station
$v$	speed of the radar station
$r$	distance of the range bin of interest
$c$	speed of light
$\rho_t$	amplitude of the signal reflected by the target
$\rho_i$	amplitude of the signal reflected by the $i$ th clutter
$\theta_t$	looking direction of the target
$\theta_i$	looking direction of the $i$ th clutter
$N_c$	number of clutter signals
$y_n^{(J)}$	jammer signal in the $n$ th antenna output
$y_n^{(w)}$	white noise in the $n$ th antenna output

first term in the equation of the received signal represents the signal reflected by the target. The second term is the signal reflected by the clutter. The last two terms represent the jammer signal and white noise. We assume there is no internal clutter motion (ICM) or antenna array misalignment.[9] The phase differences in the reflected signals are caused by the Doppler shift, the differences of the receiving antenna locations, and the differences of the transmitting antenna locations. In the MIMO radar, the transmitting waveforms  $\phi_m(\tau)$  satisfy orthogonality:

$$\int \phi_m(\tau)\phi_k^*(\tau)d\tau = \delta_{mk} \dots\dots\dots (7)$$

The sufficient statistics can be extracted by a bank of matched filters as shown in Fig. The extracted signals can be expressed as

$$\begin{aligned}
 y_{n,m,l} &\triangleq \int y_n(lT + \tau + \frac{2r}{c})\phi_m^*(\tau)d\tau = \\
 &\rho_t e^{j\frac{2\pi}{\lambda}(\sin\theta_t(2vTl+d_Rn+d_Tm)+2v_tTl)} + \\
 &\sum_{i=0}^{N_c-1} \rho_i e^{j\frac{2\pi}{\lambda}(\sin\theta_i(2vTl+d_Rn+d_Tm))} + y_{n,m,l}^{(J)} + y_{n,m,l}^{(w)},
 \end{aligned}
 \tag{8}$$

for  $n = 0; 1; \dots; N- 1$ ,  $m = 0; 1; \dots; M- 1$ , and  $l = 0; 1; \dots; L - 1$ , where  $y_{n,m,l}^{(J)}$  is the corresponding jammer signal,  $y_{n,m,l}^{(w)}$  is the corresponding white noise, and  $L$  is the number of the pulses in a coherent processing interval (CPI). To simplify the above equation, we define the following normalized spatial and Doppler frequencies

$$\begin{aligned}
 f_s &\triangleq \frac{d_R}{\lambda} \sin\theta_t, f_{s,i} \triangleq \frac{d_R}{\lambda} \sin\theta_i \\
 f_D &\triangleq \frac{2(v \sin\theta_t + v_t)}{\lambda} T.
 \end{aligned}$$

One can observe that the normalized Doppler frequency of the target is a function of both target looking direction and speed. Throughout this chapter we shall make the assumption  $d_R = \lambda/2$  that spatial aliasing is avoided. Using the above definition we can rewrite the extracted signal as

$$\begin{aligned}
 y_{n,m,l} &= \rho_t e^{j2\pi f_s(n+\gamma m)} e^{j2\pi f_D l} + \\
 &\sum_{i=0}^{N_c-1} \rho_i e^{j2\pi f_{s,i}(n+\gamma m+\beta l)} + y_{n,m,l}^{(J)} + y_{n,m,l}^{(w)},
 \end{aligned}
 \tag{9}$$

for  $n = 0; 1; \dots; N - 1$ ,  $m = 0; 1; \dots; M - 1$ , and  $l = 0; 1; \dots; L - 1$ , where  $\gamma \triangleq d_T/d_R$ , and  $\beta \triangleq 2vT/d_R$ .

**Fully Adaptive MIMO-STAP**

The goal of space-time adaptive processing (STAP) is to find a linear combination of the extracted signals so that the SINR can be maximized. Thus the target signal can be extracted from the interferences, clutter, and noise to perform the detection. Stacking the MIMO STAP signals we obtain the NML vector

$$\mathbf{y} = \left( \begin{matrix} y_{0,0,0} & y_{1,0,0} & \cdots & y_{N-1,M-1,L-1} \end{matrix} \right)^T
 \tag{10}$$

Then the linear combination can be expressed as  $\mathbf{w}^\dagger \mathbf{y}$ , where  $\mathbf{w}$  is the weight vector for the linear combination. The SINR maximization can be obtained by minimizing the total variance under the constraint that the target response is unity. It can be expressed as the following optimization problem:

$$\begin{aligned} & \min_{\mathbf{w}} \mathbf{w}^\dagger \mathbf{R} \mathbf{w} \\ & \text{subject to } \mathbf{w}^\dagger \mathbf{s}(f_s, f_D) = 1 \end{aligned} \tag{11}$$

