

Optimal and water-Filling Algorithm approach for power Allocation in OFDM Based Cognitive Radio System

Dr. A. N. Jadhav

*Professor, Department of Electronics & Telecommunication,
D. Y. Patil College of Engineering & Technology
Kasba Bawada, 416006,
Kolhapur, Maharashtra, India.*

S. R. Mujawar

*Department of Electronics & Telecommunication,
D. Y. Patil College of Engineering & Technology
Kasba Bawada, 416006,
Kolhapur, Maharashtra, India.*

Mrs. P. S. Pise

*Associate Professor, Department of Electronics & Telecommunication,
D. Y. Patil College of Engineering & Technology
Kasba Bawada, 416006,
Kolhapur, Maharashtra, India.*

Abstract

Technology plays vital role in today's world. Growing technology in wireless communication and demand for spectrum is growing day by day. Many applications in wireless are sharing same medium and due to strain which leads to lack of spectrum in given limited band. As per the report published by (FCC) Federal Communication Commission the spectrum band are not utilized efficiently where as some of spectrum bands are used heavily. The potential solution for this problem is to allocate the spectrum dynamically through means of cognitive radio. In this paper water-filling and optimal power allocation algorithm has been investigated in simulation environment. Present numerical result show that optimal power allocation algorithm can achieve higher transmission compare to water-filling (i.e. classical) power allocation algorithm. The above algorithm is compared to total transmission power, interference and individual peak constraints.

Keywords- Cognitive Radio, Primary user, secondary user Orthogonal frequency division multiplexing.

Introduction

With the advance development in wireless communications, frequency spectrum has becoming a valuable natural resource, and limitation of the spectrum is a serious issue. On other hand Federal Communication Commissions (FCC) has reported the underutilization of electromagnetic radio spectrum [1]. In some cases, the spectrum band is underutilized by licensed users (primary user) and unlicensed users (secondary user) are not allowed to operate in licensed spectrum bands. This leads to unbalanced spectrum utilization.

Spectral efficiency can be increased significantly by giving access of the frequency bands to a group of unlicensed users (referred to as secondary or CR users). Cognitive radio(CR) system is unique softer-ware defined radio proposed to

improve spectrum efficiency by exploiting unused spectrum in dynamically changing environments. The cognitive radio is an innovative system design which involves smartly sensing the holes of spectrum and then determining the transmission characteristics with respective to symbol rate, power, bandwidth, latency etc. of a group of secondary users based on the nature of the users to whom the spectrum has been licensed (referred to as primary users).

A Cognitive radio (CR) network is a technology in which a frequency band used by one or multiple primary users in a primary network can be operated by a secondary user's network which consists of one or multiple secondary users. To guaranty the quality of service (QoS) of the Primary User (PU) and to maximize the transmission rate of the secondary users is one of the most crucial roles for a Cognitive radio system [1]. Orthogonal frequency division multiplexing (OFDM) is a promising tech for cognitive radio systems. With OFDM, the SU has the ability to smoothly flexible fill the spectral gaps left unused by PUs. It also can determine its location; sense spectrums of other devices, and change in frequency, adjust output power and even alter transmission parameters and characteristics. It can fulfill the SUs communication needs without altering the FCC rules. Cognitive radio mainly works on three tasks including: Radio scene analysis in which CR can detect spectrum holes, lightly used band, interference and Channel state estimation, in which CR determine the channel capacity and the state of the channel; Spectrum management, in which CR make the spectrum sharing efficient. The CR design is an featuring new methods of radio design philosophy which involves smoothly sensing the vacant

spectrum and then determining the transmission characteristics (e.g., symbol rate, power, bandwidth, latency) of a group of secondary users based on the behavior of the users to whom the spectrum has been licensed (referred to as primary users). Although opportunistic spectrum access would allow CR user to discover and access available spectrum

