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Rotating Flow of Magnetite-Water Nanofluid over a Stretching Surface Inspired By Non-Linear Thermal Radiation and Mass Transfer

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Abstract

We have investigated MHD three dimensional rotating flow of magnetite-water nanofluid over a stretching surface inspired by non-linear thermal radiation and mass transfer. In this problem, the Maxwell-Garnett model for effective thermal conductivity and electrical conductivity of nanofluid are taken into account. A mathematical formulation has designed for velocity, momentum, temperature and concentration profiles. The governing partial differential equations are reduced to a system of ordinary differential equations using Shooting technique together with Runge -Kutta sixth order iteration scheme. The numerical results of the flow characteristics are presented graphically.

Keywords: Nanofluid, Magneto hydrodynamics (MHD), Magnetic parameter (M), ratio rate (λ), Radiation parameter (Rd), Thermal Radiation, Mass transfer.

1. INTRODUCTION

Research of Nanofluid have more attention in various scientific and engineering applications. Some actual and potential applications are transportation, defense, nuclear, space, and biomedical. It is being observed that the rotating flow of magnetite – water nanofluid over a stretching surface inspired by non-linear thermal radiation and mass transfer has got remarkable importance in every concern field. Rotation flow theory is helpful in determining the viscosity of the fluid. In this study, we have observed that An unsteady MHD boundary layers in a rotating flow investigated by Debnath [1]. Wang [2] reported three dimensional flow due to a stretching sheet being stretched in two lateral directions and rotate like a rigid body. Takhar *et al.* [3] considered an unsteady flow over a stretching surface with a magnetic filed in a rotating fluid. Sheikholeslami *et al.* [4] studied the nanofluid flow, ferrofluid flow and heat transfer in a rotating system in the presence of magnetic field. In this study, they found that an increase in the magnetic field, volume reaction of nanoparticle decreases the velocity of the fluid. Mahmood *et al.* [5] investigated analytical treatment of three dimensional MHD flow due to a stretching surface in a rotating fluid. Sheikholeslami and Ganji [6] analyzed Effect of non-uniform magnetic field on forced convection heat transfer of Fe3O4- water nanofluid. Wahiduzzaman *et al.* [7] established Viscous Dissipation and Radiation effects on MHD Boundary Layer Flow of a Nanofluid past a Rotating Stretching Sheet. Jawad and Azizah [8] examined MHD three dimensional flow of a nanofluid in a rotating channel. Mahanthesh *et al.* [9] Discussed the influence of non-linear thermal radiation on three dimensional steady flow of a nanofluid past a non linear stretching in the presence of soret and dufours effect. Mustafa *et al.* [10] investigated the Rotating flow of Magnetite-Water Nanofluid over a Stretching Surface inspired by Non-linear thermal radiation. Apart from the existence, effect of mass transfer is studied and Shooting technique together with Runge -Kutta sixth order iteration scheme is used to analyze the research problem numerically.

2. MATHEMATICAL MODEL

Consider the three dimensional stretching of a surface in a rotating fluid. Let (u, v, w) be the velocity components in the direction of Cartesian axes (x, y, z) respectively, with the axes rotating at an angular velocity Ω in the z- direction. The Magnetic field B_0 is imposed in the z- direction. Since the flow is induced by stretching the surface in the x- direction with rate a. At a constant temperature T_w whereas T_∞ denotes the temperature outside the thermal boundary layer (Fig.1). C_w, C_∞ are concentrations at the surface and far away from the sheet, the equations embodying the conservation of mass, momentum and energy, species are expressed as below:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\rho_{nf}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}-2\Omega v\right)=\mu_{nf}\left(\frac{\partial^2 u}{\partial z^2}\right)-\sigma_{nf}B_0{}^2u \tag{2}$$

$$\rho_{nf}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+2\Omega u\right)=\mu_{nf}\left(\frac{\partial^2 v}{\partial z^2}\right)-\sigma_{nf}B_0{}^2 v \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha_{nf}\left(\frac{\partial^2 T}{\partial z^2}\right)-\frac{1}{\left(\rho c_p\right)_{nf}}\frac{\partial q_r}{\partial z}+\frac{Q}{\left(\rho c_p\right)_{nf}}(T-T_\infty) \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=D\left(\frac{\partial^2 C}{\partial z^2}\right)+k(\mathrm{C}-\mathrm{C}_\infty) \tag{5}$$