Where  $\mathbf{R} \triangleq E[\mathbf{y}\mathbf{y}^\dagger]$  and,  $\mathbf{s}(f_s, f_D)$  is the size-NML MIMO space-time steering vector which consists of the elements

$$e^{j2\pi f_s(n+\gamma m)} e^{j2\pi f_D l} \tag{12}$$

For  $n = 0; 1; \dots; N - 1$ ,  $m = 0; 1; \dots; M - 1$ , and  $l = 0; 1; \dots; L - 1$ . This  $\mathbf{w}$  is called minimum variance distortion less response (MVDR) beam former. The covariance matrix  $\mathbf{R}$  can be estimated by using the neighboring range bin cells. In practice, in order to prevent self-nulling, a target-free covariance matrix can be estimated by using guard cells. The well-known solution to the above problem is

$$\mathbf{w} = \frac{\mathbf{R}^{-1} \mathbf{s}(f_s, f_D)}{\mathbf{s}(f_s, f_D)^\dagger \mathbf{R}^{-1} \mathbf{s}(f_s, f_D)} \tag{13}$$

However, the covariance matrix  $\mathbf{R}$  is NML x NML. It is much larger than in the SIMO case because of the extra dimension. The complexity of the inversion of such a large matrix is high. The estimation of such a large covariance matrix also converges slowly. To overcome these problems, partially adaptive techniques can be applied. The methods described before are examples of such partially adaptive techniques. In SIMO radar literature such partially adaptive methods are commonly used.[9]

**Virtual Array**

Observing the MIMO space-time steering vector defined previously one can view the first term  $e^{j2\pi f_s(n+\gamma m)}$  as a sampled version of the sinusoidal function  $e^{j2\pi f_s x}$ . Recall that  $\gamma$  is defined as the ratio of the antenna spacing of the transmitter and receiver. To obtain a good spatial frequency resolution, these signals should be critically sampled and have long enough duration. One can choose  $\gamma = N$  because it maximizes the time duration while maintaining critical sampling[12]. Sorting the sample points  $n + \gamma m$  for  $n = 0; 1; \dots; N - 1$ , and  $m = 0; 1; \dots; M - 1$ , we obtain the sorted sample points  $k = 0; 1; \dots; NM - 1$ . Thus the target response can be rewritten as

$$e^{j2\pi f_s k} e^{j2\pi f_D l} \tag{14}$$

for  $k = 0; 1; \dots; NM - 1$ , and  $l = 0; 1; \dots; L - 1$ . It is as if we have a virtual receiving array with NM antennas. However, the resolution is actually obtained by only M antennas in the transmitter and N antennas in the receiver.

**Observations**

Figure 2 compares the SINR performance of the MIMO system and the SIMO system in the array looking direction of zero degree, that is,  $f_s = 0$ . The optimal space-time beam former is used.

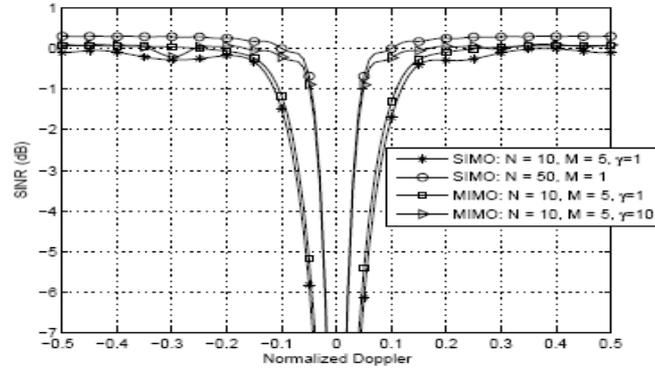


Figure: 2 The SINR at looking direction zero as a function of the Doppler frequencies for different SIMO and MIMO systems.

The parameter  $L$  equals 16, and  $\beta$  is 1.5 in this example. In all plots it is assumed that the energy transmitted by any single antenna element to illuminate all angles is fixed. The SINR drops near zero Doppler frequency because it is not easy to distinguish the slowly moving target from the still ground clutter. The MIMO system with  $\gamma = 1$  has a slightly better performance than the SIMO system with the same antenna structure. For the virtual array structure where  $\gamma = N$ , the MIMO system has a very good SINR performance and it is close to the performance of the SIMO system with  $NM$  antennas because they have the same resolution for the target signal and the clutter signals. The small difference comes from the fact that the SIMO system with  $NM$  antennas has a better spatial resolution for the jammer signals. This example shows that the choice of  $\gamma$  is very crucial in the MIMO radar. With the choice  $\gamma = 10 = N$ , radar with 51 array elements. This example also shows that the MIMO radar system has a much better spatial resolution for clutter compared to the traditional SIMO system with same number of physical antenna elements. The figure describes the input signal from the  $m$ th MIMO transmitting antenna. The amplitude of the signal is taken as a function of time.