resources, one of the main aim is to utilize the available spectrum resources in an efficient manner. OFDM base CR cognitive Radio:-OFDM is a multi-carrier modulation technique that can overcome many problems that arise with high bit-rate communications, the most serious of which is time dispersion. The data containing symbol stream is split into several low rate streams, and these streams are transmitted on different carriers. Because this splitting increases the symbol duration by orthogonally overlapping carriers (subcarriers), multipath echoes affect only a small part of the neighboring symbols. The remaining inter-symbol interference (ISI) is removed by stretching of the OFDM symbol with a cyclic prefix (CP). Other advantages of OFDM include high spectral efficiency and robustness against narrowband interference (NBI).

Orthogonal frequency division multiplexing (OFDM), because of its smooth flexibility in allocating the spectrum, has been recognized as a best air interface technology for CR systems [2]. Because of the coexistence of CR and primary users is near to each other of bands, mutual interference between these users is the limiting factor in order to achieve a better performance for CR systems . we propose a power loading algorithm that maximized the downlink transmission rate while keeping the total interference introduced to different PU receivers below a threshold level .A dispense algorithm for optimal resource allocation in OFDMA based CR systems has been proposed. When using orthogonal frequency Division Multiplexing in cognitive radio network, the power allocation schemes for spectrum resources will be convenient and very flexible. However, it become challenging Role to allocate power to individual sub channels in the OFDM-based cognitive radio networks. Due to the above reasons, OFDM-based CR systems have more attention and the related resource allocation problems have become good research topics. This paper focuses on investigating the research challenges involved in the power allocation for OFDM based cognitive radio system. Hence, the design problem is that given an interference threshold prescribed by the primary users, how much power should be transmitted into each CR user's subcarrier such that the transmission rate of the CR user is maximized.

In this paper optimal power allocation algorithm is proposed as compare to water-filling algorithm which maximize the downlink transmission capacity of cognitive radio users while keeping the interference introduce to PU below a specified threshold.

SYSTEM MODEL

We consider a wireless system consisting of L sub-channels licensed to different primary users. Each of these primary users behaves differently or has uncorrelated activity in their band. All the subchannels are divided into multiple subcarriers as shown in **figure1** and they are opportunistically available to some secondary or cognitive user which uses the band in OFDM fashion. the total number of subcarriers are N with M sub-channels licensed to different primary users. In this, first, we discuss the subcarrier grouping strategy that is used to maximize the transmission rate and minimize interference. Second, we outline in details the system model used in this thesis. This includes a description of the transmitter and

receiver, the adaptive subcarrier allocation scheme used in our work, as well as the channel model used in the analysis.

Underlay Model

We consider a downlink transmission scenario. It is assumed that the frequency bands of bandwidth B1, B2... BL have been occupied by PU1, PU2... PUL. As in Figure 3.1, SUs can occupy either the spectrum of PUs or the adjacent spectrum of PUs. The available bandwidth for CR transmission is divided into N subcarriers based OFDM system, and the bandwidth for each subcarrier is $\Delta f H z$.

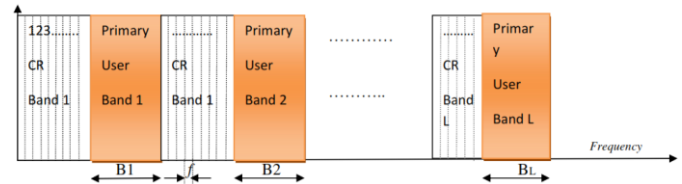


Figure 1: Distribution of primary and CR users.

In the downlink transmission scenario, there are three instantaneous fading gains: between the SUs transmitter and SUs receiver for the i th subcarrier denoted as h_i^{ss} ; between the SUs transmitter and l th PU receiver denoted as h_i^{sp} ; between l th PUs transmitter and SUs receiver denoted as h_l^{ps} . We assume that these instantaneous fading gains are perfectly known at the SUs transmitter.