With the boundary conditions

$$u=u_w=ax \quad v=0 \quad w=0 \quad T=T_w \quad \text{at} \quad z=0$$

$$u=0 \quad v=0 \quad T=T_\infty \qquad \text{as} \quad z\to\infty \tag{6}$$

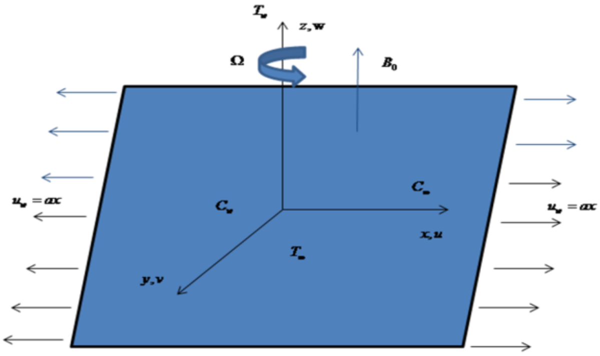

Fig. 1 Geometry of the problem

In which u and v are the velocity components along the x and y directions respectively, $q_r=-\left(\frac{4\sigma^*}{3k^*}\right)\frac{\partial T^4}{\partial z}$ is the Roseland radiative heat flux in which σ^* is the Stefan- Boltzmann constant and k^* is the mean absorption coefficient. The dynamic viscosity of nanofluid μ_{nf} as

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}} \tag{7}$$

The effective density ρ_{nf} and effective heat capacity $\left(\rho c_p\right)_{nf}$ are expressed as

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s \tag{8}$$

$$\left(\rho c_p\right)_{nf}=(1-\phi)\left(\rho c_p\right)_f+\phi\left(\rho c_p\right)_s \tag{9}$$

We take into account the Maxwell-Garnett model for effective thermal conductivity of nanofluid k_{nf} given below

$$\frac{k_{nf}}{k_f}=\frac{\left(k_s+2k_f\right)-2\phi\left(k_f-k_s\right)}{\left(k_s+2k_f\right)+\phi\left(k_f-k_s\right)} \tag{10}$$

Moreover the electrical conductivity of nanofluid σ_{nf} is

$$\frac{\sigma_{nf}}{\sigma_f}=1+\frac{3\left(\sigma_s-\sigma_f\right)\phi}{\left(\sigma_s+2\sigma_f\right)-\left(\sigma_s-\sigma_f\right)\phi} \tag{11}$$

In equations (7)-(11), ϕ denotes the nanoparticle volume fraction and the subscripts s and f correspond to the solid and fluid phases respectively. We look for similarity solution of equations (1)-(5) in the following forms

$$\eta=\sqrt{\left(\frac{a}{v_f}\right)}z \qquad u=axf'(\eta) \qquad v=axg(\eta)$$
$$w=\sqrt{av_f}\,f(\eta) \qquad T=T_\infty+\left(T_w-T_\infty\right)\theta(\eta) \qquad \phi(\eta)=\frac{c-c_\infty}{c_w-c_\infty} \tag{12}$$

In view of the above quantities, the continuity Equation (1) is identically satisfied while Equations (2)-(5) become

$$\frac{1}{(1-\phi)^{2.5}\left(1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right)\right)}f'''-f'^2+ff''+2\lambda g-\frac{\sigma_{nf}/\sigma_f}{\left(1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right)\right)}Mf'=0 \tag{13}$$

$$\frac{1}{(1-\phi)^{2.5}\left(1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right)\right)}g''+fg'-f'g-2\lambda f'-\frac{\sigma_{nf}/\sigma_f}{\left(1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right)\right)}Mg=0 \tag{14}$$

