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Some Dual Series Equations Involving Laguerre Polynomial

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Abstract

In the present paper a solution of dual series equations involving Laguerre polynomials have been obtained by using multiplying factor technique used by Noble and Lowndes.

1. INTRODUCTION

The problem considered in this paper is that of determining the sequence $\{A_{ni}\}$ such that

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} a_{ij}\frac{A_{nj}}{\Gamma(\alpha+n_i+p+1)}L_{ni+p}^{\alpha}(x)= f_i(x),\ 0\le x<d \tag{1.1}$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij}\frac{A_{nj}}{\Gamma(\alpha+\beta+ni+p)}L_{ni+p}^{\alpha}(x)= g_i(x),\ d<x<\infty \tag{1.2}$$

where $0<\beta+m,\ 0<\alpha+\beta<\alpha+1,\ p$ and m are non negative intgers and $j=1,2,3,......,s$.

$$L_{n+p}^{\alpha}(x)=\binom{\alpha+n+p}{n+p}\,{}_1F_1[-n-p;\alpha+1;x] \tag{1.3}$$

is the Laguerre polynomial, $f_i(x)$ and $g_i(x)$ are prescribed functions. $n=0,1,2,......;$ $j=1,2,......,s;$ and a_{ij},b_{ij} are known constants.

The solution presented in this paper is obtained by employing a multiplying factor technique similar to that used by Noble[3] or Lowndes [5]. Eqs (1.1)and (1.2) can also be solved by a technique used by Sneddon and Srivastava [ ] in solving dual series equations involving Bessel's functions.

2. PRILIMINARY RESULTS:

Some of the results which will be required in the course of the analysis are given below.

From Erdelyi [2] (p. 293 (5), p.(405(20)) it can be deduced that

$$\int_0^y x^{\alpha}(y-x)^{\beta+m-1} L_{n+p}^{(\alpha)}(x)\,dx = \frac{\Gamma(\beta+m)\Gamma(\alpha+n+p+1)}{\Gamma(\alpha+\beta+m+n+p+2)} y^{\alpha+\beta+m} L_{n+p}^{\alpha+\beta+m}(y) \tag{2.1}$$

where $0 < y < d$, $-1 < \alpha$, $0 < \beta + m$ and

$$\int_y^{\infty} e^{-x}(x-y)^{-\beta} L_{n+p}^{(\alpha)}(x)\,dx = \Gamma(1-\beta).e^{-y}.L_{n+p}^{\alpha+\beta-1}(x) \tag{2.2}$$

where $d < y < \infty$, $\alpha + 1 > \alpha + \beta > 0$.

From Erdelyi [2] (p. 292) (3), p. 293 (3) it is easy to derive the following orthogonality relation for the Laguerre polynomial.

$$\int_0^{\infty} e^{-x} x^{\alpha} L_m^{\alpha}(x) L_n^{\alpha}(x)\,dx = \frac{\Gamma(\alpha+n+1)}{n!}\delta_{m,n} \tag{2.3}$$

where $\alpha > -1$ and $\delta_{m,n}$ is kronecker delta.

The differential formula

$$\frac{d^{m+1}}{dx^{m+1}}\left\{x^{\alpha+m+1} L_n^{\alpha+m+1}(x)\right\} = \frac{\Gamma(\alpha+m+n+2)}{\Gamma(\alpha+n+1)} x^{\alpha} L_n^{\alpha}(x) \tag{2.4}$$

Follows from Erdelyi [1] (p. 190(27)).

The analysis in the next section will be formal and no attempt to justify the various limiting process will be made.

3. SOLUTION OF THE PROBLEM:

Multiply equation (1.1) by $x^{\alpha}(y-x)^{\beta+m-1}$, integrat with respect to x over $(0, y)$ and then use (2.1) to find

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} a_{ij} \frac{A_{nj}}{\Gamma(\alpha+\beta+m+n_i+p+2)} y^{\alpha+\beta+m} L_{ni+p}^{\alpha+\beta+m}(y)$$

$$= \frac{1}{\Gamma(\beta+m)} \int_0^y x^{\alpha} (y-x)^{\beta+m-1} f_i(x)\,dx \tag{3.1}$$

where $0 < y < d,\ -1 < \alpha,\ 0 < \beta + m$ and m is a non-negative integer.

Differentiate (3.1) (m+1) times with respect to y and use (2.4) to find

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij} \frac{A_{nj}}{\Gamma(\alpha+\beta+n_i+p)} L_{ni+p}^{\alpha+\beta+1}(y)$$

$$= \frac{y^{1-n-\beta}}{\Gamma(\beta+m)} \frac{d^{m+1}}{dy^{m+1}} \int_0^y x^{\alpha} (y-x)^{\beta+m-1} f_i(x)\,dx \tag{3.2}$$

where $0 < y < d, -1 < \alpha, 0 < \beta + m$ and m is a non-negative integer. Again multiply (1.2) by $e^{-x}(x-y)^{-\beta}$, integrate with respect to x over (y,∞) and then use (2.2) to find

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij} \frac{A_{nj}}{\Gamma(\alpha+\beta+m+n_i+p)} L_{ni+p}^{\alpha+\beta-1}(y)$$

$$= c_{ij} \frac{e^{-y}}{\Gamma(1-\beta)} \int_y^{\infty} (x-y)^{-\beta} e^{-x} g_i(x)\,dx \tag{3.3}$$

where $d < y < \infty,\ \beta < 1$ and $0 < \alpha + \beta,\ c_{ij}$ are the elements of the matrix $[b_{ij}][a_{ij}]^{-1}, i = 1,2,3,\ldots,s$, The left hand sides of eqs. (3.2) and (3.3) are now identical and the following solution of eqs. (1.1) and (1.2) can therefore be obtained by virtue of orthogonality relation (2.3).

For $\alpha + 1 > \alpha + \beta > 0,\ \ \beta + m > 0$ any two non-negative integers m and p ,

$$A_{nj} = \sum_{j=1}^{s} d_{ij} \frac{(n+p)!}{\Gamma(\beta+m)} \left[\sum_{j=1}^{s} c_{ij} \int_0^d e^{-y} L_{ni+p}^{\alpha+\beta-1}(y) F_i(y)\,dy + \frac{(n_i+p)!}{\Gamma(1-\beta)} \int_d^{\infty} y^{\alpha+\beta-1} L_{ni+p}^{\alpha+\beta-1}(y) G_i(y)\,dy \right] \tag{3.4}$$

with

$$F_i(y) = \frac{d^{m+1}}{dy^{m+1}} \int_0^y x^{\alpha} (y-x)^{\beta+m-1} f_i(x)\,dx \tag{3.5}$$

and

$$G_i(y) = \int_y^{\infty} (x-y)^{-\beta} e^{-x} g_i(x)\, dx \tag{3.6}$$

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