Interference introduced to PU by SU

We assume that the signal transmitted on the subcarrier is an ideal Nyquist pulse. The power spectrum density of the i th subcarrier can be written as:

$$X_i^{(f)} = P_i T_s \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2 \quad (1)$$

Where ,

P_i : total transmits power in the i th subcarrier.

T_s : the symbol duration.

Then the interference introduced to the l th PU band by the i th subcarrier is:

$$I_i^{(l)} = P_i |h_l^{sp}|^2 T_s \int_{d_{il} - \frac{B_l}{2}}^{d_{il} + \frac{B_l}{2}} \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2 df \quad (2)$$

Where,

d_{il} : distance in frequency between the i th subcarrier and the l th PU band,

B_l : represents occupied bandwidth by the l th PU.

Interference introduced to SU by PU

The power spectrum density of the PU signal after M-fast Fourier transform (FFT) processing can be expressed as:

$$E[I_N(\omega)] = \frac{1}{2\pi M} \int_{-\pi}^{\pi} X_{PU}(e^{j\omega}) \left(\frac{\sin \frac{(\omega - \gamma)M}{2}}{\sin \frac{(\omega - \gamma)}{2}} \right)^2 dy \quad (3)$$

Where,

$X_{PU}(e^{j\omega})$: power spectrum density of the PU signal.

The PU signal has been taken to be an elliptically filtered white noise process with amplitude P_{PU} . According to the

interference introduced to the i th subcarrier by the l th PU band can be written as:

$$J_i^{(l)}(P_{PU}) = |h_i^{sp}|^2 \int_{d_{il}-\frac{\Delta f}{2}}^{d_{il}+\frac{\Delta f}{2}} E[I_N(\omega)] d\omega \quad (4)$$

Optimal Power Loading Algorithm

According to Shannon capacity formula, the transmission rate at the i th subcarrier is given by:

$$R_i(P_i) = \log \left(1 + \frac{|h_i^{ss}|^2 p_i}{\sigma^2 + \sum_{l=1}^L J_i^{(l)}} \right) \quad (5)$$

Where,

σ^2 = additive white Gaussian Noise (AWGN) variance.

Our objective is to maximize the total transmission rate of SU_s, expressing mathematically as:

$$C = \max_{P_i} \Delta f \sum_{i=1}^N \log_2 \left(1 + \frac{|h_i^{ss}|^2 p_i}{\sigma^2 + \sum_{l=1}^L J_i^{(l)}} \right) \quad (6)$$

Subjected to:

$$P_r \left(\sum_{i=1}^N I_i^{(l)}(d_{il}, P_i) \leq I_{th}^{(l)} \right) \geq a, \quad \forall l, \quad (7)$$

$$P_i \geq 0, \quad \forall i, \quad (8)$$

$$\sum_{i=1}^N P_i \leq P_T \quad (9)$$

$$P_i \geq G_i \quad (10)$$

Where,

- G_i = individual peak power constraint.
- C = total transmission capacity of SU.
- N = number of OFDM subcarriers.
- P_r = the probability.

Now the Probability interference constraint in Eq. (7) can be written as

$$P_r \left(|h_i^{sp}|^2 \sum_{i=1}^N K_i^{(l)} P_i \leq I_{th}^{(l)} \right) \geq a, \quad \forall l, \quad (11)$$

Where,

$$K_i^{(l)} = T_s \int_{d_{il}-B_i/2}^{d_{il}+B_i/2} \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2 df.$$

Since $|h_i^{sp}|$ is assumed to be Rayleigh distributed with known parameter λ_i , the distribution of $|h_i^{sp}|^2$ corresponds to an exponential distribution with the parameter λ_i^2 . The constraint in Eq. (11) can be evaluated in closed form for the Rayleigh fading case as follows

$$1 - e^{-\frac{I_{th}^{(l)}}{2\lambda_i^2 \sum_{i=1}^N P_i K_i^{(l)}}} \geq a, \quad \forall l \quad (12)$$

