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Some Results on Primes of the Form (K+1)(K+2)(K+3)+-1

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Abstract

The Prime number theorem[1] conjectured by Carl Gauss states that the number of primes below a fixed number is

$$\pi(x) \sim \int_2 \frac{1}{\log t}$$

asymptotic to x/log x or more specifically

where $\pi(x)$ is the prime counting function counting number of primes below a fixed number. The prime number theorem is a very wonderful theorem providing deep insights in understanding prime numbers. An application of Prime number theorem states that the probability of a random number being prime is around the order of 1/log n [2]. We use this idea and other ideas to develop the heuristic proof that there are infinitely many primes of the form (k-1)(k)(k+1)+-1 (A293861)[3] We also provide some facts that give us some evidence for infinitude of our primes and show some path to find the number of such primes below a fixed number. Also, We find the number of our primes between any x and 2x (heuristic proof)

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1. INTRODUCTION

At the beginning, we should start looking at the table of number of our primes below fixed powers of 10.(The serial numbers represents 10^k)

10^k	Number of our primes
1	2
2	5
3	10
4	21
5	39
6	66
7	118
8	213
9	419
10	770

It is quite evident to see and note that the number of our primes below fixed powers of 10 are quite plentiful and not only that, the rate of increase of number of our primes below a number does not decrease rapidly, this suggests that there may be infinitely many primes of our form and that the list never becomes stagnant.

Anyways, this is a evidence based idea not a proof, so let’s look ahead!

Now, Let us look at the graph of A293861(n) versus n

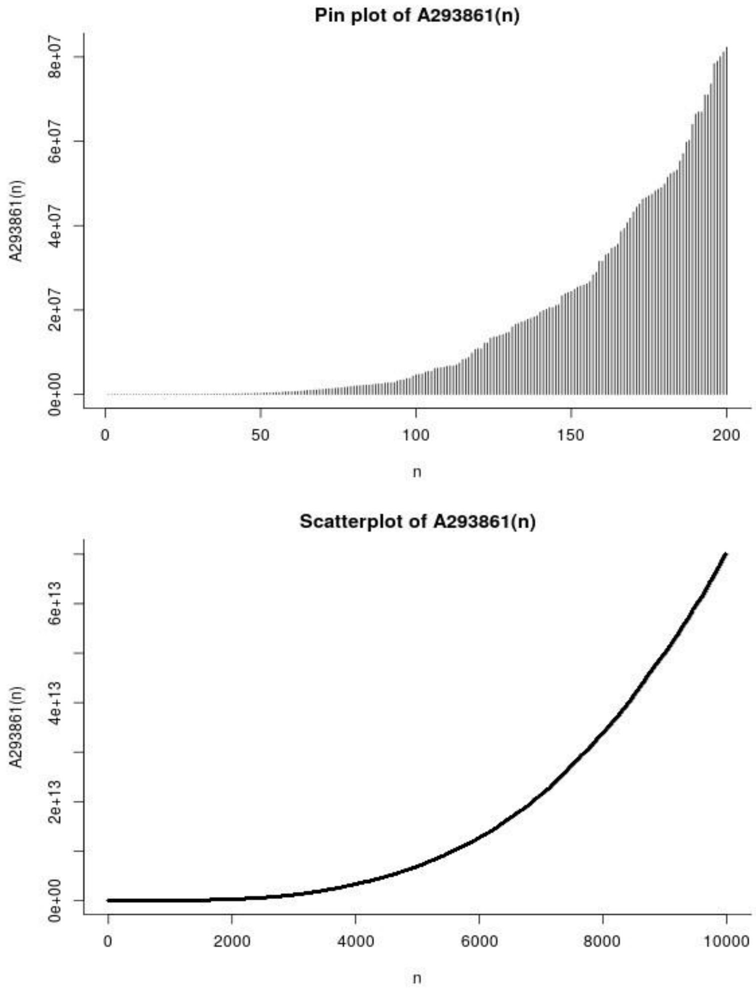

Figure 1. Pin plot and Scatter plot of A293861

This is just a reference for future readers to get some ideas over this.

2. HEURISTIC PROOF

By Prime Number Theorem , It is known that the probability of a ”random number” less than n being a prime is around the order of 1/log n (asymptotic to) . [1]

Hence, the probability that (k-1)(k)(k+1) +-1 is a prime is of the size of $\frac{1}{\log((k-1)(k)(k+1))}$

Is easy to see that this value, written above is greater than $\frac{1}{\log(k^3)}$

Now, the number of our primes below a number can be found to be

$$\sum_{k=2}^{n} \frac{1}{\log((k-1)(k)(k+1))}$$

Now, since to know if there are infinite primes of our form, we need to take the sum from 2 to ∞

Now, since $\sum \frac{1}{log(k^3)} \to \infty$ (Because, $\sum \frac{1}{log(k)} \to \infty$) and that being said that $\sum_{k=2}^{\infty} \frac{1}{\log((k-1)(k)(k+1))}$ is greater than $\sum \frac{1}{\log(k^3)}$

Hence, $\sum_{k=2}^{\infty} \frac{1}{\log((k-1)(k)(k+1))} \to \infty$

This, Heuristically shows that the number of our primes below a number is not bounded and hence is infinite in number.

3. NUMBER OF OUR PRIMES BELOW A FIXED NUMBER

Now, by the previous section, it is evident that the number of our such primes below a number is asymptotically $\sum_{k=2}^{n} \frac{1}{\log((k-1)(k)(k+1))}$

We know, that the terms are not independent thus we require some constant (say a2) that substitutes the work for making the estimated conjecture true !

So, we end the paper stating a2 as some ”dependence constant” which by some means links (probably) Merten’s theorem[4] or some other theorem to estimate more accurately :

$$\mu(n) \sim \sum_{k=2}^{n} \frac{a2}{\log((k-1)(k)(k+1))}$$

This can be expressed as integral as follows:

$$\mu(n) \sim \int_{2}^{n} \frac{a2}{\log((k-1)(k)(k+1))}$$

4. NUMBER OF OUR PRIMES BETWEEN N AND 2 N

Since, we obtain number of our primes below x and similarly for 2 x, we just subtract the number of our primes below x from the number of our primes below 2x. This is simply $\mu(2n)$ - $\mu(n) \sim \sum_{k=2}^{2n} \frac{a2}{\log((k-1)(k)(k+1))} -$

$$\sum_{k=2}^{n} \frac{a2}{\log((k-1)(k)(k+1))} \text{ or } \int_{2}^{2n} \frac{a2}{\log((k-1)(k)(k+1))} - \int_{2}^{n} \frac{a2}{\log((k-1)(k)(k+1))}$$

REFERENCES

[1] C. F. Gauss. Werke, Bd 2, 1st ed, 444447. Gttingen 1863.

[2] AN AMAZING PRIME HEURISTIC, https://www.utm.edu/staff/caldwell/preprints/Heuristics.pdf

[3] A293861, OEIS , http://oeis.org/A293861

[4] F. Mertens. J. reine angew. Math. 78 (1874), 4662