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Comparative Analysis of Singular Value Decomposition (SVD) and Wavelet Difference Reduction (WDR) based Image Compression

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Abstract
This paper presents study of two lossy image compression techniques. The two techniques are Singular Value Decomposition (SVD) based image compression and Wavelet Difference Reduction (WDR) based image compression. SVD based compression reduces the psychovisual redundancies present in the image through rank reduction method. WDR is a lossy image compression technique. It gains compression by taking the discrete wavelet transform of the input image and then encodes the transform values using difference reduction method. Various image compression parameters like PSNR, MSE and compression ratio are evaluated for the two techniques. The two techniques are compared on the basis of same compression parameters and visually similar compressed images.

Keywords: Lossy Image Compression, Singular Value Decomposition, Wavelet Difference Reduction.

Introduction
Compression is an important image processing technique that removes the redundant information present in an image without much affecting the quality of that image. It maps a higher dimensional space into a lower dimensional space [11]. It is applied in different fields like signal processing, artificial intelligence, communication, etc. The aim of compression is to save space so that huge data can be stored, transmitted and retrieved efficiently. Data to be compressed can be multimedia, documents, videoconferencing information, medical images, etc. The uncompressed images require not only more storage capacity but also more transmission bandwidth. Compression will reduce the size of the file and allow more number of images to be stored in a given memory space.

Image compression deals with reducing the extra irrelevant information present in an image. Redundancies present in an image can be:
1. Psycho visual redundancies: This information does not appear sensitive to human eye. It belongs to visually non-essential information of an image. Such an information can be discarded.
2. Inter pixel redundancy: This redundancy occurs due to presence of similar neighboring pixels. Such type of redundancy can be removed.
3. Coding redundancy: This redundancy occurs due to use of longer code words to encode information of an image. Coding redundancies can be removed by assigning shortest code words to most frequently occurring information. [11]
Removing the redundancy will reduce the number of bits required to represent an image. This can be achieved by different compression techniques.

There are two types of compression techniques- lossless compression and lossy compression [9]. In lossless image compression there is no loss of information as the original image is perfectly recovered from compressed one. This is needed in case of data like executables, documents, some medical images, etc. which need to be exactly reproduced when decompressed. In lossy compression, some amount of loss of information is tolerable and reconstruction of the original data is possible even after removing some amount of redundant information [10]. Some images need not have to be reproduced exactly after compression. An approximation of original image is enough for most of the purpose, as long as the error between the original and compressed image is not too high. Such lossy compression techniques are used to lessen the amount of data in order to store, handle, and transmit the represented content in effective manner.

In this paper two lossy image compression techniques are discussed. The two techniques are Singular Value Decomposition (SVD) based image compression and Wavelet Difference Reduction (WDR) based image compression.

Methods

Singular Value Decomposition based image Compression

Singular Value Decomposition (SVD) decomposes an image matrix into product of three matrices. Out of the three matrices, two are orthogonal matrices and third is diagonal matrix. The diagonal matrix contains the diagonal elements which are the singular values of image matrix. [14][1]

Let A be an image matrix, then singular value decomposition of matrix A is:

$$\mathbf{A_{yxz} = U_{yxy}\, S_{yxz}\, (V_{zxz})^T} \qquad \ldots(1)$$

where, A is a y×z matrix,
S is a y×z diagonal matrix. The elements on the diagonal of S are singular values of A.
U is a y×y matrix containing left singular vectors of A.
V^T is a z×z matrix containing right singular vectors of A.
U and V are orthonormal matrices i.e. $UU^T = I$ and $VV^T = I$.
In matrix form the equation (1) becomes:

$$A=[\, u_1, u_2, \ldots u_y] \begin{bmatrix} \sigma_1 & 0 & 0 & 0\ldots..0 \\ 0 & \sigma_2 & 0 & 0\ldots..0 \\ 0 & 0 & \sigma_3 & 0\ldots..0 \\ . & . & & . \\ . & & . & . \\ . & & . & . \\ 0 & 0 & 0 & 0\ldots\sigma_z \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ . \\ . \\ . \\ v_z \end{bmatrix}$$

In the above form, u_i (for i=1,2…y) represents columns of matrix U and are eigen vectors of AA^T with eigen value σ_i^2. u_i is the left singular vector of image matrix A. The rows v_i (for i=1,2,…z) of V are eigen vectors of A^TA, with eigen value σ_i^2. v_i is right singular vector of image matrix A. σ_i (for i=1, 2,…….z) are the singular values of matrix A. The singular values are such that $\sigma_1 \geq \sigma_2 \geq \sigma_3 \ldots\ldots\ldots.. \geq \sigma_z \geq 0$.