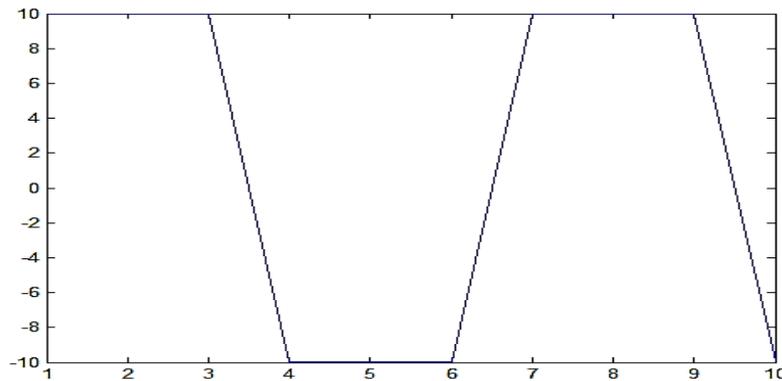


Figure: 3 Input signal from any of the transmitting antenna of MIMO Transmitter array

For different wind powers, the computed zRMS and threshold values for the double path model application are shown in figure

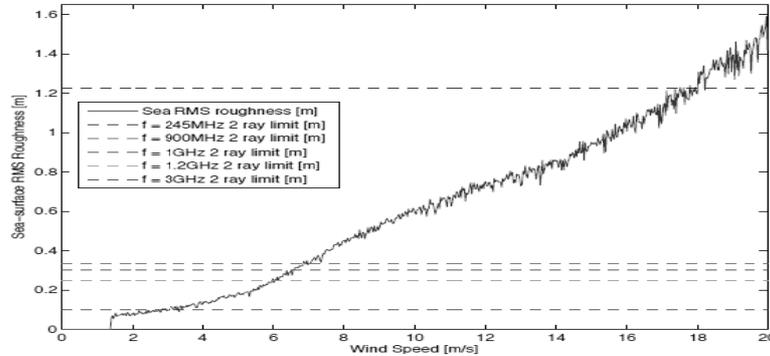


Fig:4 RMS roughness depending on the wind power for different candidate frequencies.

The double-path model application thresholds are summed up in the following table for several candidates frequencies. In that table, we give the maximum wind speed where the double path model is valid depending on the frequency.

Here in the above calculation the SINR is designed as a function of normalized Doppler. So that the Doppler shifted signal is important for the signal design. The figure 5 and 6 gives the waveform of the Doppler shifted signal. By taking the phase shifted amplitude as a function of time.

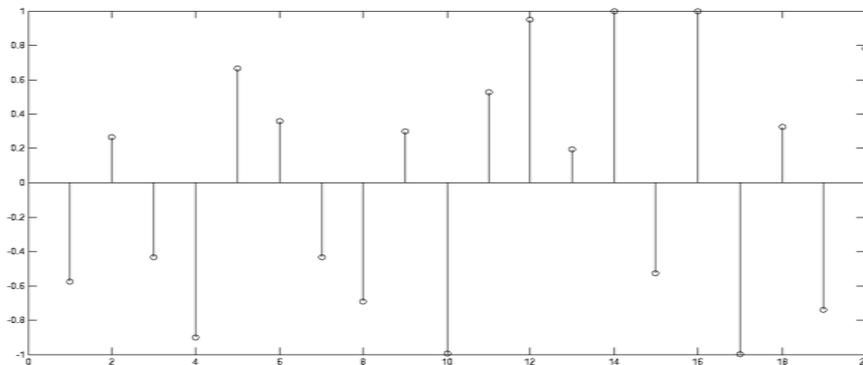


Figure: 5 The Doppler shifted signal for MIMO STAP in Discrete form

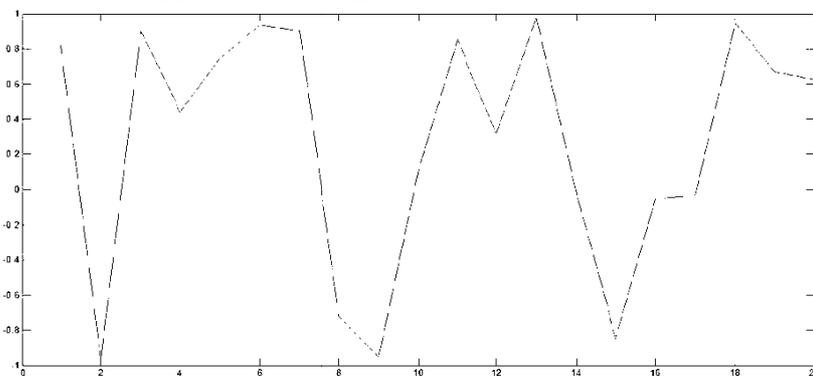


Figure: 6 The Doppler shifted signal of MIMO in analog form

**Conclusion**

The above work takes into account, the actual sea state condition in the proposed STAP for the MIMO radar. The K-distribution based Sea state condition actually reflects the electromagnetic proportion of sea water and the white tooth generated on top of the sea wave and its helps in actually modeling the STAP to reduce interference based on sea clutter for proper radar detection. This also helps in detecting small targets in asymmetric warfare.

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