After some mathematical Manipulation, Eq (12) can be written as

$$\sum_{i=1}^N P_i K_i^{(l)} \leq \frac{I_{th}^{(l)}}{2\lambda_i^2 (-\ln(1-a))}, \quad \forall l \quad (13)$$

It is clear that this is a problem of convex optimization. (9) is a concave function, $I_i^{(l)}(P_i)$ is a convex function. According to convex optimization theory, when the total transmission capacity is maximized, the optimal power of the i th subcarrier is given by:

$$P_i^* = \left[\frac{1}{\theta + \omega + \sum_{l=1}^L \xi_l K_i^{(l)}} - \frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} \right] \forall i, \quad (14)$$

Where,

$\xi_l, v_i, \theta, \omega$ = Lagrange multipliers.

Water-Filling Loading Algorithm

In water-Filling algorithm, which is optimal power allocation algorithm in conventional OFDM system, we use the total power allocation by uniform loading as the power constraint. the allocated power in the i th subcarrier because of the l th interference because of the l th interference constraint is written as

$$P_i^{(l)} = P/K_i^{(l)}, \quad \forall l, \quad (15)$$

where P can be calculated by assuming strict equality in the l th interference constraint in Eq. (9). Using Eq. (15), in this equality constraint

$$\sum_{i=1}^N P_i K_i^{(l)} = \frac{I_{th}^{(l)}}{2\lambda_i^2 (-\ln(1-a))}, \quad (16)$$

We can write

$$P = \frac{I_{th}^{(l)}}{2N\lambda_i^2 \ln \frac{1}{(1-a)}} \quad (17)$$

Using Eq. (15) and (17), we can calculate $P_i^{(l)}$ as

$$P_i^{(l)} = \frac{I_{th}^{(l)}}{2K_i^{(l)} N\lambda_i^2 \ln \frac{1}{(1-a)}}, \quad \forall i. \quad (18)$$

Now we need to calculate power values $P_i^{(L+1)}$ due to the total power constraint. In order to meet the total power constraint, we use the standard water-filling algorithm to distribute total power PT among N CR subcarriers. According to the water-filling algorithm with a total power constraint PT , the power values can be written as

$$P_i^{(L+1)} = \max \left\{ 0, \frac{1}{\alpha} - \frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} \right\} \quad \forall i, \quad (19)$$

Where the lagrange constant α can be calculated from the following equation

$$\sum_{i=1}^N \max \left\{ 0, \frac{1}{\alpha} - \frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} \right\} = P_T \quad (20)$$

The power value for i th subcarrier, denoted by $P_i^{(WF)}$ are obtained using the standard water-filling algorithm as mentioned in Eq (18) and (19) considering the total power constraint equal to the total power allocated by uniform loading algorithm. The power values will satisfy the total power constraint given in Eq. (12) however it is checked that if the power values satisfy the interference constraints specified in Eq (12). If a particular interference constraint is not satisfied, the power value in each subcarrier $P_i^{(WF)}$ is reduced such that the all interference constraints are satisfied. Also, if none of these interference constraints is met strictly,

the power value $P_i^{(WF)}$ is increased until one of these interference constraints is met strictly.

SIMULATION RESULTS

System Speciation.

Performance of optimal power allocation algorithm are compared with performance of classical power loading algorithms i.e. water-filling power allocation schemes that are used for conventional OFDM-based cognitive radio system parameters and network configurations used for experiments is described as.

Performance Parameters

The four important performance parameters for evaluation are as:

- Power budget is the allocation of available power among the available user.
- Interference: This term is used for interference between Primary User and Secondary User.
- Probability: it is transmission rate of CR user C is maximized while the probability that interference to PU's is kept below threshold
- Individual Peak Power: it is defined as the power constraint to CR to protect primary user by restricting the secondary user

System parameter specification.