$$\frac{1}{\left(1-\phi+\phi\left(\frac{\left(\rho c_p\right)_s}{\left(\rho c_p\right)_f}\right)\right)}\frac{1}{\Pr}\left(\left(k_{nf}/k_f+Rd\left(1+(\theta_w-1)\theta\right)^3\right)\theta''+\left(\frac{Q}{k_f}\right)\theta\right)+f\theta'=0 \tag{15}$$

$$\phi''+Sc\gamma\phi+Sc\phi'=0 \tag{16}$$

$$f(0)=0 \qquad g(0)=0 \qquad f'(0)=1 \qquad \theta(0)=1$$
$$f'(\infty)=0 \qquad g(\infty)=0 \qquad \theta(\infty)=0 \tag{17}$$

Where, $\Pr=\frac{\left(\mu c_p\right)_f}{k_f}$ is the Prandtl number of the base fluid, $Rd=\frac{16\sigma^* T_\infty^{\ 3}}{3k^* k_f}$ denotes the Radiation parameter, $M=\frac{\sigma B_0^2}{\rho_f\Omega}$ is the magnetic field parameter and $\lambda=\frac{\Omega}{a}$ is the ratio of rotation rate to the stretching rate. $Sc = \frac{\nu_f}{D}$ is Schmidt Number and $\gamma = \frac{k}{a}$ is chemical reaction parameter.

3. NUMERICAL ANALYSIS

In this study the set of Non-dimensional Non-linear couple boundary layer equations with boundary conditions does not possess a closed form analytical solution. The governing partial differential equations can be converted to closed form equations by using Shooting method then it has been solved numerically by Runge-Kutta sixth order integration technique. The entire numerical analysis is done by using Mathematica computer language. From the process of numerical computation the fluid velocity, the temperature, the concentration, the Skin friction coefficient, the Nusselt number and Sherwood number are proportional to $f'(\eta), \theta(\eta), \phi(\eta), f''(\eta), \theta'(\eta), \phi'(\eta)$.

4. RESULTS AND DISCUSSION

The effect of Magnetic field parameter (M) on the velocity, temperature and concentration profiles are shown in fig.4.1, 4.2, 4.3 .It is observed from these figures the velocity, concentration field decreases while the temperature field increase with the increase of Magnetic parameter (M).The effects of various parameters on the components of Skin friction (C_f), the Nusselt number (Nu) and the Sherwood number (Sh) are shown in fig.4.4, 4.5, 4.6. It is observed from these figures the Skin friction (C_f), Nusselt number (Nu) increases while the Sherwood number (Sh) decrease with the increase of Magnetic parameter (M).

The effect of Ratio rate (λ) on the velocity, temperature and concentration profiles are shown in fig.4.7, 4.8, 4.9 .It is observed from these figures the velocity, concentration field decreases while the temperature field increase with the increase of ratio rate (λ).The effects of various parameters on the components of Skin friction (C_f), the Nusselt number (Nu) and the Sherwood number (Sh) are shown in fig.4.10, 4.11, 4.12. It is observed from these figures the Skin friction coefficient (C_f), Nusselt number (Nu) increases while the Sherwood number (Sh) decrease with the increase of ratio rate (λ).

The effect of Radiation parameter (Rd) on the velocity, temperature and concentration profiles are shown in fig.4.13, 4.14, 4.15.It is observed from these figures the velocity, concentration field decreases while the temperature field increase with the increase of Radiation parameter (Rd).The effects of various parameters on the components of Skin friction coefficient (C_f), the Nusselt number (Nu) and the

Sherwood number (Sh) are shown in fig.4.16, 4.17, 4.18. It is observed from these figures the Skin-Friction coefficient (C_f), Nusselt number (Nu) increases while the Sherwood number (Sh) decrease with the increase of radiation parameter (Rd).