In order to bring out compression, the dimensions of diagonal matrix are reduced to S_{pxq} where p≤y and q≤z. After application of SVD, only some singular values from matrix S are kept while the lower singular values are removed. This can be done because of the fact that singular values are arranged in descending order and that first singular value contains much information than the following singular values that contain decreasing amount of image information. So, the lower singular values that contains less important information can be discarded.[4]

The number of non-zero singular values present in diagonal matrix S specifies the rank of the matrix A. If the singular values after a certain rank are not zero, they are considered as redundant and can be removed.

Equation (1) above can also be written as:

$$A=u_1\sigma_1v_1^T + u_2\sigma_2v_2^T \ldots + u_r\sigma_rv_r^T \ldots +u_z\sigma_zv_z^T \qquad \ldots (2)$$

where r is the rank of A. Reducing equation (2) till 'r' values does not give much change in the image. The amount of compression achieved will be very less and the image quality will also remain nearly same as quality of original image. For better compression, only the first 'k' singular values before 'r' of equation (2) are taken so that equation (2) becomes

$$A = u_1\sigma_1v_1^T + u_2\sigma_2v_2^T \ldots\ldots\ldots\ldots+ u_k\sigma_kv_k^T \qquad \ldots (3);$$

where k < r .

The value of k is chosen such that good amount of compression is achieved while maintaining the image quality. Ignoring lower singular values removes the psychovisual redundancies present in an image without reducing visual quality of an image. [5]

Algorithm for SVD based image compression:

1. Read the input image
2. Convert it into graylevel image.
3. Decompose the image using singular value decomposition.
4. Discard the singular values not required for compression.
5. Reconstruct the image.
6. Compute compression ratio, mse and psnr.

Wavelet Difference Reduction based Image Compression

The Wavelet Difference Reduction (WDR) is an encoding technique which is based on the difference reduction method. It gains compression by taking the discrete wavelet transform of the input image and then encodes the transform values using difference reduction method. [3]

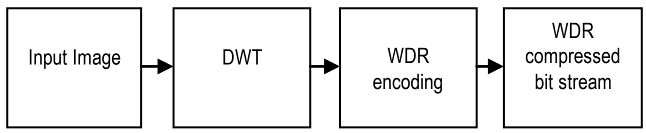


Figure 1: Block diagram for WDR based compression

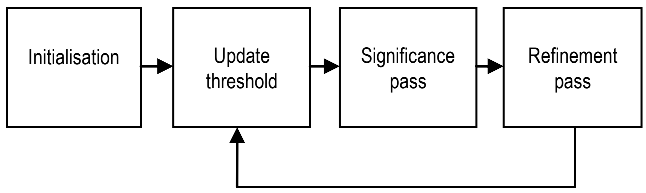


Figure 2: WDR encoding

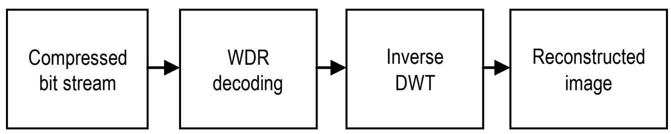


Figure 3: Block diagram for WDR decompression

Discrete wavelet transform divides image into four subbands- LL, LH, HL, HH. Wavelet Difference Reduction encoding uses four steps for encoding: Initialisation, Thresholding, Significant pass and refinement pass.

1. Initialisation: In this the scan order is decided. The scan order goes through sub-bands from higher level to lower levels in zig-zag manner. A threshold T_0 is selected.
2. Update Threshold: Threshold is updated to $T_k=T_{k-1/2}$, for k=1,2…p and 'p' is the number of pixels in an image.
3. Significance Pass: Here, values of wavelet transform are compared to a specific threshold value. A value is significant if it is greater than or equal to threshold value. If an index is found to be significant then it is

removed from the scan order. Next, difference of these index values is taken and binary expansion of successive difference is done. Since the MSB in these expansions is always 1, we can ignore this bit and use the signs of the significant transform values in its place in the symbol stream. The stream consists of four symbols that can be encoded using probabilistic model.

4. Refinement Pass: In this, standard bit plane quantization is carried out to give refinement bit. Refined value gives better approximation of transform value.
5. Repeat steps (2) to (4) until you get desired bit budget.