Sr no.	Parameters	Values
1	No of Sub-channels	5 No's
2	Band width	1 Mhz
3	Power in dB	-20dBm
4	Noise in db	-110 dB

Table 1: System Parameters

Transmission rate vs. Power budget.

The **Figure 2** is a graph of maximum transmission rate for the CR user versus the total power budget for various algorithms. The value of $I_{th}^{(2)}$ has been fixed to the individual peak power constraint. From this **figure 2** and **table 2**, it is observed that the optimal algorithm is able to achieve higher transmission rate for a given power budget than the water-filling algorithm.

For optimal power allocation algorithm as increased in the power budget from 0.1 mw to 0.5 mw the maximum transmitted data rate reach up to 6.8914×10^6 (bps) and remain constant at average value of 7.7×10^6 (bps) and for water-filling power allocation algorithm maximum transmitted data rate reach up to 1.70268×10^6 (bps) and

remain constant at average value of 2.6×10^6 (bps) because as we increase power budget for CR user, the interference constraint become dominant and transmission rate of CR user does not increases. In this region the CR system operates in an interference limited scenario.

Power Budget (mWatt)	Maximum Transmitted Data Rate (bps) ($\times 10^6$)	
	Water filling	Optimal
0.1	1.70268	6.8914
0.2	2.64768	7.8364
0.3	2.62428	7.813
0.4	2.60268	7.7914
0.5	2.42268	7.6114

Table 2: Power Budget Vs transmitted data rate

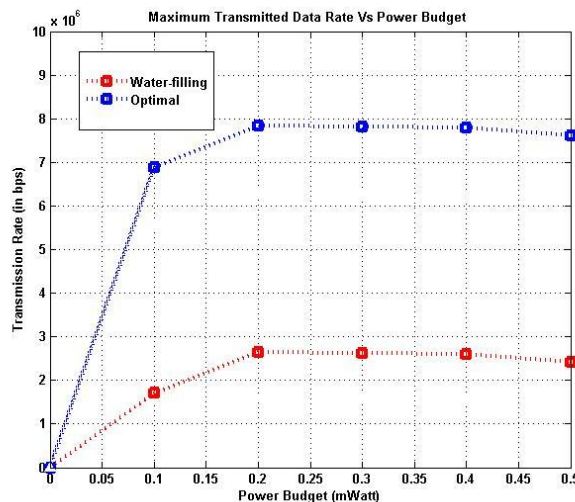


Figure 2: Maximum Transmitted data rate vs. power budge (PT) for CR user for different power allocation algorithm

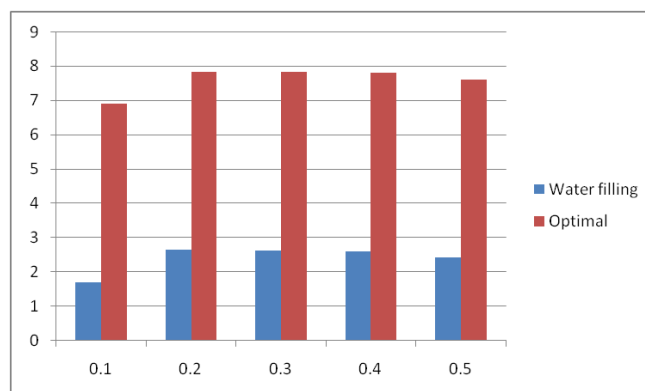


Figure 3: Bar-graph for Maximum Transmitted data rate vs. power budge for different power allocation algorithm (PT) for CR user

Transmission data rate versus interference.