The effect of Magnetic field parameter (M):

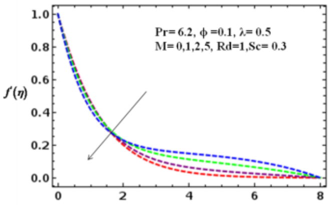

Fig.4.1.Effect of Magnetic field parameter (M) on velocity profile f′(η)

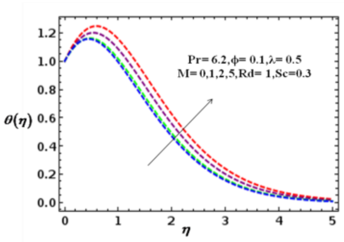

Fig.4.2.Effect of Magnetic field parameter(M) on temperature profile θ(η)

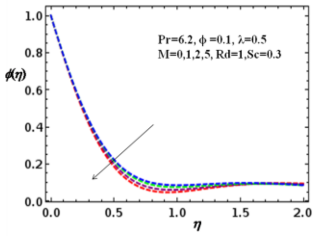

Fig.4.3.Effect of Magnetic field parameter (M) on concentration profile ϕ(η)

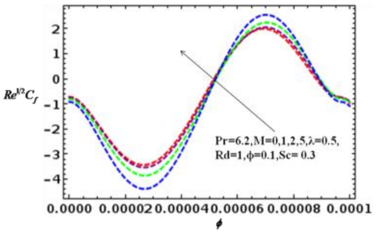

Fig.4.4.Effect of Magnetic field parameter (M) on Skin friction coefficient $Re^{1/2}C_f$

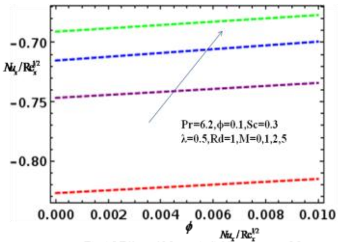

Fig.4.5.Effect of Magnetic field parameter (M) on Nusselt number

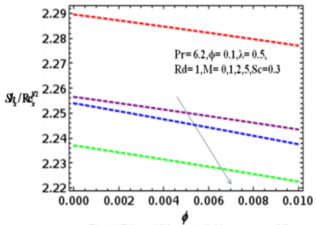

Fig.4.6.Effect of Magnetic field parameter (M) on Sherwood number $Sh_x/Re_x^{1/2}$

The effect of Ratio rate (λ):

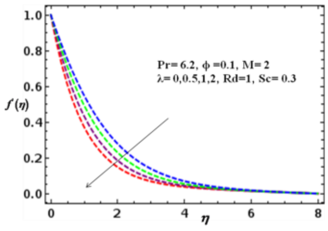

Fig.4.7.Effect of ratio (λ) on velocity profile f′(η)

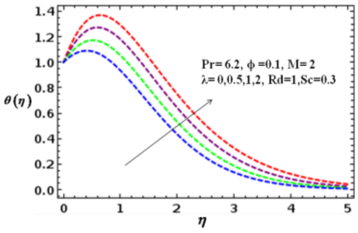

Fig.4.8.Effect of ratio (λ) on temperature profile θ(η)

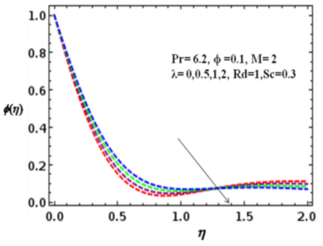

Fig.4.9.Effect of ratio (λ) on concentration profile ϕ(η)

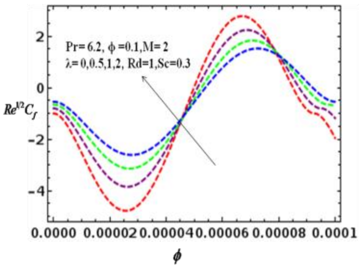

Fig.4.10.Effect of ratio (λ) on Skin friction coefficient $Re^{1/2}C_f$

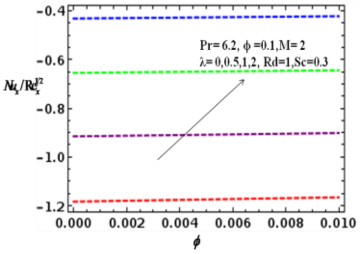

Fig.4.11.Effect of ratio (λ) on Nusselt number $Nu_x/Re_x^{1/2}$

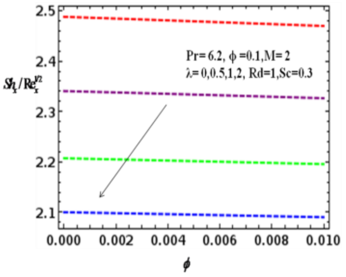

Fig.4.12.Effect of ratio (λ) on Sherwood number $Sh_x/Re_x^{1/2}$

The effect of Radiation parameter (Rd):