To reconstruct the image, WDR decoding and inverse DWT is performed on compressed bit stream. [12]

The property of WDR is that it gives perceptually better image at high compression ratio while retaining the desirable features.

Compression parameters

The results of different compression techniques can be compared by using different compression parameters like compression ratio, mean square error (MSE) and peak signal to noise ratio (PSNR). These parameters are used to measure the degree to which an image is compressed and estimate the quality of compressed image. [19]

1. Compression Ratio: It is defined as the ratio of original size of the image to compressed size of the image. It gives measure of the degree to which an image is compressed.

   Compression ratio = (Size of original image/Size of compressed image)

2. Mean Square Error (MSE): It gives the measure of degradation of compressed image quality as compared to the original image. It is defined as square of the difference between original image pixel values and the corresponding pixel value of the compressed image, averaged over the entire image.

$$MSE = (\sum\sum(g(x,y) - g'(x,y))^2) / (m*n);$$

   where m*n gives total number of pixels present in original image;
   g(x,y) and g'(x,y) are the pixel value of original and compressed image respectively.

3. Peak Signal to Noise Ratio (PSNR): It is the ratio of maximum signal power to the noise power. In Image compression, noise refers to the deviation of the compressed image from the original one. Thus, PSNR gives the quality of the reconstructed images after compression.

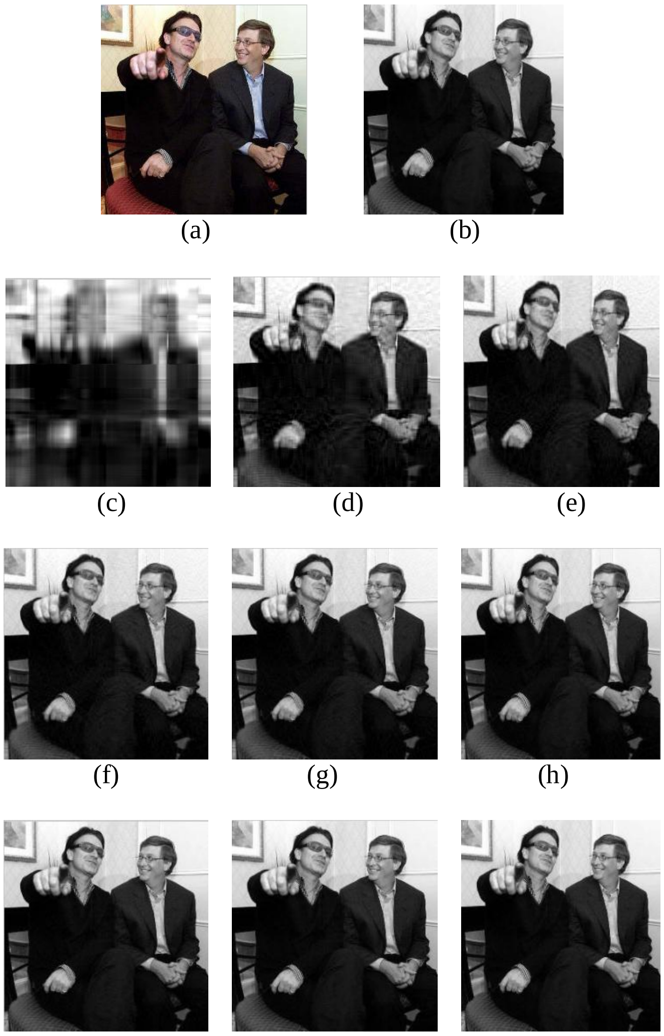
$$PSNR = 10*\log_{10} [255*255/MSE]$$

Results

The experimental results for the two techniques are shown below. The codes of these algorithms are executed in MATLAB software. For this MATLAB version 2013a was used. MATLAB provides hundreds of built-in functions for technical computation, graphics, etc.

For experimenting, a jpeg image is selected whose matrix size is 256x256.

SVD based image compression results for different number of singular values (k):



The figures shown above are:
(a) Original image (b) Gray scale image (c) Image for k=5
(d) Image for k=25 (e) Image for k=45 (f) Image for k=65
(g) Image for k= 85 (h) Image for k=105 (i) Image for k=125
(j) Image for k=145 (k) Image for k=165