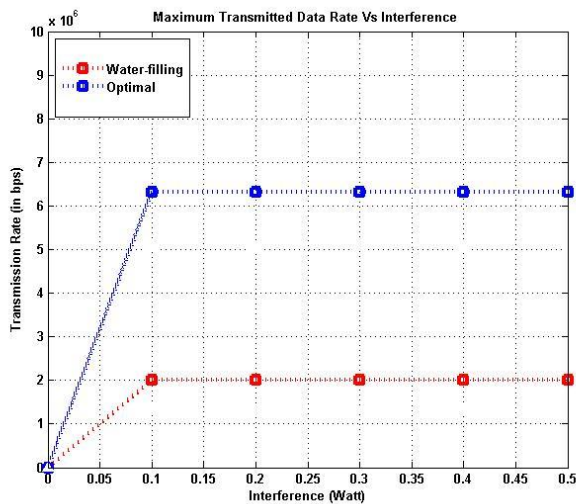


Figure 4: Transmission data rate for the CR user versus Interference threshold for second PU

The **figure 4** is graph of maximum transmission rate for CR user versus the interference threshold for second PU user band. It is observed from **figure 4 and table 3** that the proposed optimal power allocation algorithm achieves higher transmission rate as compare to waterfilling power allocation algorithm.

The proposed optimal algorithm achieves higher transmission rate 6.32393×10^6 bps than that of other algorithms and water-filling algorithm achieves higher transmission rate upto 2×10^6 bps. The transmission rate versus interference threshold curve saturates after a certain value of $(I_{th}^{(2)})$. The reason is that although $(I_{th}^{(2)})$ is relaxed by increasing its value, other constraints $((I_{th}^{(1)}), (I_{th}^{(3)})$ and P_T) becomes dominant. The optimal power allocation gives better results because it limits the interference by limiting power to the subcarriers near to primary user's and allocate more power to subcarriers which are far from primary users thus limiting interference.

Transmission rate versus probability.

The **Figure 6** is a graph of transmission rate for the CR user versus probability a . It is observe that the proposed optimal power allocation algorithm performs best over other water-filling power allocation algorithm.

It is observed from **figure 6 and table 4** that for probability of 0.75 the available transmission rate is 1.42714×10^8 bps for optimal power allocation algorithm and it is 4.54253×10^7 bps for water-filling algorithm for certain signal power and as we increase the probability to 0.95 the available transmission rate decrease to 3.27341×10^6 for optimal and 8.08773×10^6 for water filling . This happens because as we go on increasing signal power of secondary user to increase probability the interference toward primary user also increase so the achievable transmission rate of CR user decreases for a given power budget and interference thresholds.

Interference (mWatt)	Maximum Transmitted Data Rate (bps) (x 10 ⁶)	
	Water filling	Optimal
0.1	2	6.32393
0.2	2	6.32393
0.3	2	6.32393
0.4	2	6.32393
0.5	2	6.32393
0.6	2	6.32393
0.7	2	6.32393
0.8	2	6.32393
0.9	2	6.32393
1	2	6.32393

Table 3: Transmission rate for the CR user vs interference threshold for different power allocation algorithm.