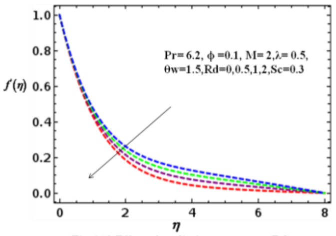

Fig.4.13.Effect of radiation parameter(Rd) on velocity profile f'(η)

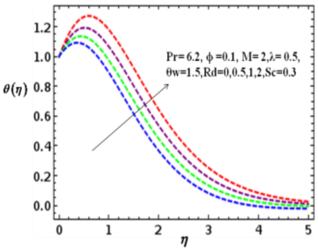

Fig.4.14.Effect of radiation parameter(Rd) on temperature profile θ(η)

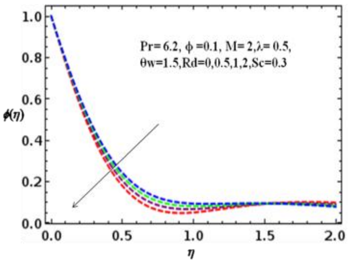

Fig.4.15.Effect of radiation parameter(Rd) on concentration profile ϕ(η)

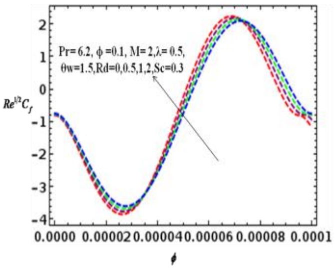

Fig.4.16.Effect of radiation parameter(Rd) on Skin friction coefficient $Re^{1/2}C_f$

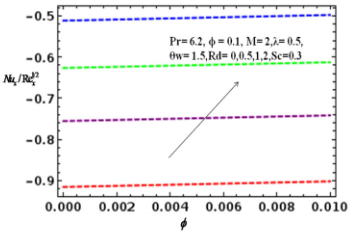

Fig.4.17.Effect of radiation parameter(Rd) on Nusselt number $Nu_x/Re_x^{1/2}$

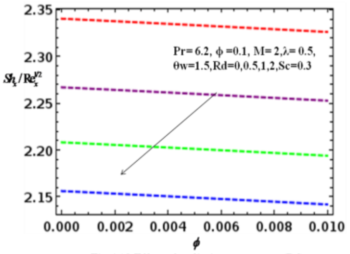

Fig.4.18.Effect of radiation parameter(Rd) on Sherwood number $Sh_x/Re_x^{1/2}$

5. CONCLUSION

In this study, the Rotating flow of Magnetite-water Nanofluid over a Stretching Surface inspired by Non-linear thermal Radiation and Mass transfer .The governing equations and boundary conditions are reduced to ordinary differential equations using Shooting technique together with Runge-Kutta sixth order iteration scheme.

- The velocity, concentration field decreases while the temperature field increase with the increase of Magnetic parameter (M).

- The Skin friction (C_f), Nusselt number (Nu) increases while the Sherwood number (Sh) decrease with the increase of Magnetic parameter (M).
- The velocity, concentration field decreases while the temperature field increase with the increase of ratio rate (λ).
- The Skin friction coefficient (C_f), Nusselt number (Nu) increases while the Sherwood number (Sh) decrease with the increase of ratio rate (λ).
- The velocity, concentration field decreases while the temperature field increase with the increase of Radiation parameter (Rd).
- The Skin friction coefficient (C_f), Nusselt number (Nu) increases while the Sherwood number (Sh) decrease with the increase of radiation parameter (Rd).

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