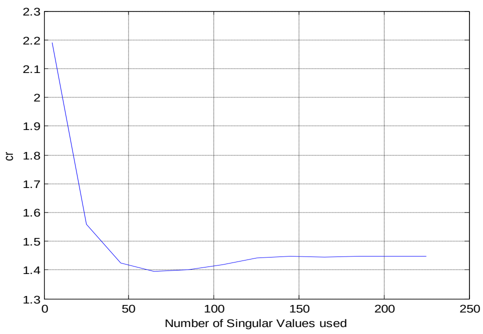


Figure 4: Graph of compression ratio V/s number of singular values used

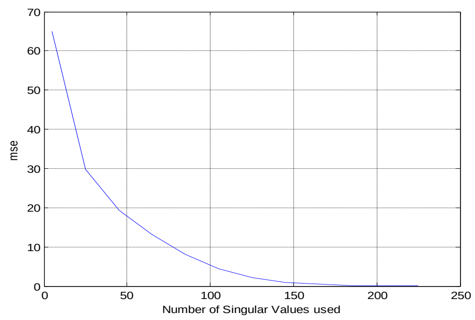


Figure 5: Graph of MSE V/s number of singular values used

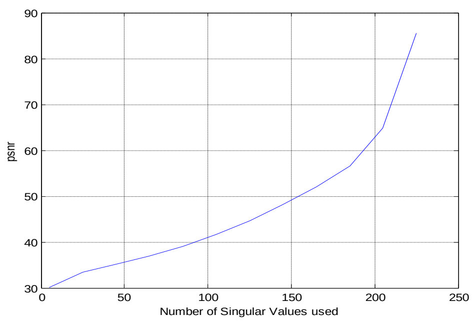


Figure 6: Graph of PSNR (in dB) V/s number of singular values used

Table 1: Compression parameters evaluated for images constructed from different number of singular values

Number of singular values used	CR	MSE	PSNR
5	2.1915	65.0023	30.0015
25	1.5595	29.7480	33.3962
45	1.4234	19.3943	35.2540
65	1.3960	13.1369	36.9458
85	1.4006	8.1005	39.0457
105	1.4190	4.5006	41.5980
125	1.4405	2.2304	44.6470
145	1.4485	1.0120	48.0788
165	1.4460	0.4147	51.9539

WDR based image compression results:

Figure 7: Original image

Figure 8: Gray scale image

Figure 9: WDR compressed image

Table 2: Compression parameters evaluated for WDR compressed image

Compression Parameters	Values
Compression Ratio	1.6875
MSE	39.3868
PSNR	32.1773

Result of comparison between two techniques with nearly equal compression parameters:

Figure 10: SVD and WDR compressed images having nearly equal compression parameters (compression ratio≈1.7, MSE≈40, PSNR≈32.1)

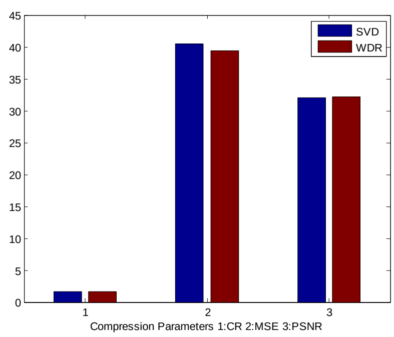


Figure 11: Bar graph representing nearly equal compression parameters for both the techniques with SVD at k=17.

Result of comparison between two techniques with nearly same visual quality of compressed image:

Figure 12: SVD (at k=52) and WDR compressed images with nearly same visual quality but different compression parameters.
For SVD, compression ratio=1.40, MSE=17.10, PSNR=35.80.
For WDR
, compression ratio=1.68, MSE=39.38, PSNR=32.17

Discussion

- SVD based image compression technique gives better visual quality at higher singular values. With increase in number of singular values used, compression ratio and MSE decreases. Hence, higher values of ‘k’ will give lower compression. With increase in number of singular values used, PSNR increases as shown in figure 6.

- WDR based image compression achieves perceptually good image with high compression.

- On comparing SVD and WDR based compression for same compression parameters, it is seen that WDR compressed image appears perceptually better than SVD as shown in figure 10. For this SVD based compression with 17 singular values was carried out.

- On comparing compressed images of the two techniques with same visual quality, it is found that compression ratio of WDR based compression is higher than SVD based compression. For this SVD based compression with 52 singular values was carried out.

- Since reconstructed images are not exactly same as that of original image, both the techniques are lossy compression techniques.

Conclusion

From the above discussion it can be concluded that WDR based compression gives good quality images with higher compression ratios. SVD based image compression gives better quality images at higher singular values. For same compression parameters, the results obtained by WDR based compression are superior than that of SVD based compression. If we integrate both these techniques then performance of WDR will get enhanced. We can get visually better images with high amount of compression if WDR is used along with SVD.

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