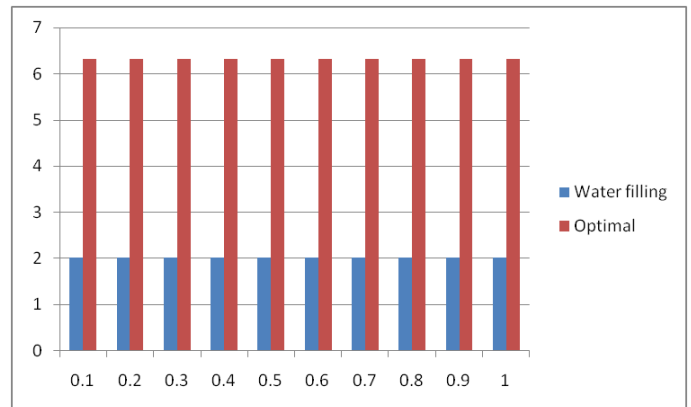


Figure 5: Bar-graph for Maximum Transmitted data rate vs. interference threshold

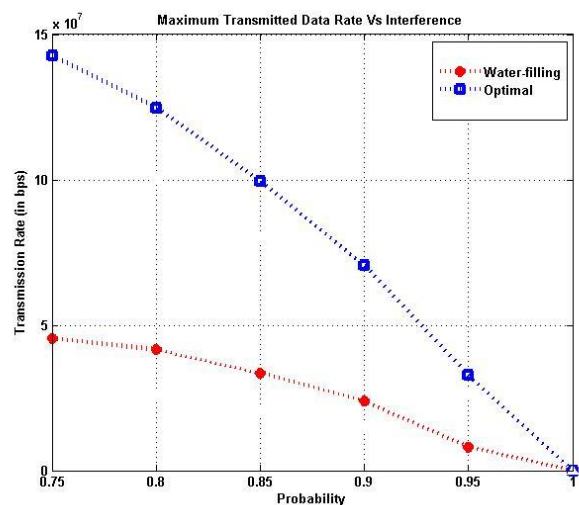


Figure 6: Transmission rate for the CR user versus probability with instantaneous interference introduced to PU

Probability	Maximum Transmitted Data Rate (bps)	
	Water - filling	Optimal
0.75	4.54253	14.2714
0.8	4.16429	12.4662
0.85	3.34596	9.96157
0.9	2.38291	7.05276
0.95	0.808773	3.27341
1	0	0

Table 4: Transmission rate for the CR user vs. probability with interference introduced to the PU.

caused by secondary user's, less power is assigned to the subcarriers which are near to primary user's band and more power is assigned to the subcarriers which are far from the primary user band.

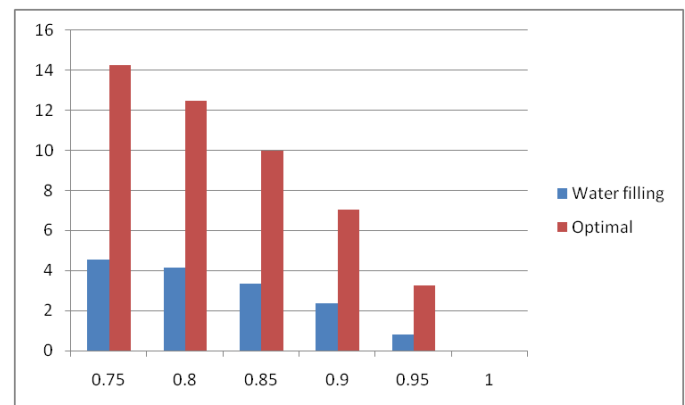


Figure 7: Bar-graph of Transmission rate for the CR user versus probability with instantaneous interference introduced to PU

Transmission vs. individual peak power Constraints.

The Figure 8 and table 5 is a plot of the achievable maximum transmission rate for the CR user versus individual power constraints for various algorithms. It is observed that the proposed optimal algorithm is able to achieve higher transmission rate for a given power budget than the water-filling power allocation algorithm.

It is observed from figure 8 and table 5 that as we increase the individual peak power constraint from 0.1×10^{-3} to 0.5×10^{-3} for CR user, the transmission rate of CR user for optimal algorithm increases from 2.87142×10^5 to 9.51425×10^5 and for water-filling it increases from 0.70945×10^5 to 3.02835×10^5 it is Because the interference to the primary user is under a threshold and the power constraint to CR protect primary user by restricting the secondary user hence the transmission rate of CR user does increase as the individual peak power constraint increases. This is expected as individual subcarrier can be allocated more power.

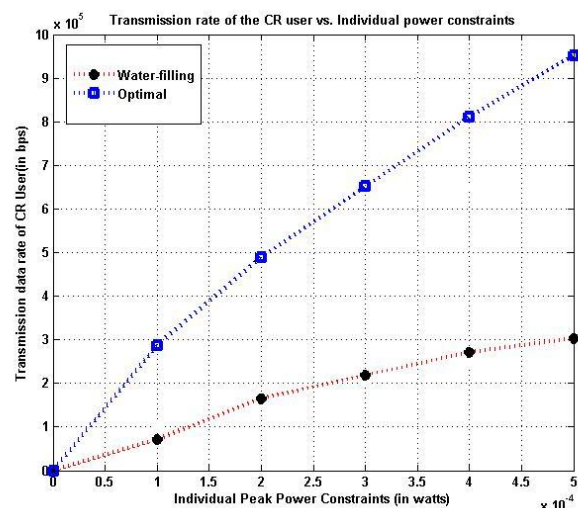


Figure 8: Transmission rate for the CR user vs. individual peak power Constraints

CONCLUSIONS

The simulation procedure is carried out for the implementation of the proposed system discussed in previously. The simulation analysis is mainly divided into two different power allocation algorithm namely water-filling and optimal (adaptive) power loading algorithm. the simulation is done between four different parameters name as power budget, interference threshold ,Probability to interference introduced to PU and Individual peak constraints to each carrier Vs. Transmission data rate of CR user(in bps)

Transmission rate vs. total power budget.

It is concluded that as the total power budget is increased then the maximum transmission data rate for CR user is found to be increased. Further it is observed that the proposed optimal power allocation algorithm performance better than water-filling and able to achieve more transmission rate about 75% .

This is because the power distributed in Optimal algorithm is like a ladder profile i.e. to reduce the interference

Peak Power (mw)	Maximum Transmitted Data Rate (bps)	
	Water filling	Optimal
0.0001	170945	287142
0.0002	165480	489775
0.0003	218690	651083
0.0004	271113	811604
0.0005	302835	951425

Table 5: Transmission rate for the CR user vs. individual peak power Constraints

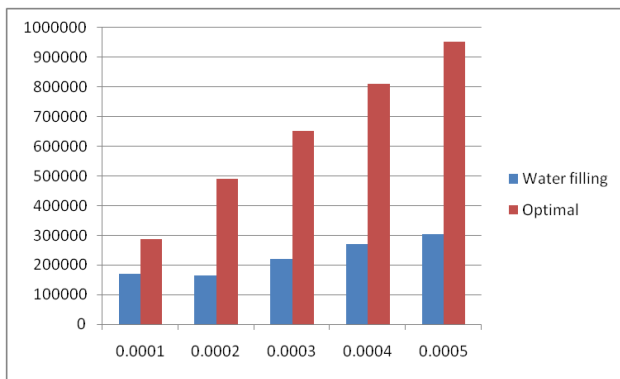


Figure 6.29: Bar-graph for Maximum Transmitted data rate vs. individual peak Constraints

Transmission data rate versus interference.

From simulation result it is concluded that transmission rate for CR user increase for a fixed specified interference Threshold. It is observed that optimal power allocation algorithm performs better than water-filling algorithms and able to achieve more transmission rate about **80%**.

This is because optimal power allocation algorithm reduces the interference ratio between the primary user's and secondary user's by limiting power ratio to the nearest cognitive user and increasing power ratio as the distance between primary and secondary is more.

Transmission rate versus probability.

It is concluded that transmission rate goes on decreasing as probability to the interference goes on increasing.

Further it is observed that at particular probability the optimal power allocation algorithm performs better than water-filling power allocation algorithm and gives more transmission rate about **70%**. Because unlike other optimal power allocation algorithm, optimal power allocation reduce the interference probability depending on distance between primary user's and secondary user's.

Transmission vs. individual peak power Constraints.

It is concluded that transmission rate is increased as increased in individual peak power constraints, with which instantaneous inference introduced to PU band remains below interference threshold.

Further it is observed that proposed optimal power allocation algorithm is able to perform better water-filling power allocation algorithms and gives more transmission rate about **60%** because unlike other optimal power allocation algorithm assign different interference threshold to different secondary subcarrier depend upon step size i.e. distance between primary user's and secondary user's.

From all above conclusion from Simulation results it is observed that our proposed optimal power allocation algorithms can achieve higher transmission rate for CR user compared to the water-filling power allocation algorithms

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